More Non-Linear Rheology

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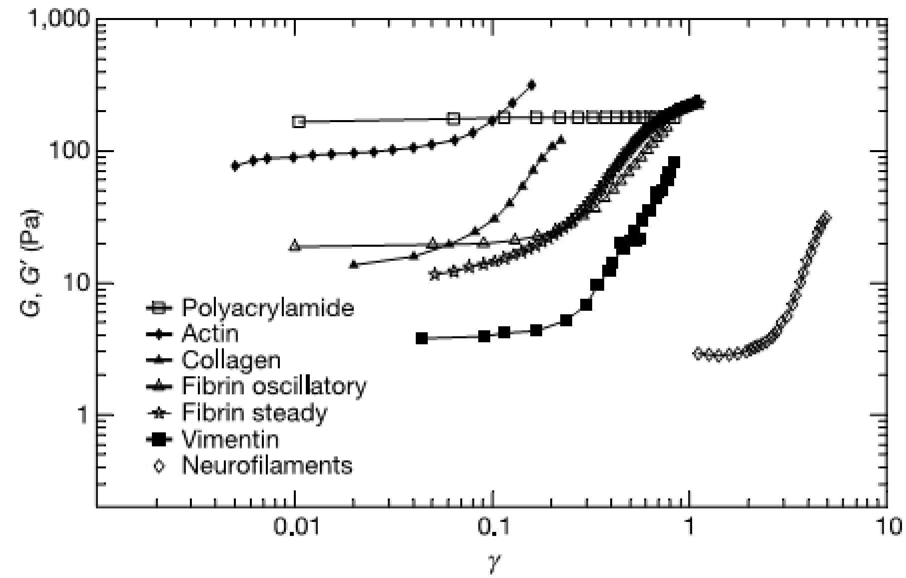
Weitzlab group meeting tutorial

Rheology of biopolymers

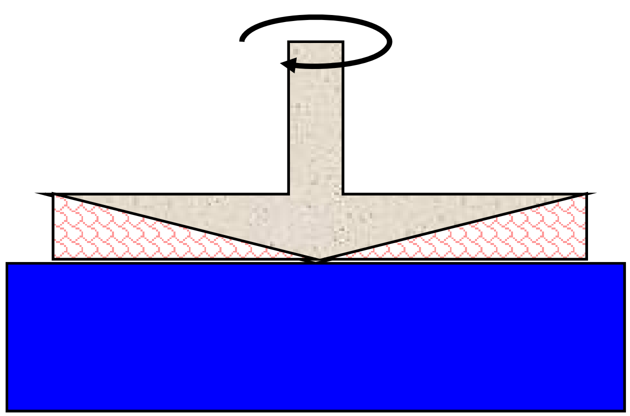
- *"Unlike simple polymer gels, many biological materials—including blood vessels, mesentery tissue, lung parenchyma, cornea and blood clots—stiffen as they are strained, thereby preventing large deformations that could threaten tissue integrity."* (Storm *et al.*, 2005)

- Tensile uniaxial and biaxial tests show similar behavior (Roeder et al., Brightman et al.) for collagen type I gels.



Oscillatory Shear in Bulk Rheology

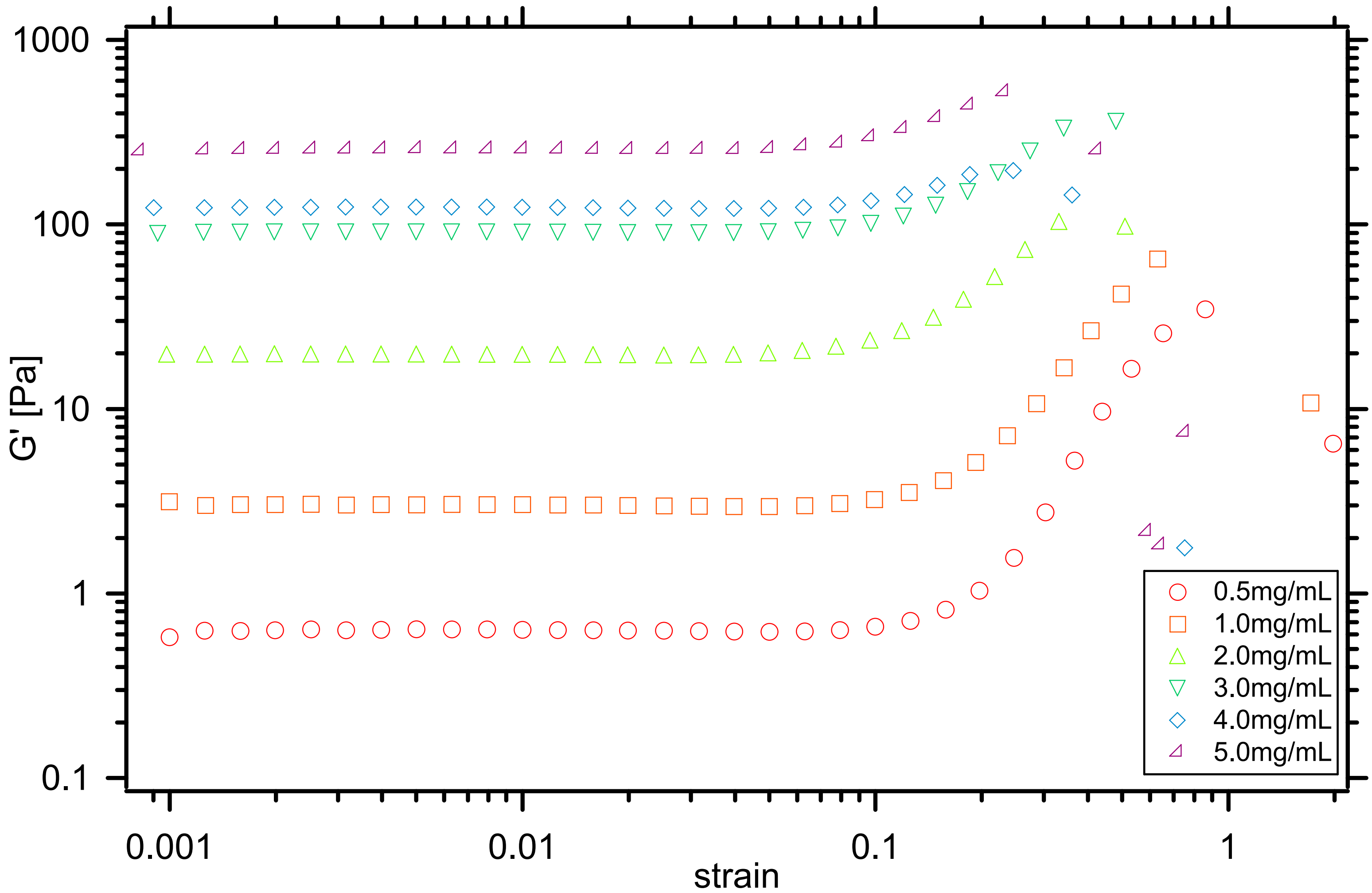
- Advantages
  - Low sample volume
  - Well-controlled geometry
  - Well-characterized methods and protocols



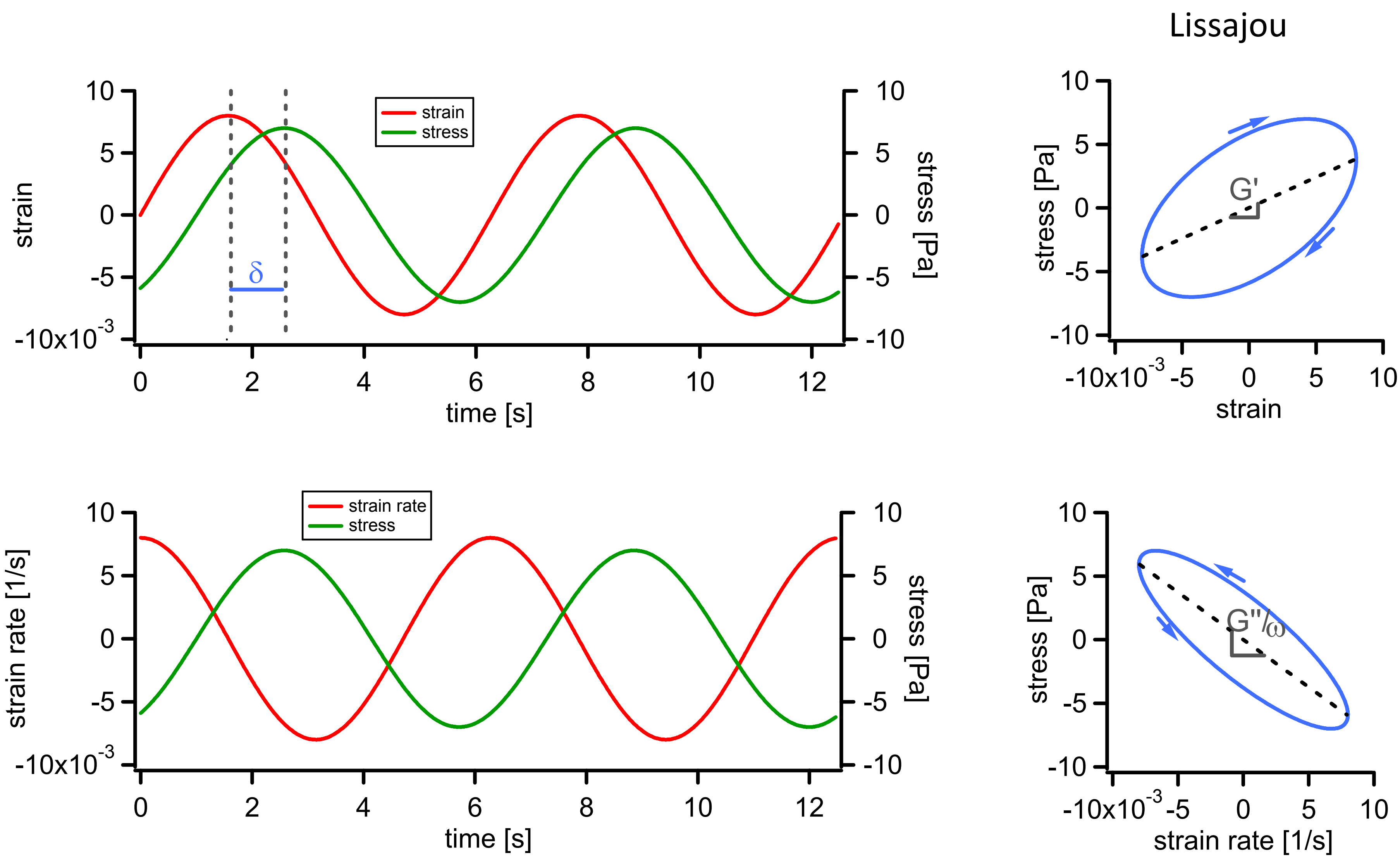
$$\gamma(t) = \gamma_0 \cdot \sin(\omega t)$$

$$\sigma(\omega, t) = G'(\omega)\,\gamma(\omega t) + G''(\omega)\,\frac{\dot{\gamma}(\omega t)}{\omega}$$

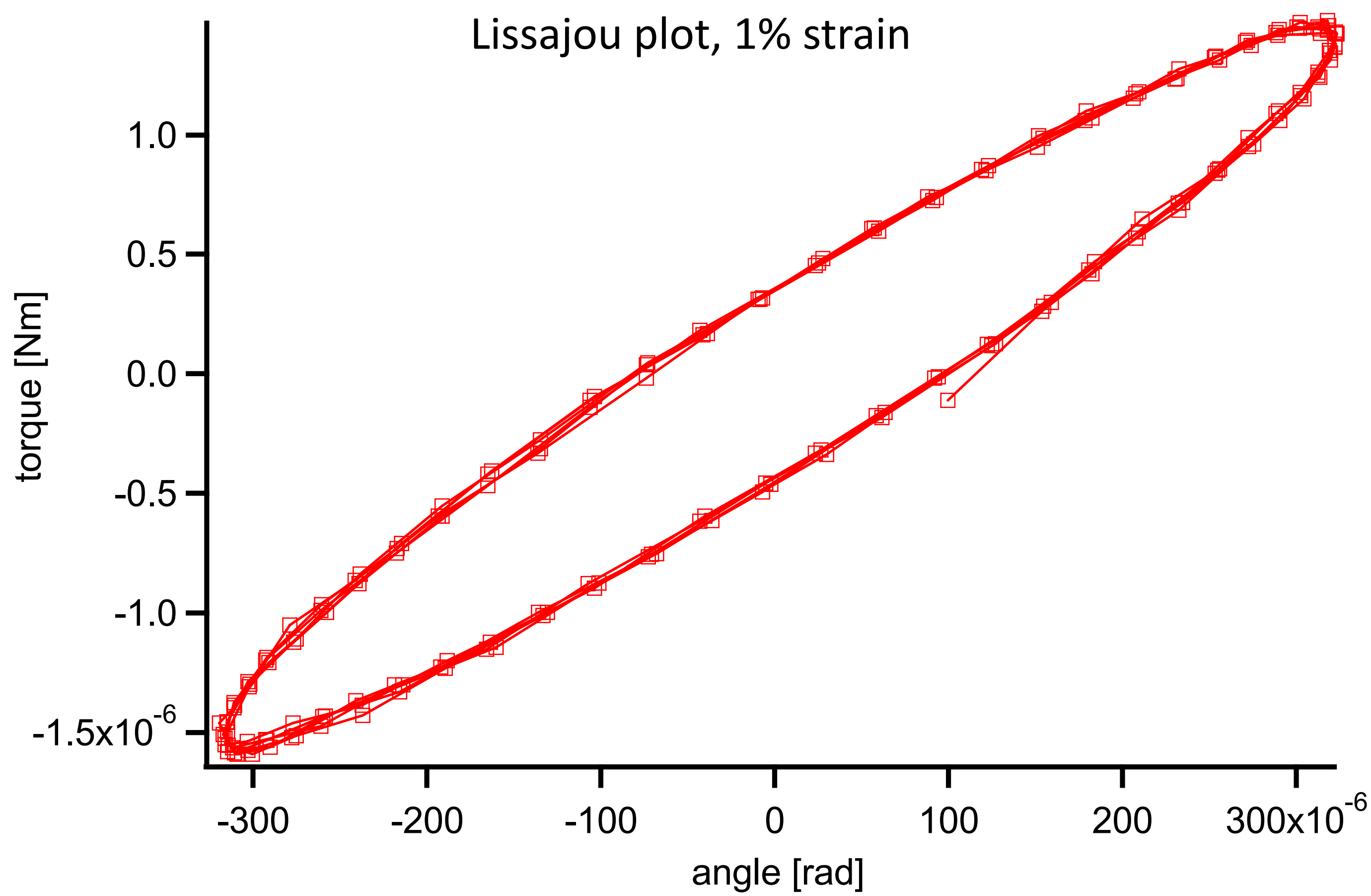
Oscillatory strain sweeps (collagen gels)



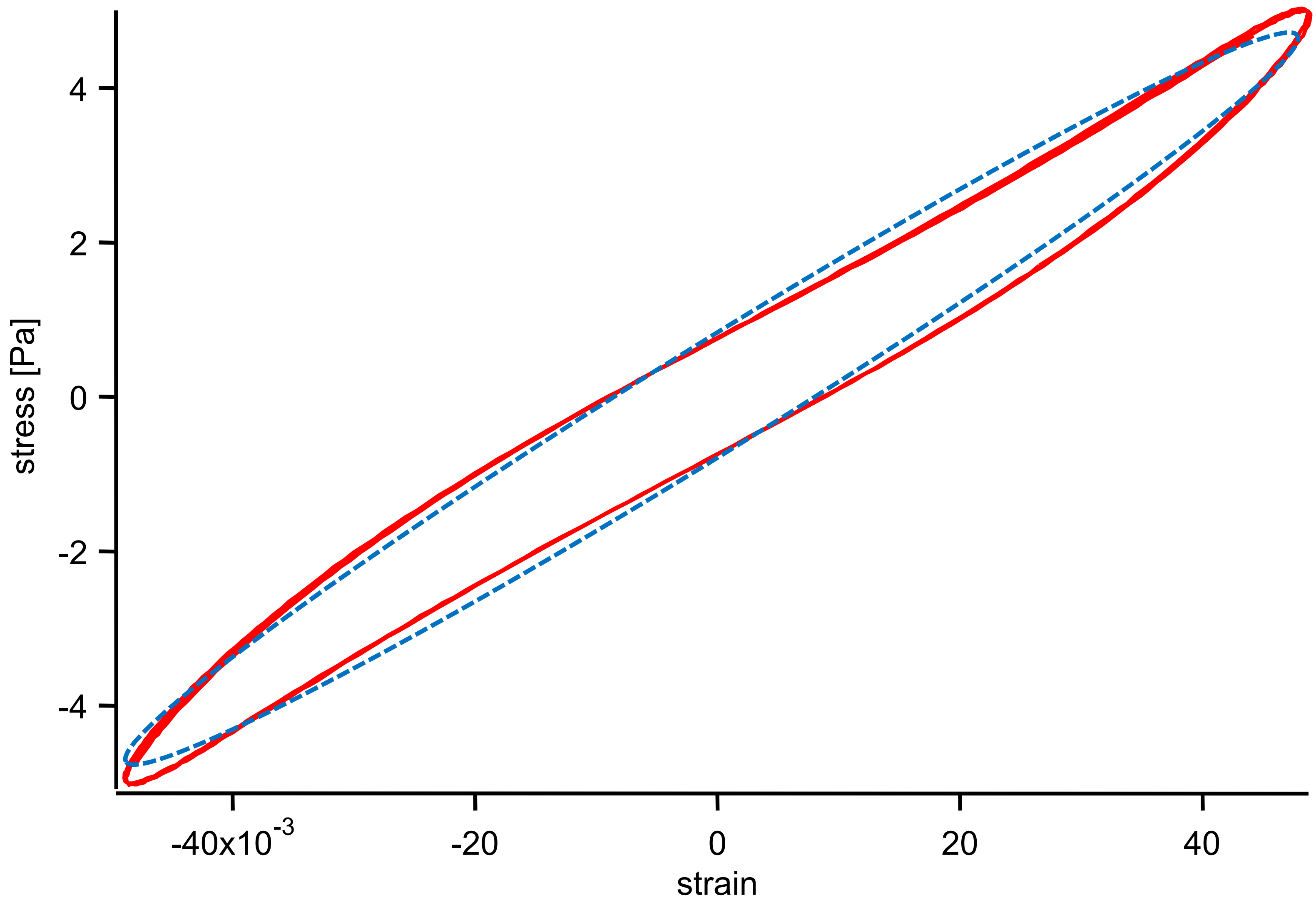
Lissajou plots



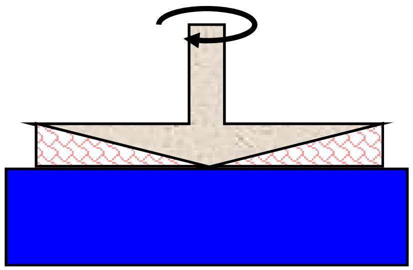
Raw data: Lissajou plots



Nonlinear Lissajou plot



Linear viscoelasticity



$$\gamma(t) = \gamma_0 \cdot \sin(\omega t)$$

$$\sigma(\omega,t) = G'(\omega)\,\gamma(\omega t) + G''(\omega)\frac{\dot{\gamma}(\omega t)}{\omega}$$

$$\sigma(x,y) = \sigma'(x,\gamma_0) + \sigma''(y,\gamma_0)$$

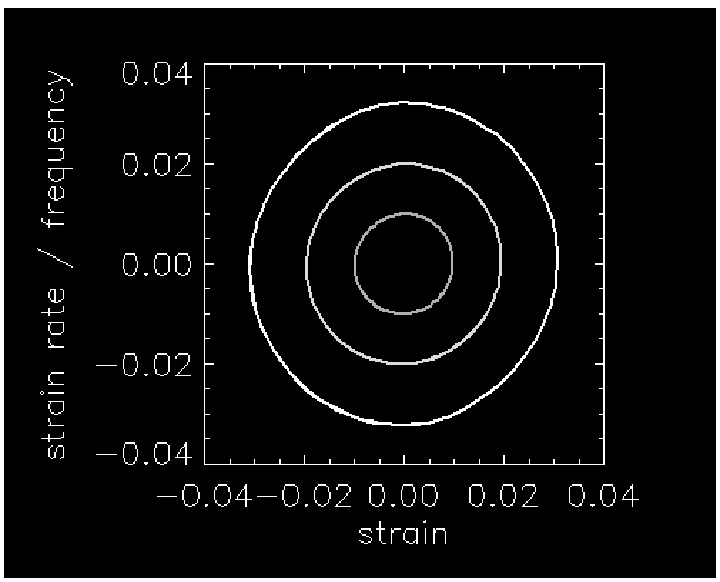
$$\sigma' = G'x(t), \quad \sigma'' = G''y(t)$$

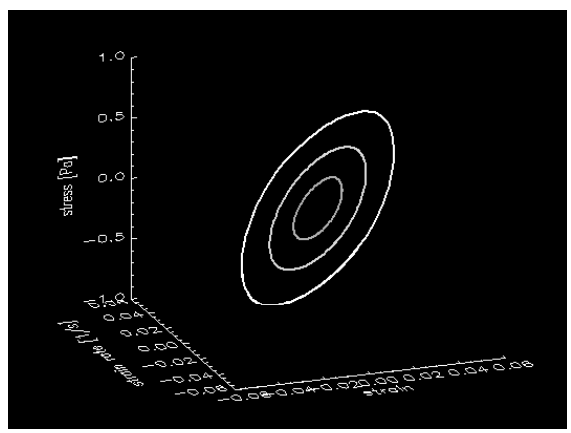
$$\oint \sigma' dx = 0, \quad \oint \sigma'' dy = 0$$

$$\oint \sigma dx = \oint \sigma'' dx, \quad \oint \sigma dy = \oint \sigma' dy$$

$$x = \gamma \text{ and } y = \dot{\gamma}/\omega$$

$$x^2(t) + y^2(t) = \gamma_0^2$$





Normal vector

$$\mathbf{n} = G'\mathbf{i} + G''\mathbf{j} - \mathbf{k}$$

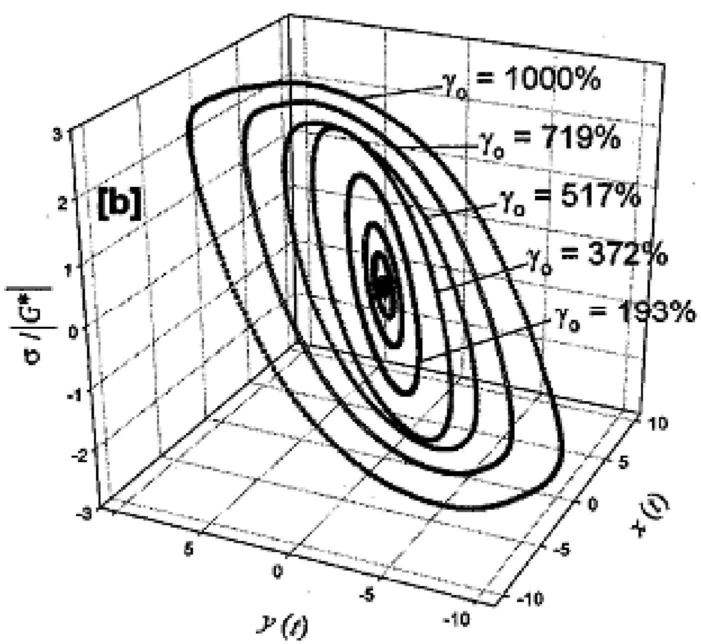
Generalized moduli

$$\sigma' = \Gamma'(x, \gamma_0)x, \quad \sigma'' = \Gamma''(y, \gamma_0)y$$

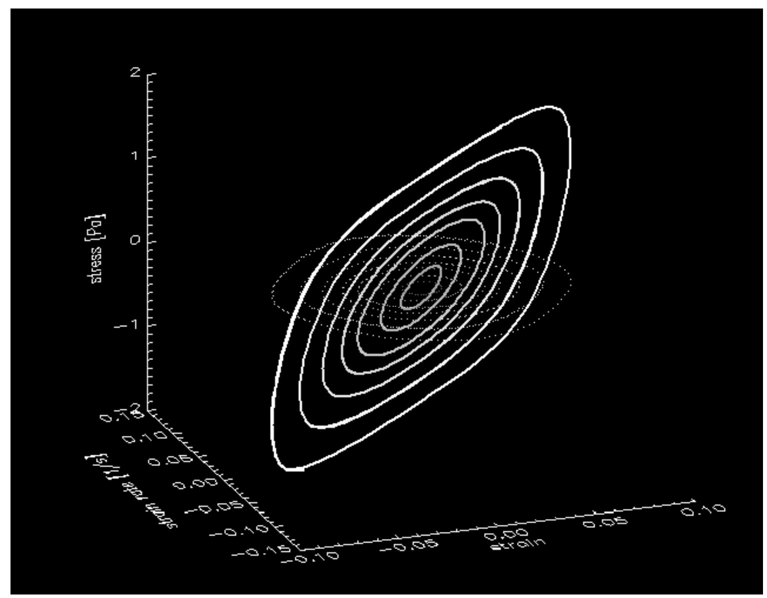
$$\lim_{\gamma_0 \to 0} \Gamma'(x, \gamma_0) = G'(\omega), \quad \lim_{\gamma_0 \to 0} \Gamma''(y, \gamma_0) = G''(\omega)$$

$$\sigma'(x, \gamma_0) = G'(\omega)x + G'_3(\omega, \gamma_0)x^3 + G'_5(\omega, \gamma_0)x^5 + \cdots$$

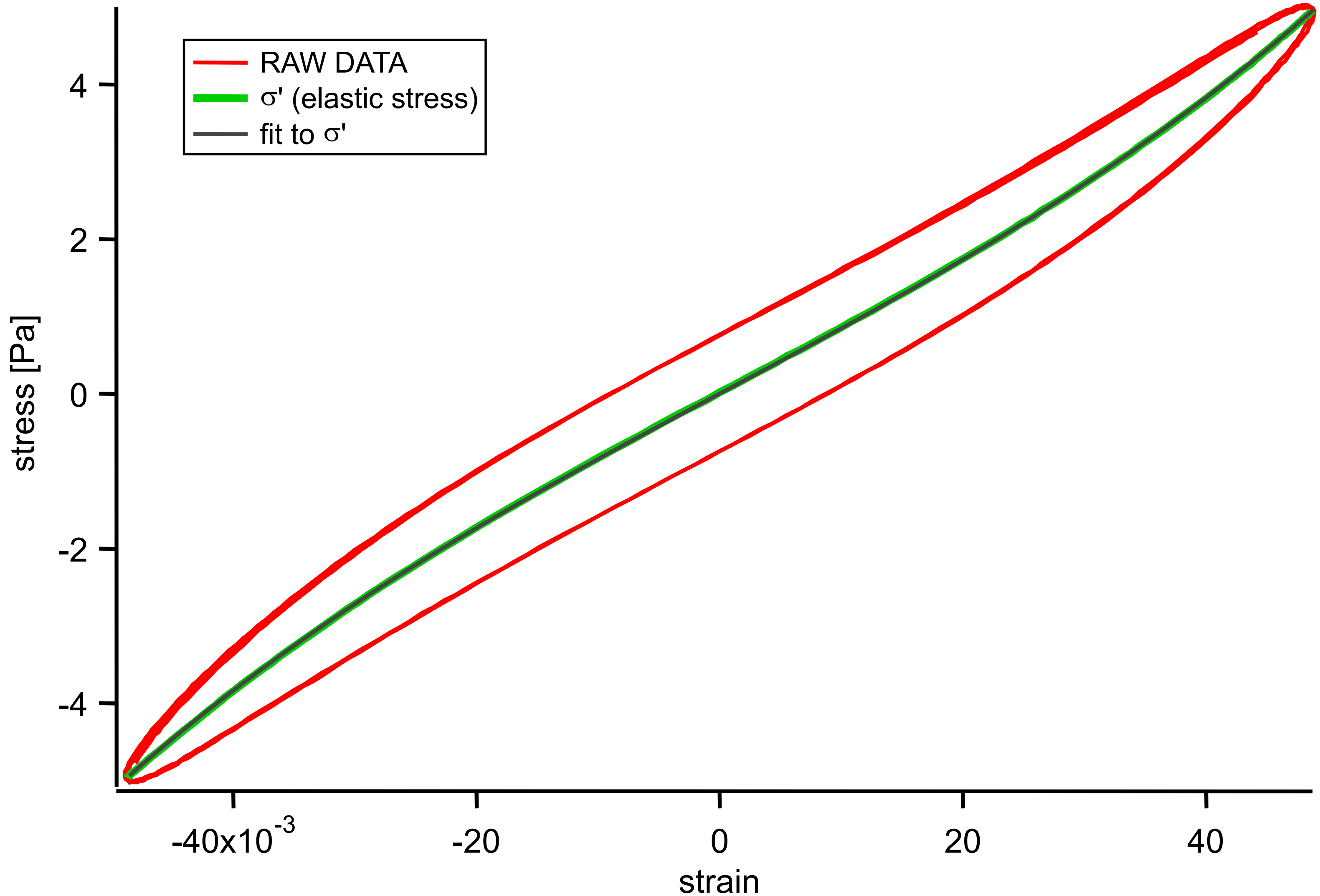
Cho data: shear thinning



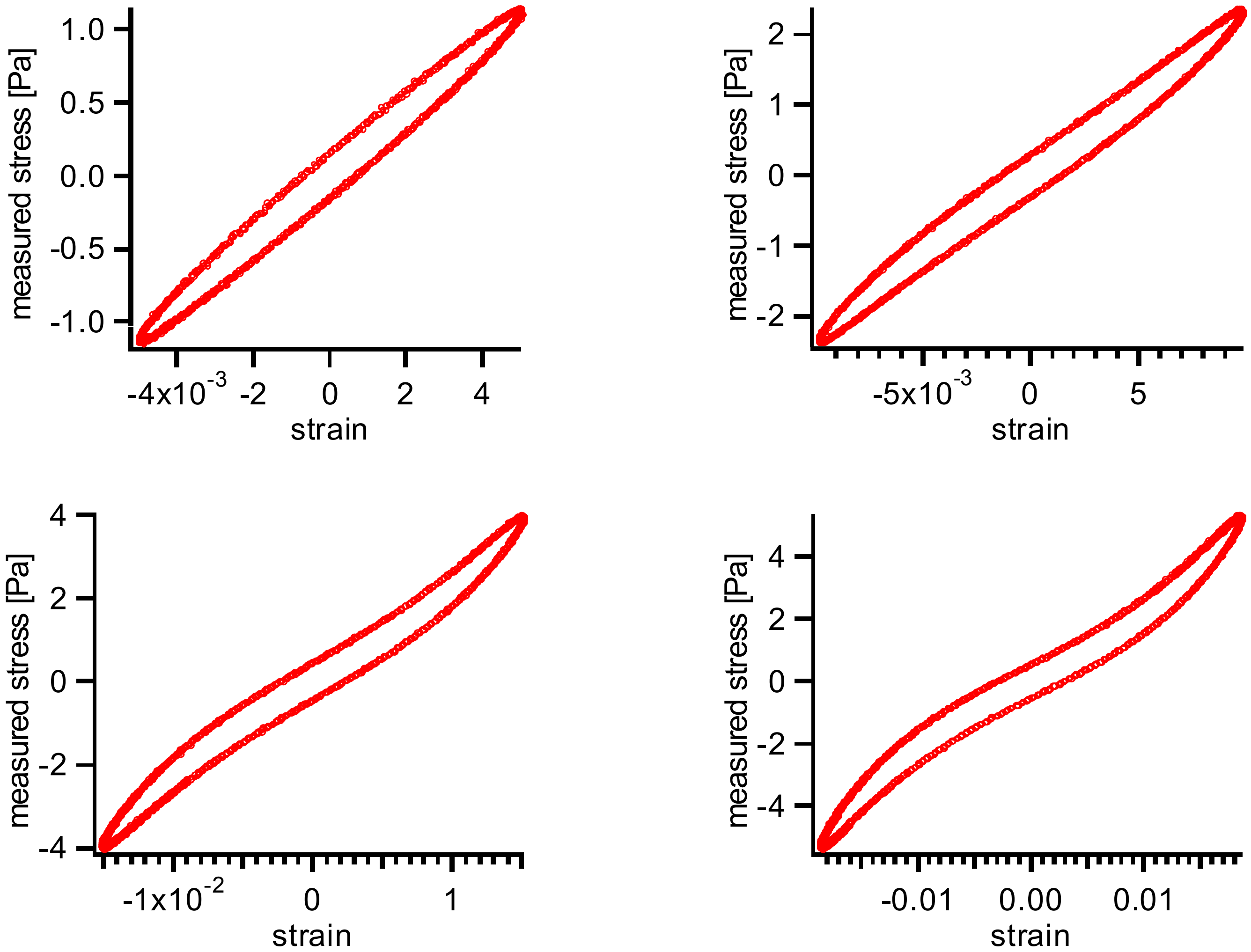
2mg/mL collagen: strain stiffening



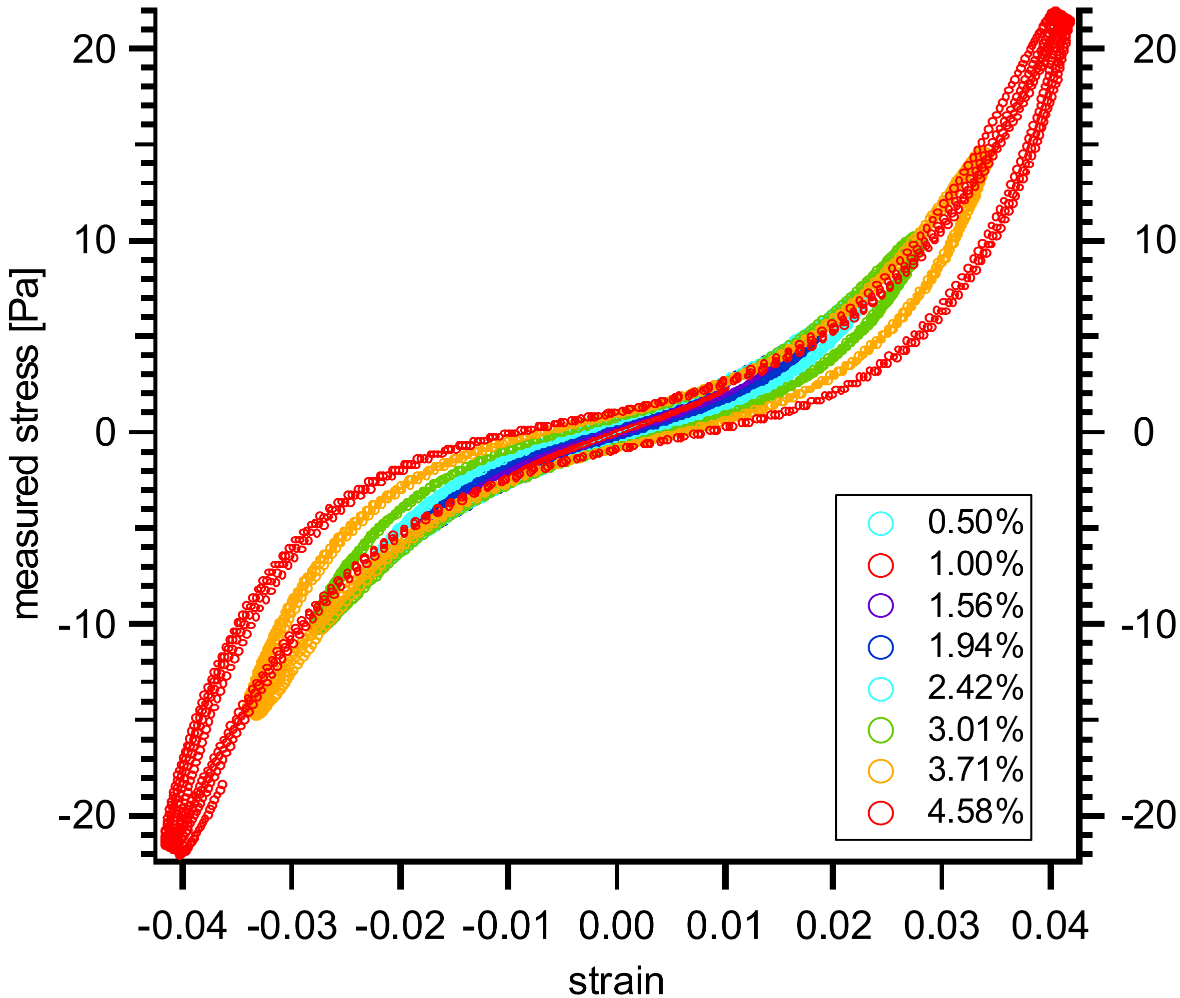
Nonlinear Lissajou plot



2.4mg/mL cone-plate

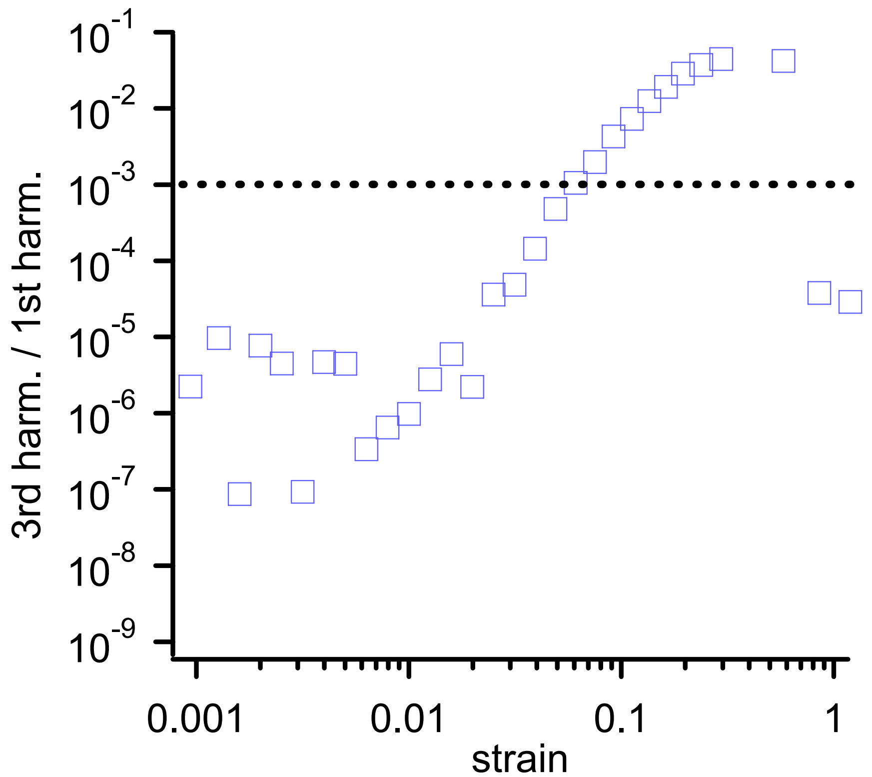


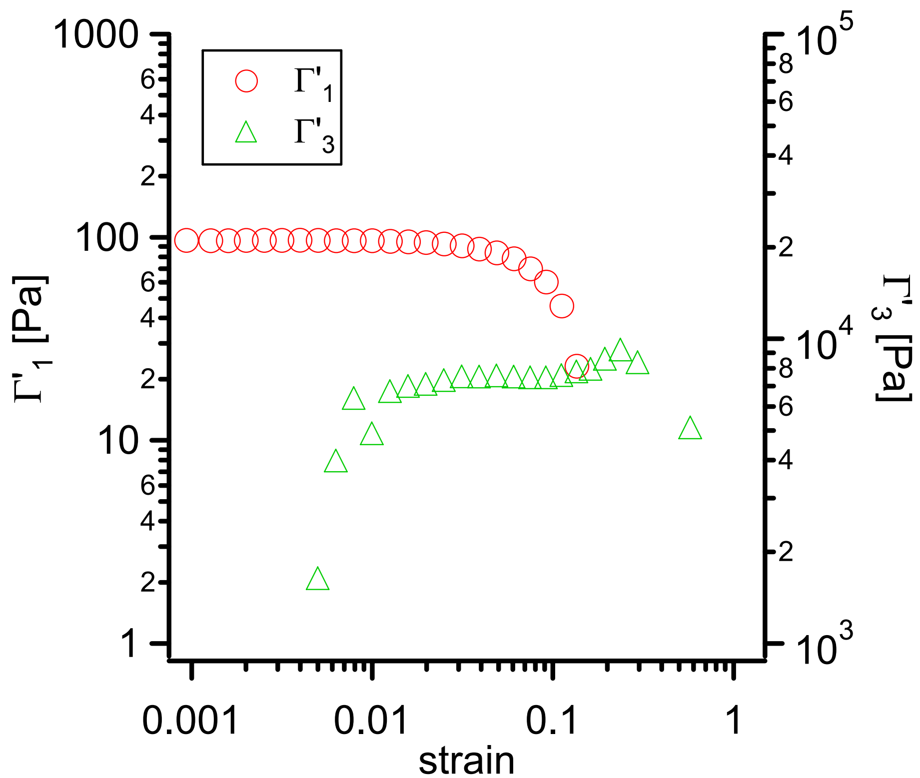
2.4mg/mL cone-plate



Self-consistency checks

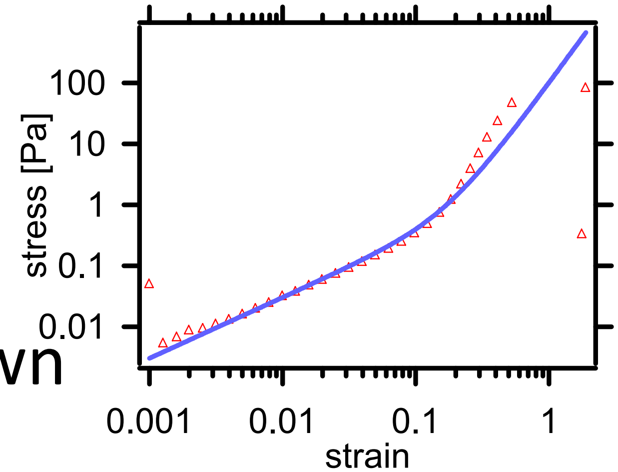
- Frequency spectrum of $\gamma(t)$ should be close to $\delta(f_0)$
- 3rd order polynomial fit done locally, i.e. for each γ:
    - Γ'_1, Γ'_3 consistent over a range of γ?
    - Can we do a global 3rd order polynomial fit of σ vs γ?

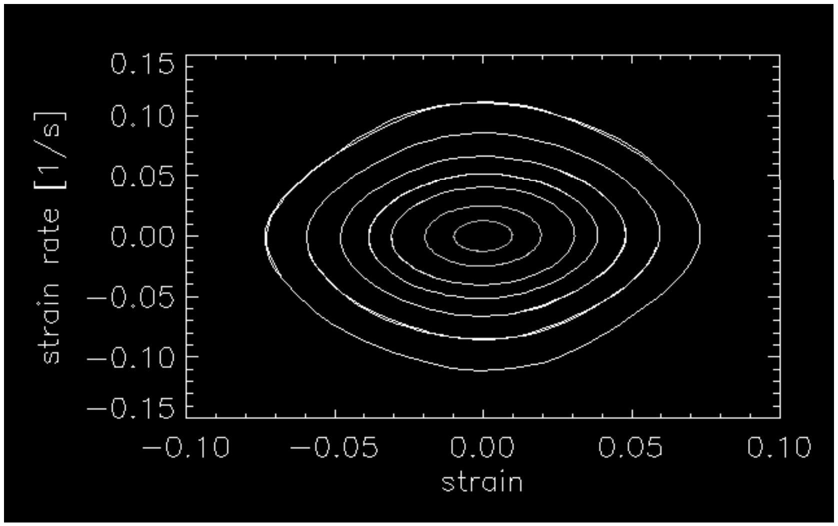


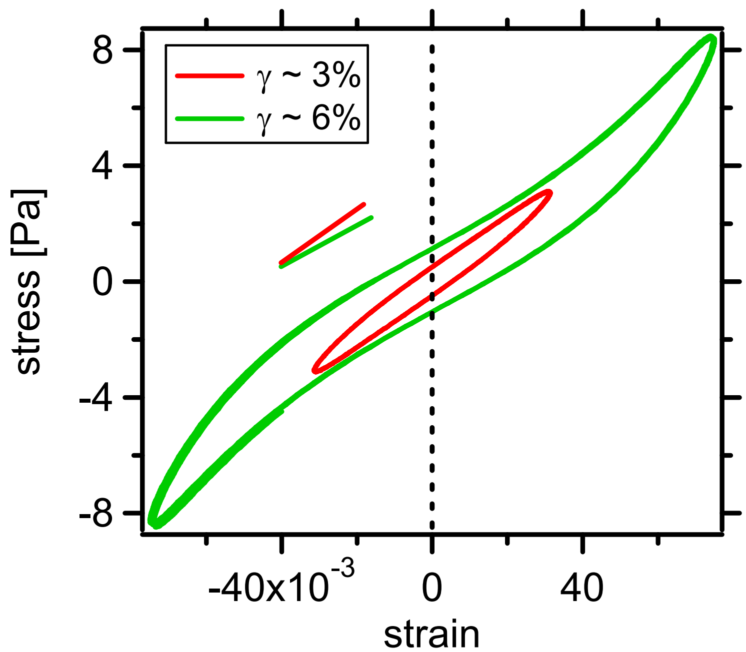


Breakdown of the model

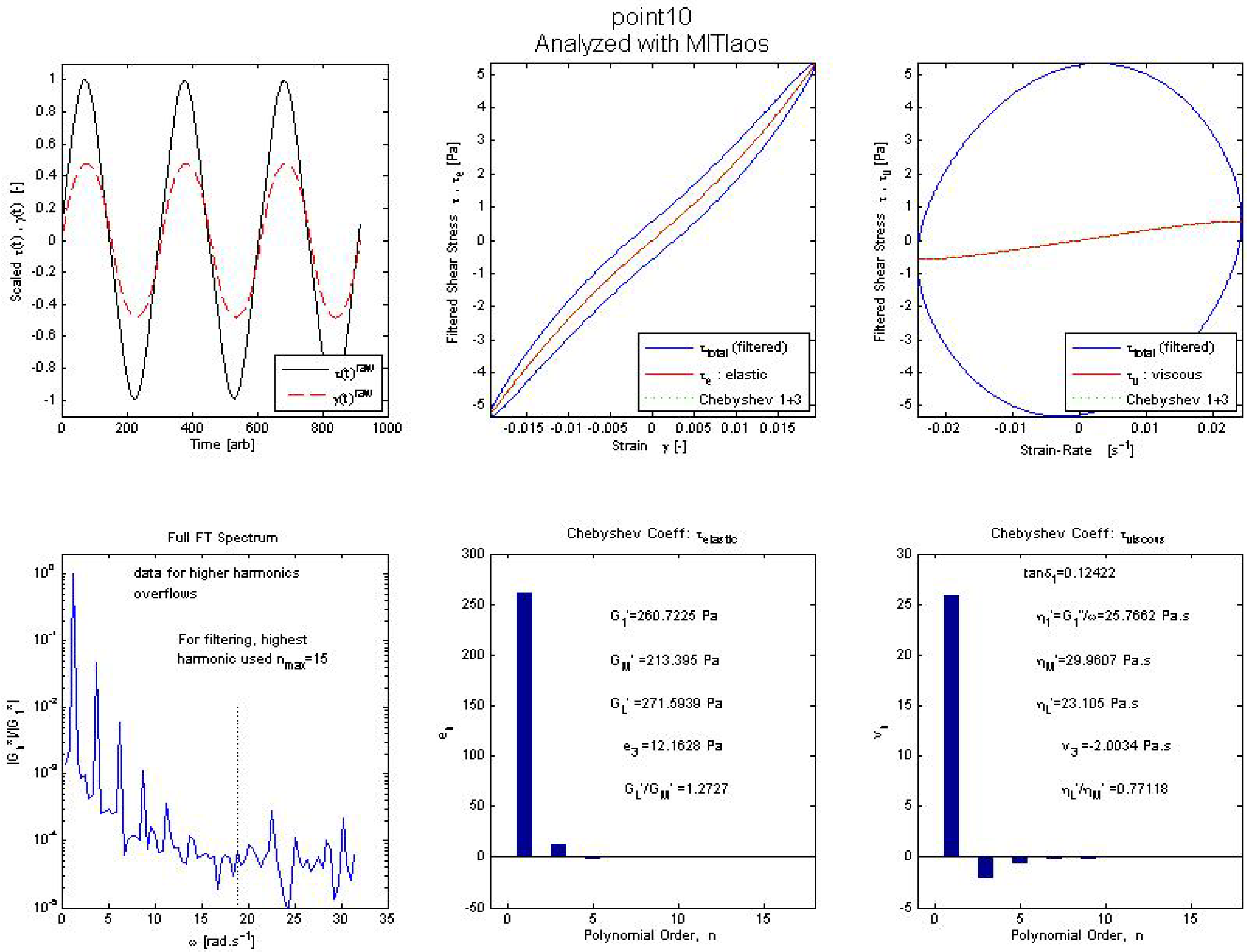
- Low concentrations
- High strains
  - Pseudo-strain control breaks down
  - Higher order terms needed
  - Shear thinning?







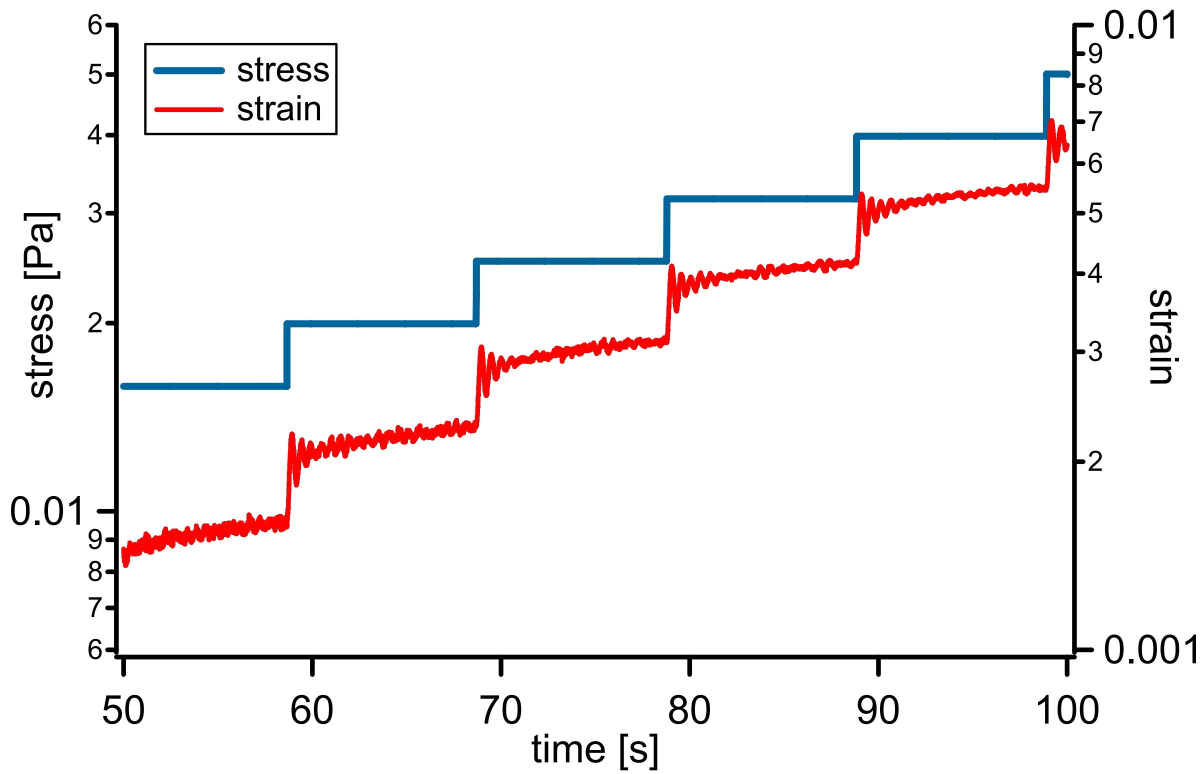
MIT LAOS MATLAB package



Creep-ringing

Creep-ringing

- Norman & Ryan's work here (fibrin, jamming)
- Good tutorial by Ewoldt & McKinley (MIT)



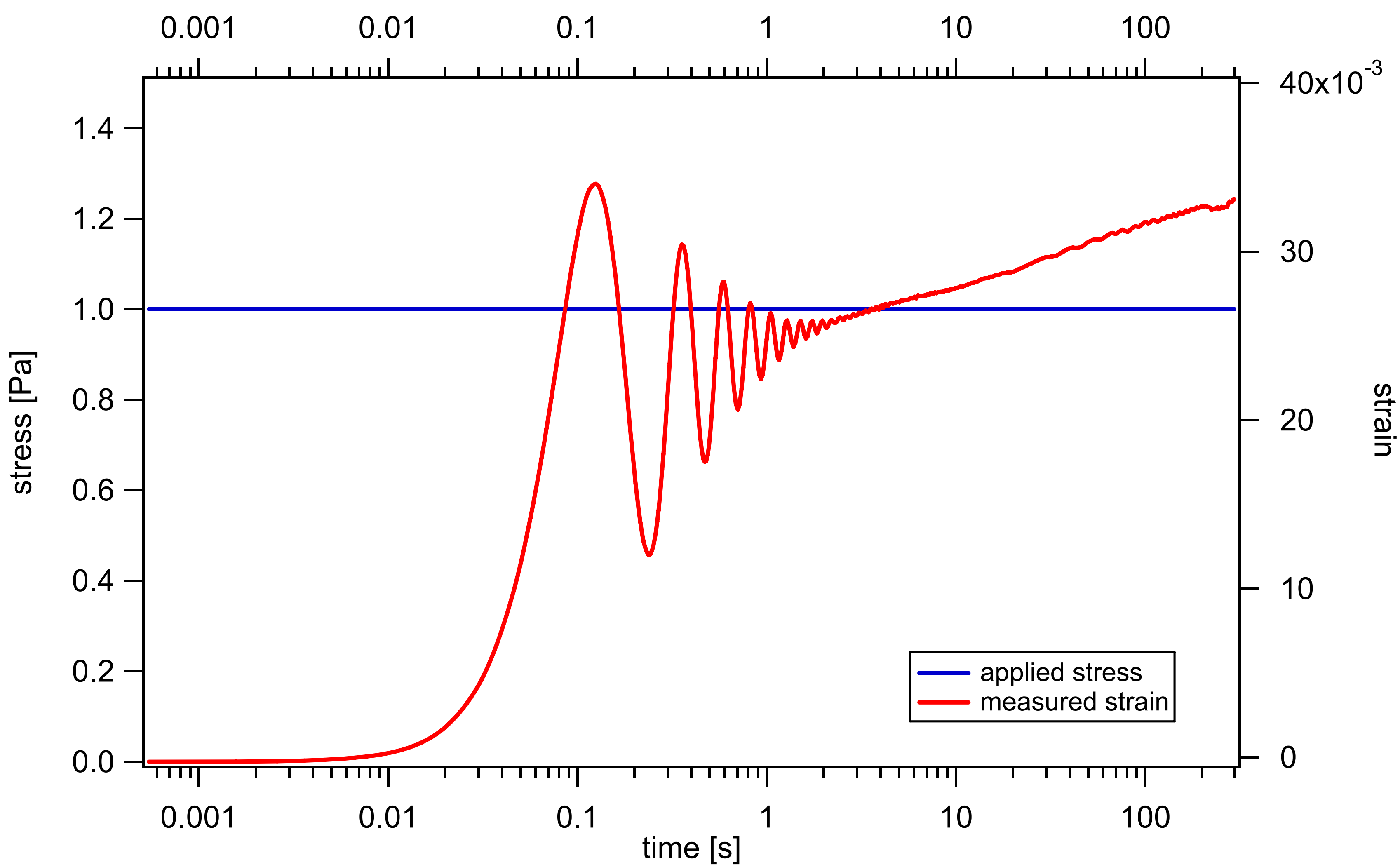
$$G' \approx \frac{I\omega^2}{b}\left(1 + (\Delta/2\pi)^2\right)$$

$$G'' \approx \frac{I\omega^2}{b}(\Delta/\pi)$$

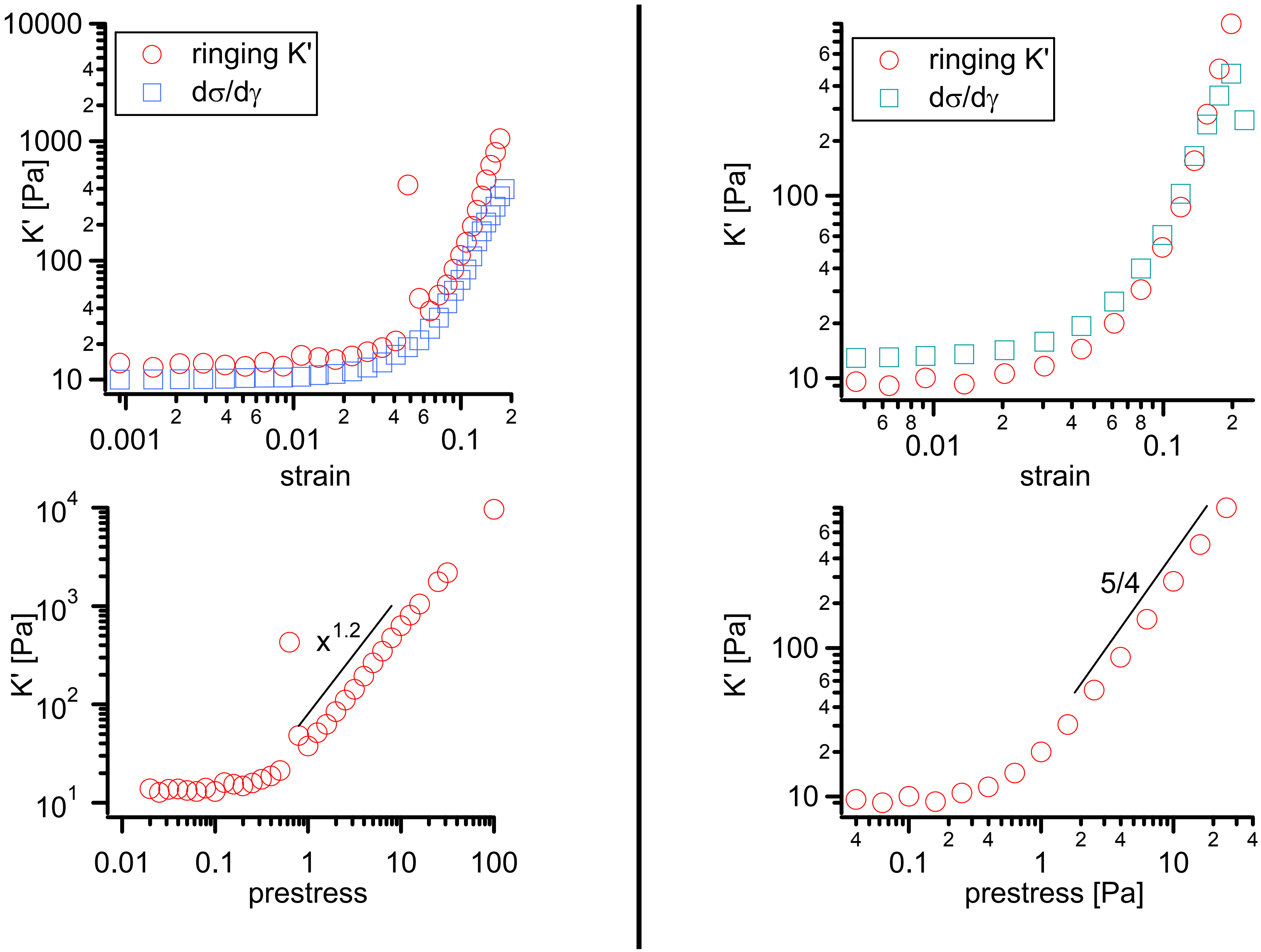
$$\Delta = \frac{1}{n}\ln\left(\frac{A_1}{A_{n+1}}\right)$$

$$b = \frac{2\pi \cdot R^3}{3\tan\theta}$$

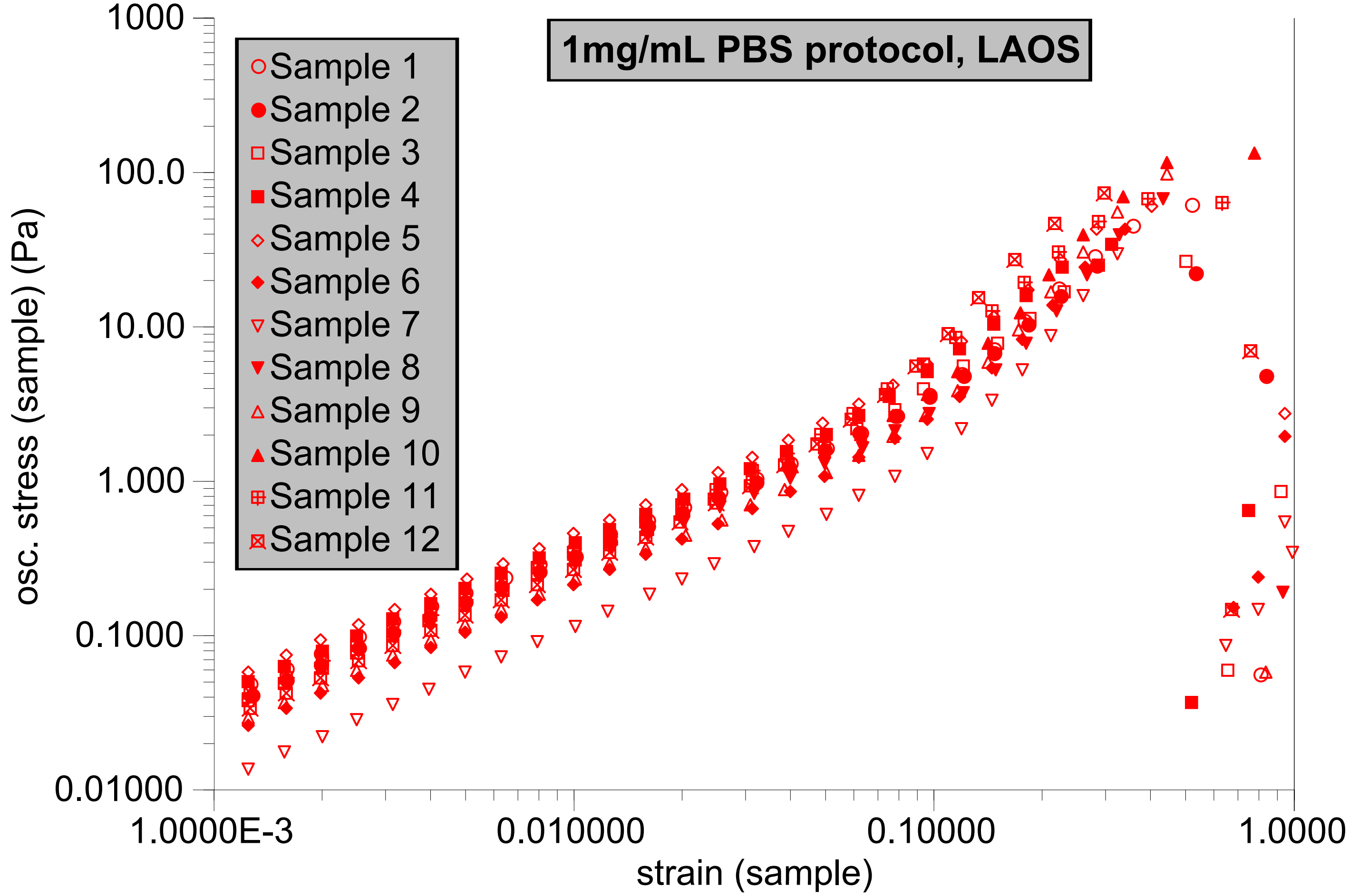
Collagen’s creep response



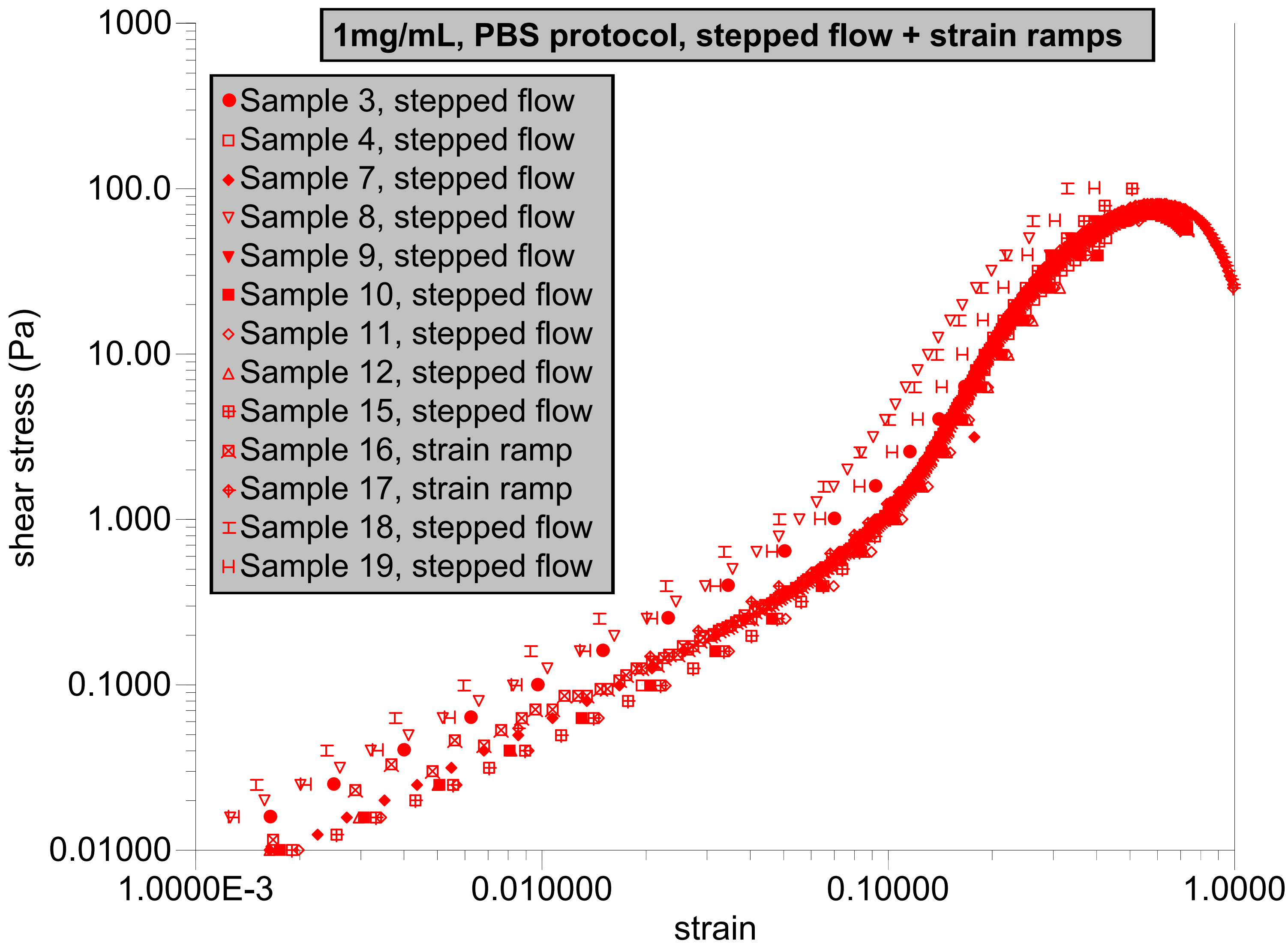
Creep-ringing results



Large amplitude oscillatory shear results

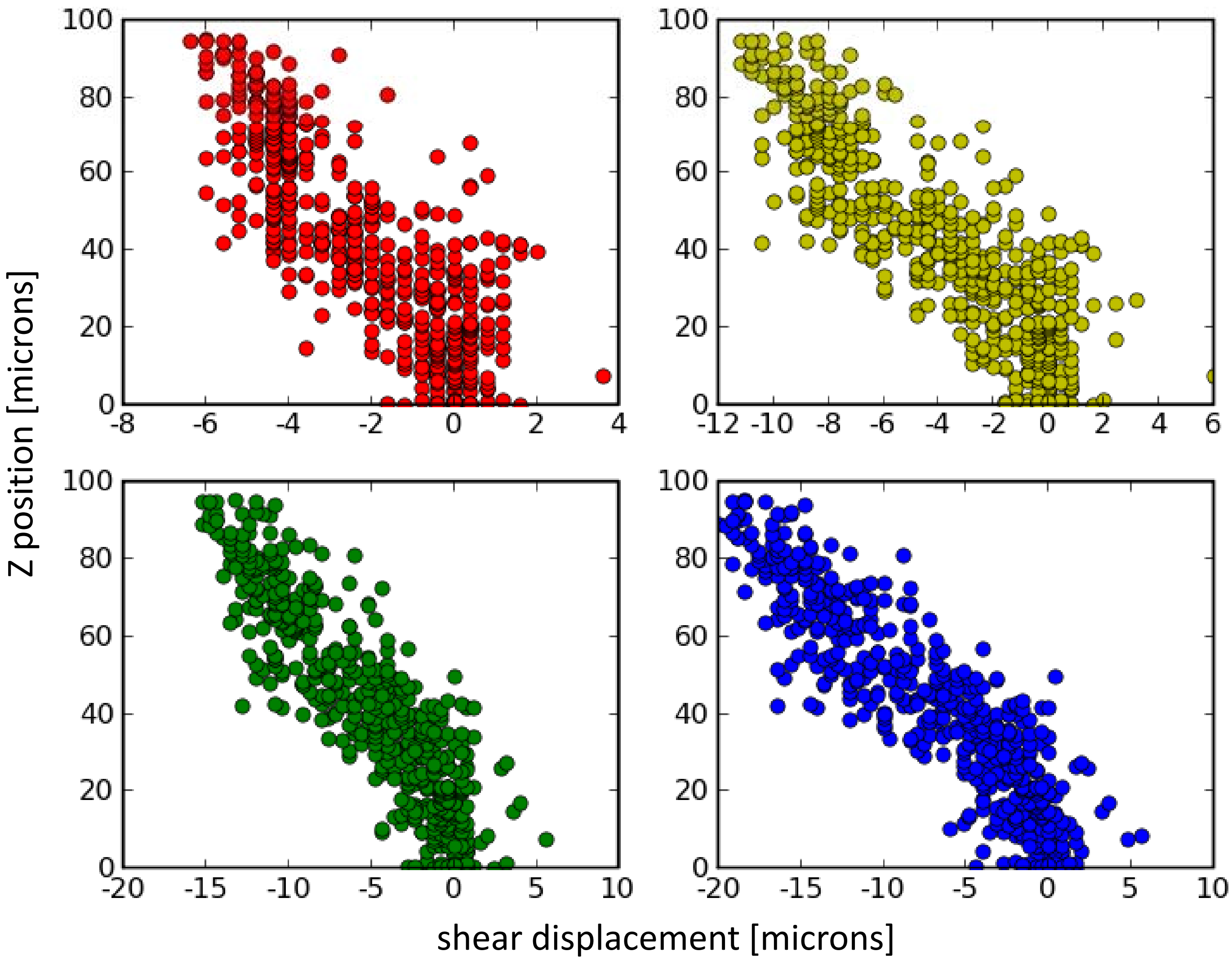


Creep-ringing results

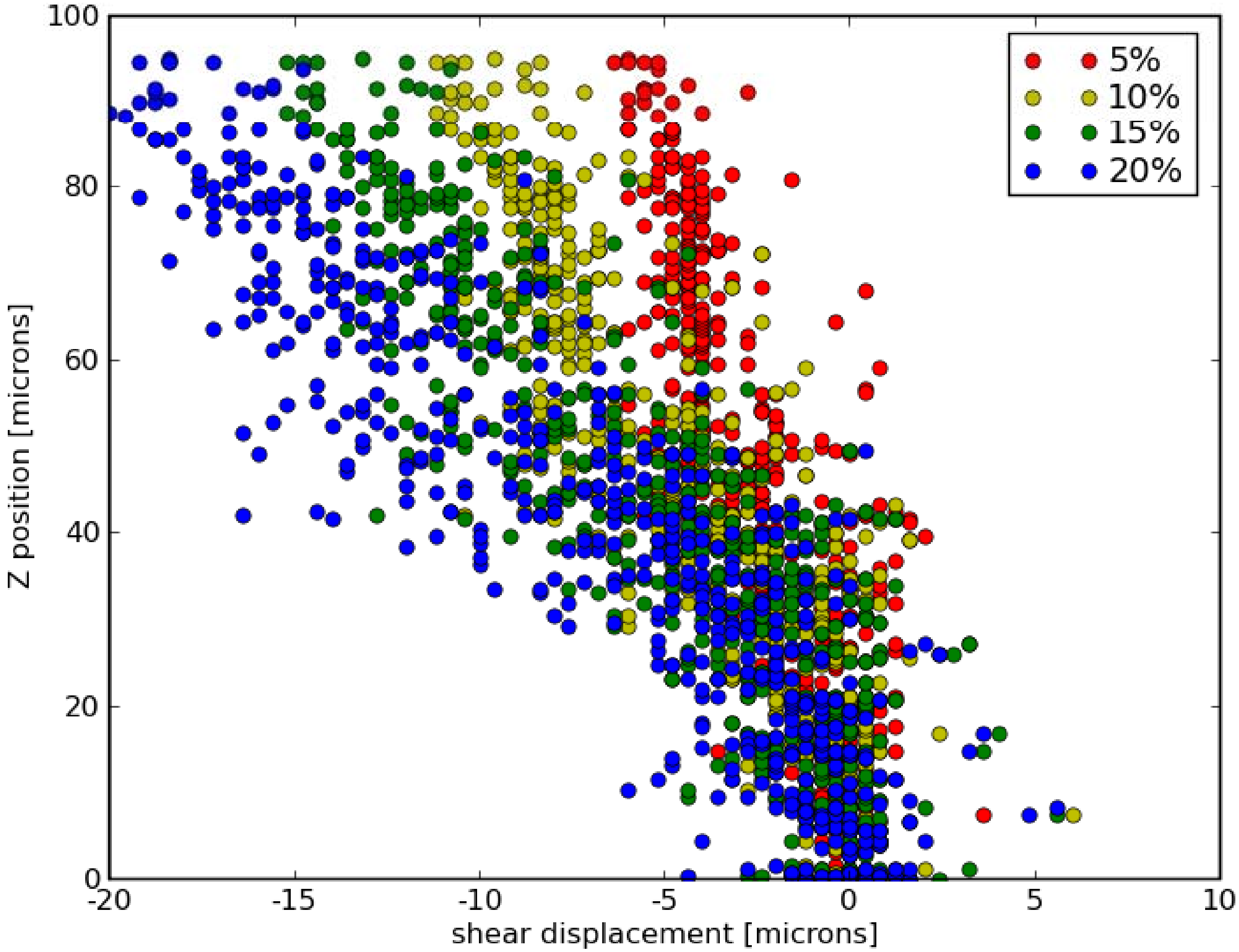


Origin of nonlinear behavior

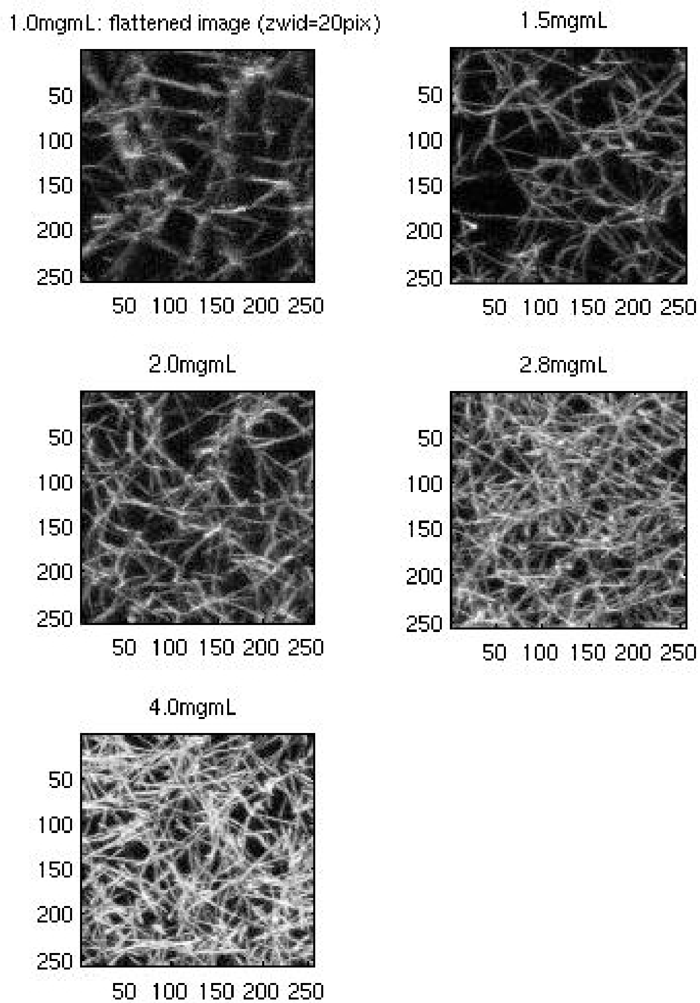
Shear profiles

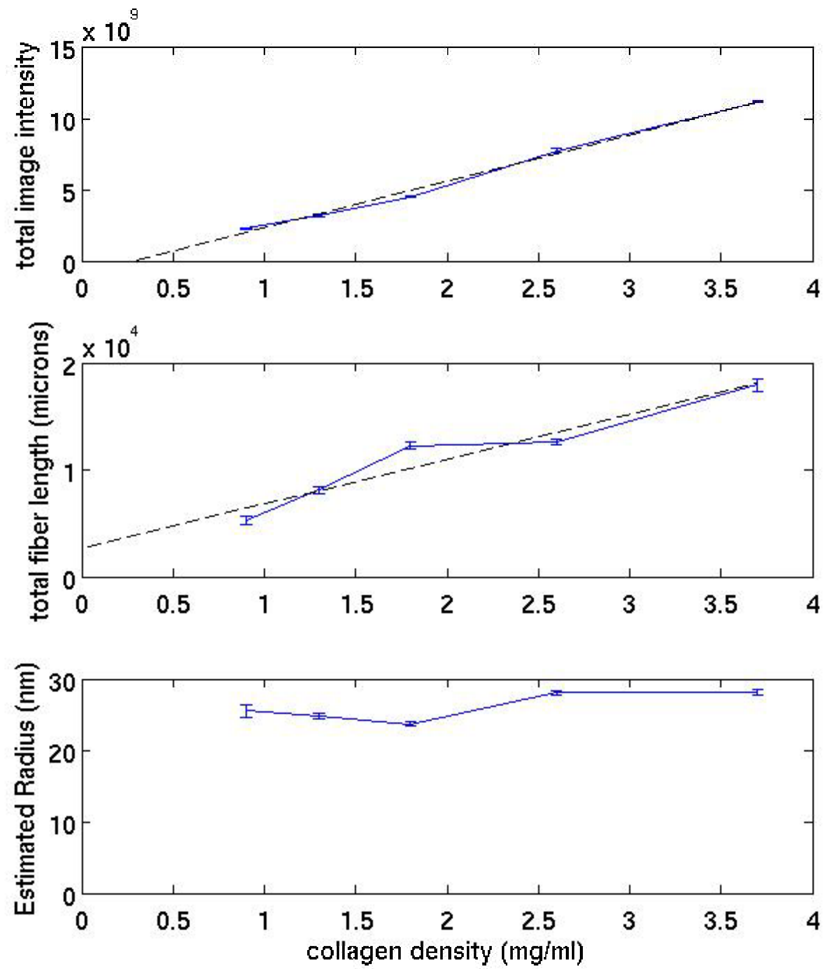


Shear profiles



Geometrical properties

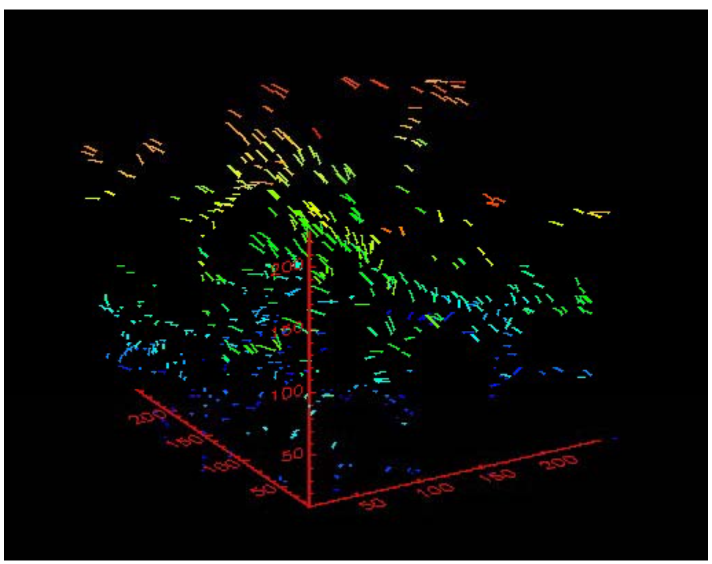


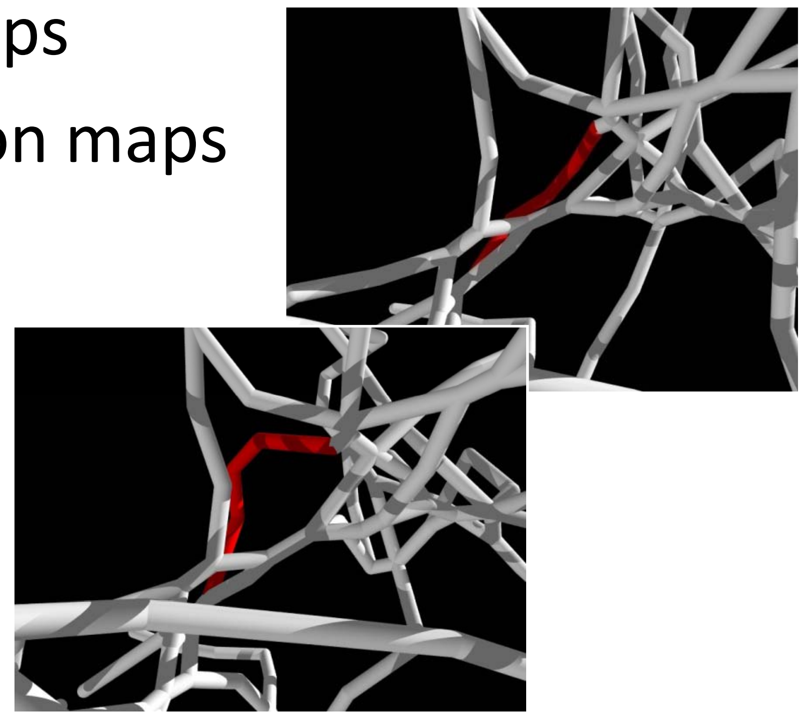


0.2μm/px => 50μm x 50μm x 4μm

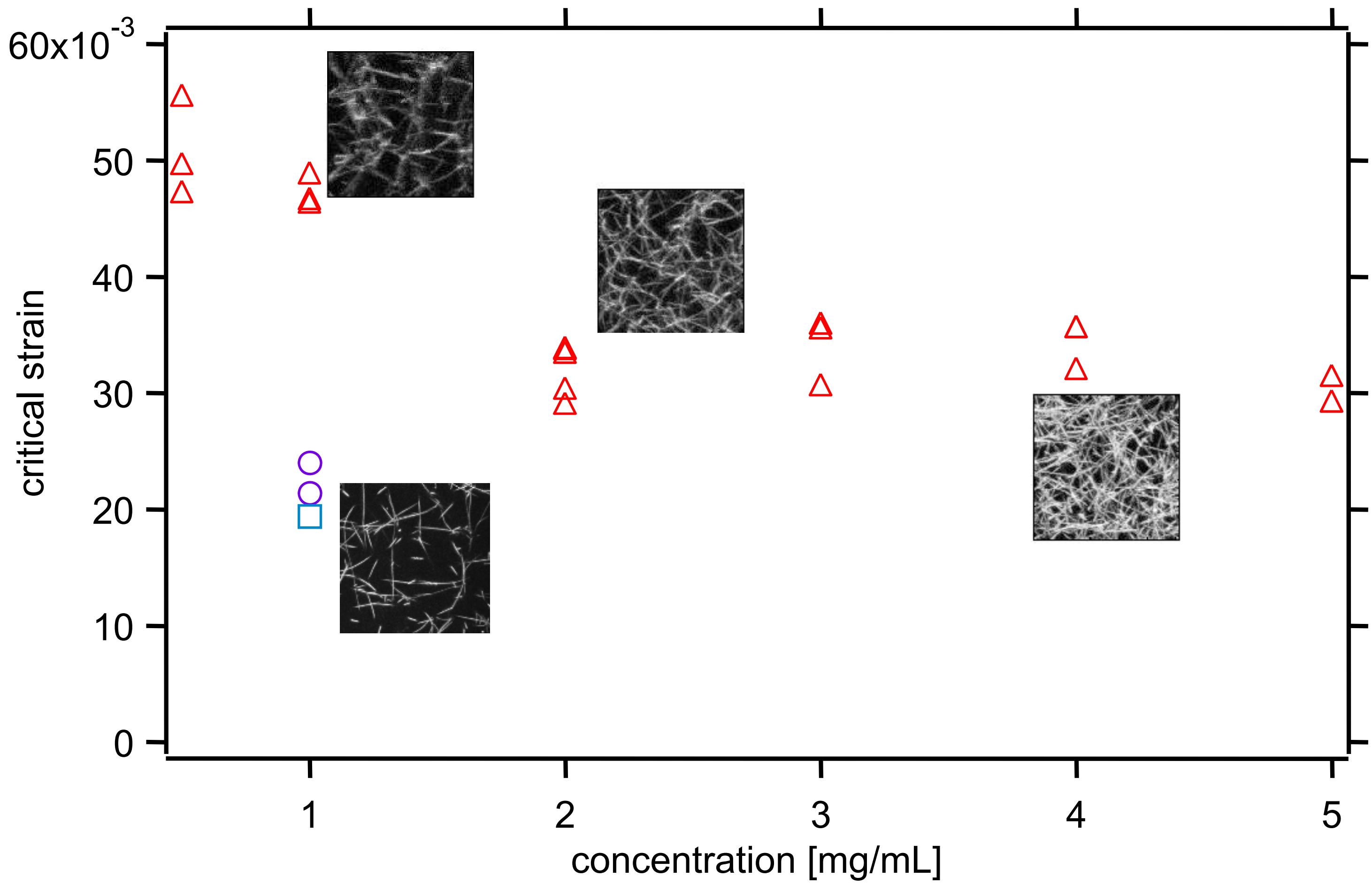
Single fiber/xlink behavior

- Bio(polymer) Fi(ber) T(racking): BioFiT
  - Use fiber skeleton + raw image
  - Track fiber crosslinks => track fibers
  - Generate diplacement maps
  - Generate fiber deformation maps



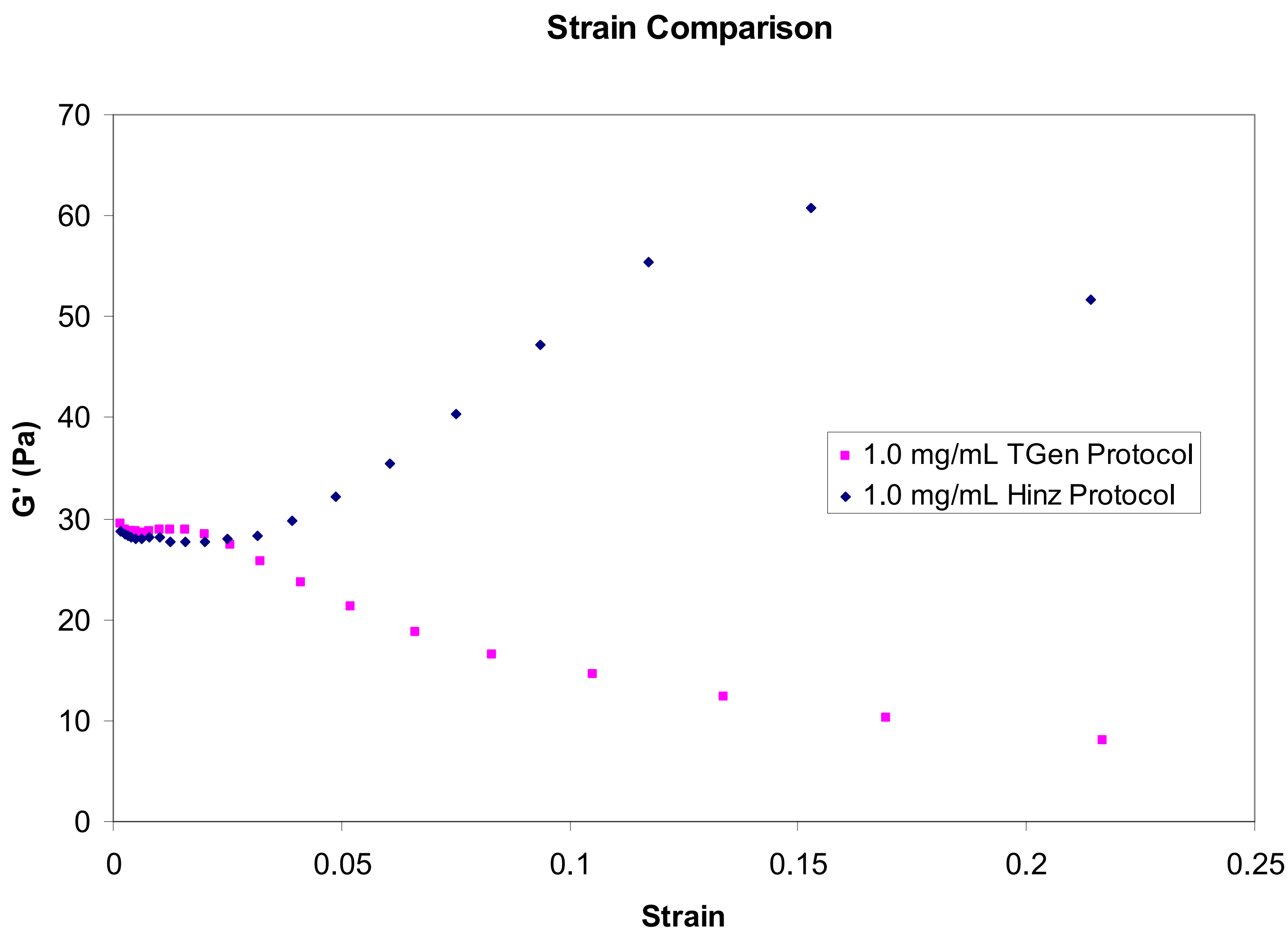


Onset of strain stiffening



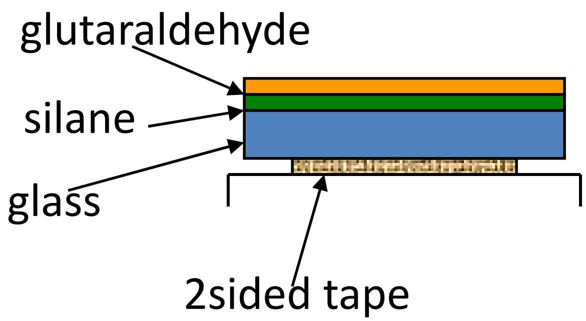
Probing surface attachment

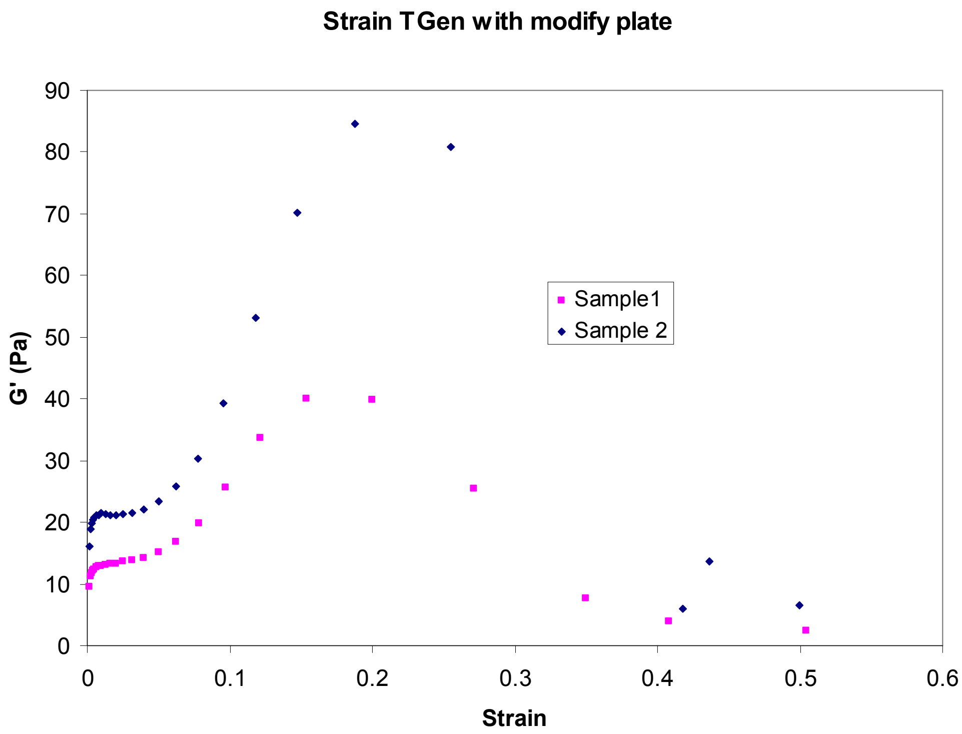
Pablo: only one of the two protocols stiffens….



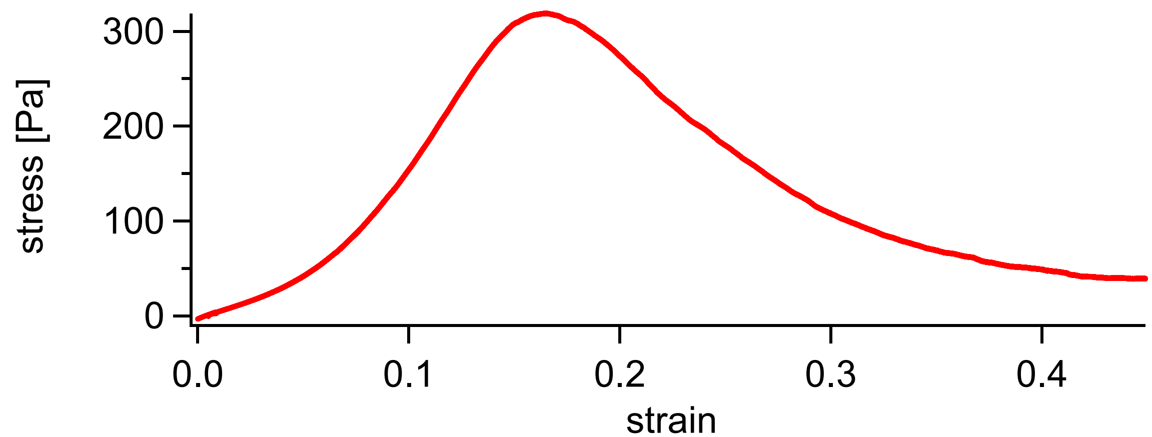
Pablo: but…

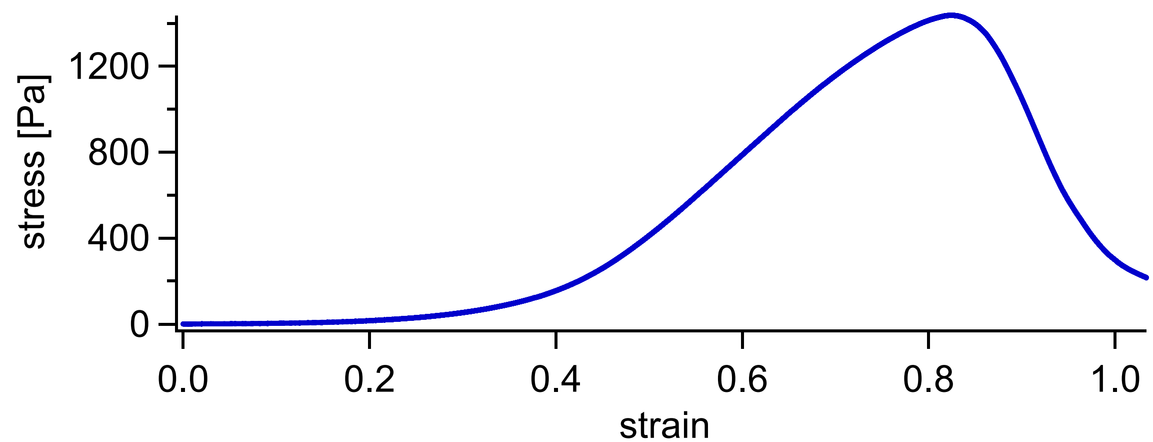
- We expect similar qualitative behavior between two protocols
- Are we really probing collagen mechanics OR surface attachment?





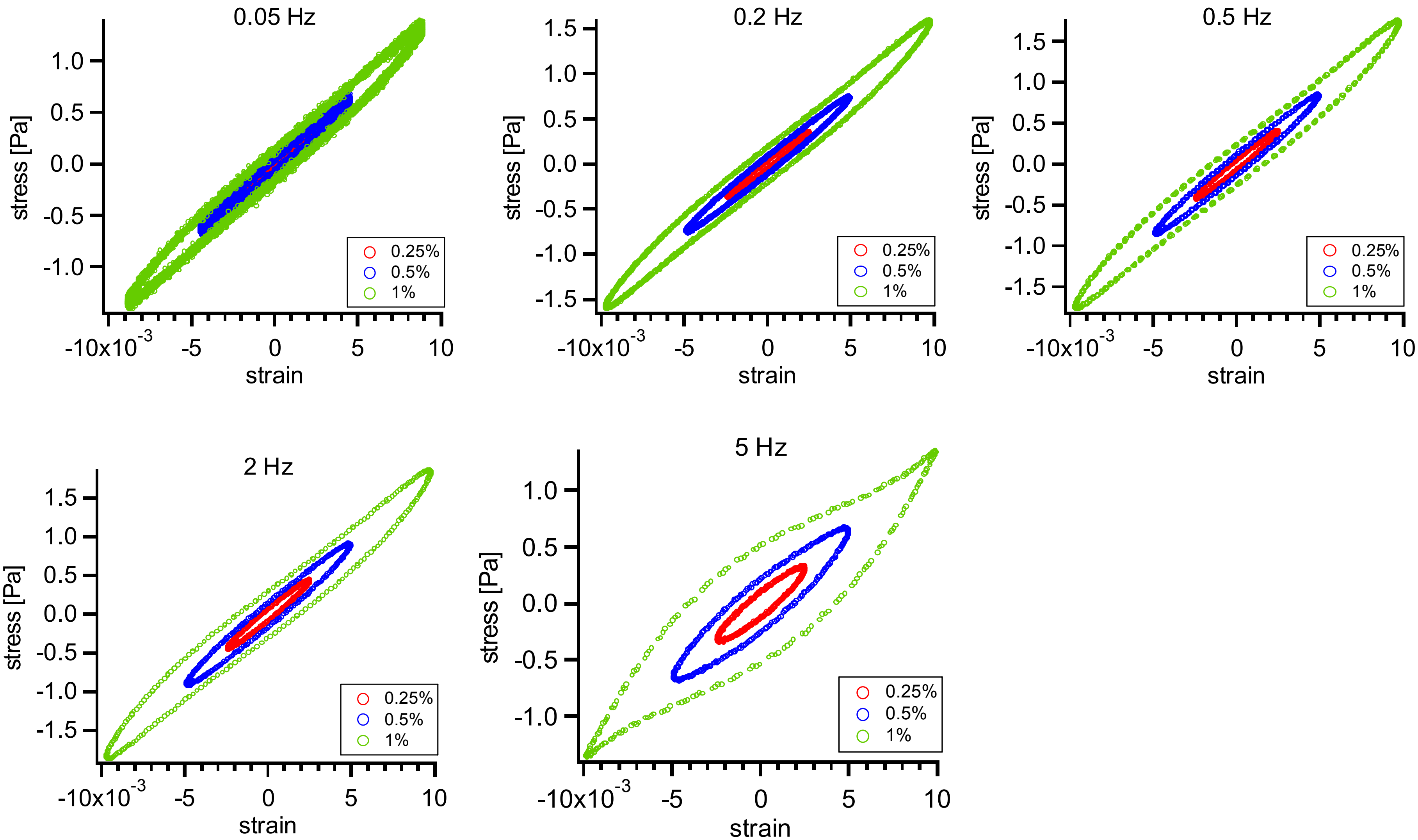
Sticking of top/bottom surface





More on LAOS

Strain rate dependence?



Nonlinear viscoelasticity (*Cho 2005*)

Important assumptions:

- periodicity, i.e. closed Lissajou curves
- steady-state, i.e. no plastic deformation

Time and frequency dependence implicit: $\sigma(\omega, \gamma_0, t) = \sigma(x, y)$

Change of sign of strain => $\sigma(-x, -y) = -\sigma(x, y)$

$$\sigma(x,y) = \frac{\sigma(x,y) - \sigma(-x,y)}{2} + \frac{\sigma(x,y) - \sigma(x,-y)}{2}$$

$$\sigma_{\mathrm{OE}} = \frac{\sigma(x,y) - \sigma(-x,y)}{2}, \quad \sigma_{\mathrm{EO}} = \frac{\sigma(x,y) - \sigma(x,-y)}{2}$$

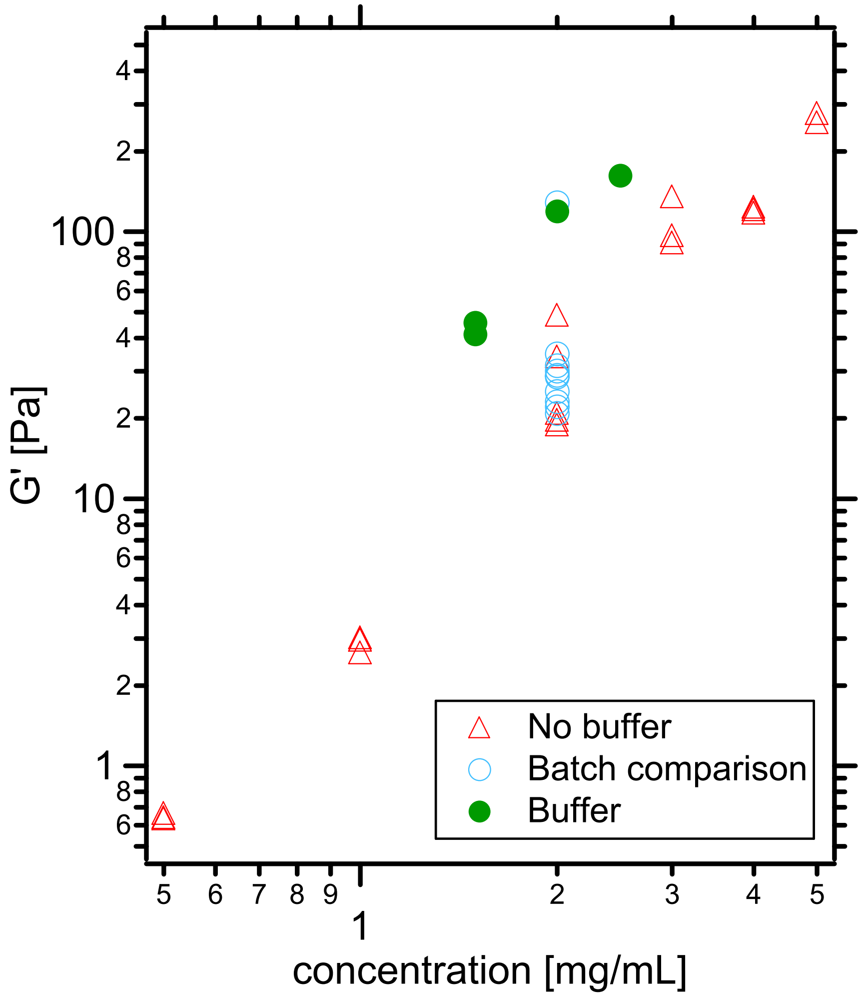
Pairity => $\oint \sigma_{\mathrm{OE}} dx = 0 \qquad \oint \sigma_{\mathrm{EO}} dy = 0$

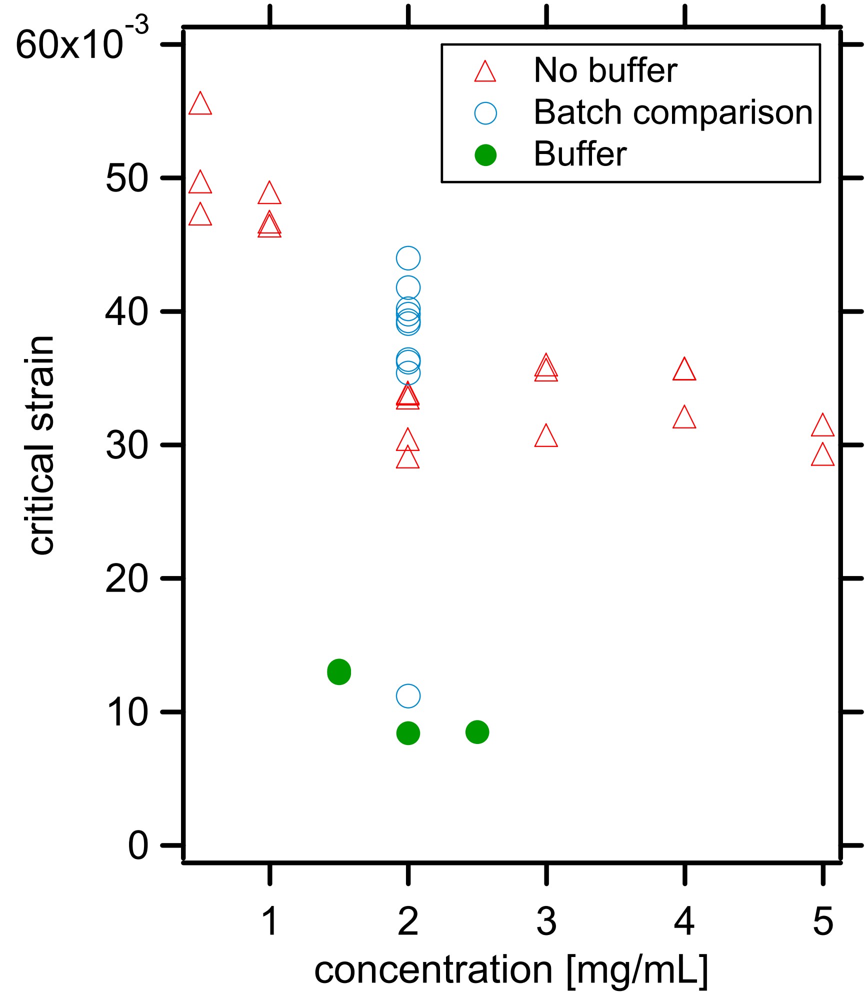
$$\oint \sigma dx = \oint \sigma_{\mathrm{EO}} dx, \quad \oint \sigma dy = \oint \sigma_{\mathrm{OE}} dy$$

Uniqueness? $\sigma'(x,y) = \sigma'(x, \gamma_0), \quad \sigma''(x,y) = \sigma''(y, \gamma_0)$

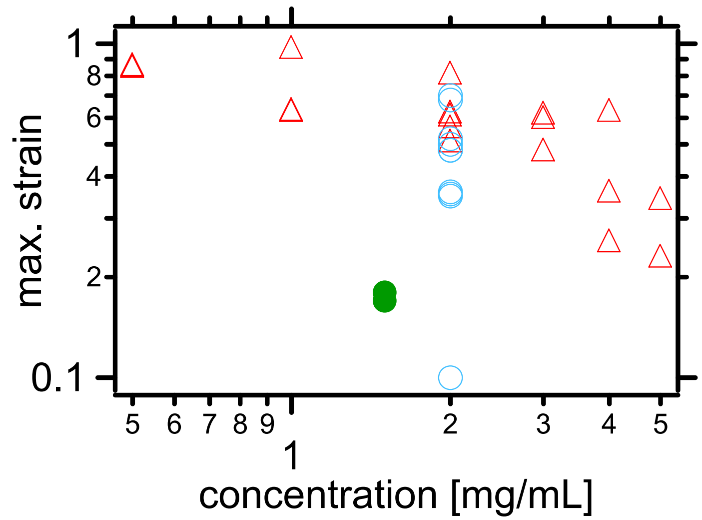
Rheology results

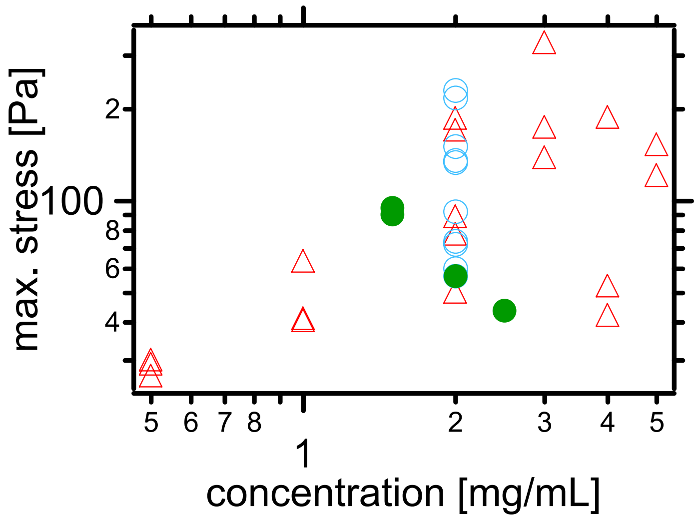
Modulus and γ_{crit}





Other variables of interest?





K’ calculated as derivative
of stress-strain curve