Robust and Clustered Standard Errors

Molly Roberts

March 6, 2013
Outline

1. An Introduction to Robust and Clustered Standard Errors
   - Linear Regression with Non-constant Variance
   - GLM’s and Non-constant Variance
   - Cluster-Robust Standard Errors

2. Replicating in R
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2. Replicating in R
Review: Errors and Residuals

Errors are the vertical distances between observations and the unknown Conditional Expectation Function. Therefore, they are unknown.

Residuals are the vertical distances between observations and the estimated regression function. Therefore, they are known.
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\[ y = X\beta + u \]
\[ u = y - X\beta \]

Residuals represent the difference between the outcome and the estimated mean.

\[ y = X\hat{\beta} + \hat{u} \]
\[ \hat{u} = y - X\hat{\beta} \]
Notation

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\begin{align*}
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u &= y - X\beta
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Variance of $\hat{\beta}$ depends on the errors

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$= (X'X)^{-1} X'(X\beta + u)$$

$$= \beta + (X'X)^{-1} X'u$$
Variance of $\hat{\beta}$ depends on the errors

\[
V[\hat{\beta}] = V[\beta] + V[(X'X)^{-1}X'u] \\
= 0 + V[(X'X)^{-1}X'u] \\
= E[(X'X)^{-1}X'uu'X(X'X)^{-1}] - E[(X'X)^{-1}X'u]E[(X'X)^{-1}X'u]' \\
= E[(X'X)^{-1}X'uu'X(X'X)^{-1}] - 0
\]
Variance of $\hat{\beta}$ depends on the errors (continued)

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$$= 0 + V[(X'X)^{-1} X'u]$$
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$$= E[(X'X)^{-1} X'uu'X (X'X)^{-1}] - 0$$
$$= (X'X)^{-1} X'E[uu']X (X'X)^{-1}$$
$$= (X'X)^{-1} X'\Sigma X (X'X)^{-1}$$
Under standard OLS assumptions,
Constant Error Variance and Dependence

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\[ u \sim N_n(0, \Sigma) \]
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\[ \Sigma = Var(u) = E[uu'] = \begin{bmatrix} \sigma^2 & 0 & 0 & \ldots & 0 \\ 0 & \sigma^2 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \sigma^2 \end{bmatrix} \]
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0 & 0 & \sigma^2 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \sigma^2
\end{bmatrix}$$

What does this mean graphically for a CEF with one explanatory variable?
Evidence of Non-constant Error Variance (4 examples)
Notation

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In this section we will allow violations of this assumption in the following heteroskedastic form.
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In this section we will allow violations of this assumption in the following heteroskedastic form.

\[
\text{Var}(u) = E[uu'] = \begin{bmatrix}
\sigma_1^2 & 0 & 0 & \ldots & 0 \\
0 & \sigma_2^2 & 0 & \ldots & 0 \\
& & \ddots & & \\
0 & 0 & 0 & \ldots & \sigma_n^2
\end{bmatrix}
\]
Consequences of non-constant error variance

- The $\hat{\sigma}^2$ will not be unbiased for $\sigma^2$.
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- For "$\alpha$" level tests, probability of Type I error will not be $\alpha$.
- "$1 - \alpha$" confidence intervals will not have $1 - \alpha$ coverage probability.

The LS estimator is no longer BLUE. However, the degree of the problem depends on the amount of heteroskedasticity.

$\hat{\beta}$ is still unbiased for $\beta$. 

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Heteroskedasticity Consistent Estimator

Suppose

\[ V[u] = \Sigma = \begin{bmatrix} \sigma^2_1 & 0 & \cdots & 0 \\ 0 & \sigma^2_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2_n \end{bmatrix} \]

then

\[ \text{Var}(\hat{\beta}) = (X'X)^{-1}X'\Sigma X(X'X)^{-1} \]

and Huber (and then White) showed that

\[ (X'X)^{-1}X'\begin{bmatrix} \hat{u}^2_1 \\ \vdots \\ \hat{u}^2_n \end{bmatrix}X(X'X)^{-1} \]

is a consistent estimator of \( V[\hat{\beta}] \).
Heteroskedasticity Consistent Estimator

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\]
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Things to note about this approach

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2. Doesn’t make \( \hat{\beta} \) BLUE

3. What are you going to do with predicted probabilities?
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2. Replicating in R
What happens when the model is not linear?

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Under certain conditions, you can get the standard errors, even if your model is misspecified.
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Under certain conditions, you can get the standard errors, even if your model is misspecified.

These are the robust standard errors that scholars now use for other glm’s, and that happen to coincide with the linear case.
What does it mean for a non-linear model to have heteroskedasticity?

- Think about the probit model in the latent variable formulation.
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What does it mean for a non-linear model to have heteroskedasticity?

- Think about the probit model in the latent variable formulation.
- Pretend that there is heteroskedasticity on the linear model for $y^\ast$.
- Heteroskedasticity in the latent variable formulation will completely change the functional form of $P(y = 1|x)$.
- What does this mean? The $P(y = 1|x) \neq \Phi(x\beta)$. Your model is wrong.
RSEs for GLMs

To derive robust standard errors in the general case, we assume that

$$y \sim f_i(y | \theta)$$

Then our likelihood function is given by

$$\prod_{i=1}^{n} f_i(Y_i | \theta)$$

and thus the log-likelihood is

$$L(\theta) = \sum_{i=1}^{n} \log f_i(Y_i | \theta)$$
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RSEs for GLMs

We will denote the first and second partial derivatives of \( L \) to be:

\[
L'(\theta) = \sum_{i=1}^{n} g_i(Y_i|\theta),
\]

\[
L''(\theta) = \sum_{i=1}^{n} h_i(Y_i|\theta),
\]

Where

\[
g_i(Y_i|\theta) = \left[ \log f_i(y|\theta) \right]' = \delta \frac{\partial}{\partial \theta} \log f_i(y|\theta)
\]

and

\[
h_i(Y_i|\theta) = \left[ \log f_i(y|\theta) \right]'' = \delta^2 \frac{\partial^2}{\partial \theta^2} \log f_i(y|\theta)
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This shouldn’t be too unfamiliar.
RSEs for GLMs

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Remember, the Fisher information matrix is $-E_\theta [h_i(Y_i|\theta)]$. 
Let's assume the model is correct – there is a true value $\theta_0$ for $\theta$. 
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Then we can use the Taylor approximation for the log-likelihood function to estimate what the likelihood function looks like around $\theta_0$:
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Then we can use the Taylor approximation for the log-likelihood function to estimate what the likelihood function looks like around $\theta_0$:

$$L(\theta) = L(\theta_0) + L'(\theta_0)(\theta - \theta_0) + \frac{1}{2}(\theta - \theta_0)^T L''(\theta_0)(\theta - \theta_0)$$
RSEs for GLMs

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  $$\text{Avar}(\hat{\theta}) = [ -L''(\theta_0)]^{-1} [\text{Cov}(L'(\theta_0))] [ -L''(\theta_0)]^{-1}$$
RSEs for GLMs

It’s the sandwich estimator.

\[
Avar(\hat{\theta}) = \left[-L''(\theta_0)\right]^{-1}[Cov(L'(\theta_0))][-L''(\theta_0)]^{-1}
\]

\[
= \left[-\sum_{i=1}^{n} h_i(Y_i|\hat{\theta})\right]^{-1} \left[\sum_{i=1}^{n} g_i(Y_i|\hat{\theta})^T g_i(Y_i|\hat{\theta})\right] \left[-\sum_{i=1}^{n} h_i(Y_i|\hat{\theta})\right]^{-1}
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\]

Bread: \( -\sum_{i=1}^{n} h_i(Y_i|\hat{\theta}) \)^{-1} \hspace{1cm} \text{Meat:} \ \left[ \sum_{i=1}^{n} g_i(Y_i|\hat{\theta})^T g_i(Y_i|\hat{\theta}) \right]
Cluster-Robust Standard Errors

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\sum_{j=1}^{n} \sum_{i \in c_j} g_i(Y_i|\hat{\theta})^T g_i(Y_i|\hat{\theta})
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Sometimes it’s difficult to figure out what is going on in Stata.
Replicating in R

- There are lots of different ways to replicate these standard errors in R.
- Sometimes it’s difficult to figure out what is going on in Stata.
- But by really understanding what is going on in R, you will be able to replicate once you know the equation for Stata.
Some Data

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Thanks to Michele Margolis and Dan Altman for their contributions to the library!
First, let’s run their model:
The Model

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```r
fmla <- as.formula(restrict ~ art8 + shift_left + flexible + regnorm + gdpgrow + resgdp + bopgdp + useimfcr + surveil + univers + resvol + totvol + tradedep + milit + termlim + parli + lastrest + lastrest2 + lastrest3)
fit <- glm(fmla, data=treaty1, family=binomial(link="logit"))
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The Meat and Bread

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bread <- vcov(fit)
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For the meat, we are going to use the estimating function to create the matrices first derivative:

```r
est.fun <- estfun(fit)
```

Note: if estfun doesn’t work for your glm, there is a way to do it using numericGradient().
The Sandwich

So we can create the sandwich
The Sandwich

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```r
meat <- t(est.fun)%*%est.fun
sandwich <- bread%*%meat%*%bread
```

And put them back in our table

```r
library(lm.test)
coeftest(fit, sandwich)
```

Note: For the linear case, `estfun()` is doing something a bit different than in the logit, so use:

```r
robust <- sandwich(lm.1, meat=crossprod(est.fun)/N)
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sandwich <- bread%*%meat%*%bread
```

And put them back in our table

```r
library(lm.test)
coeftest(fit, sandwich)
```

Note: For the linear case, estfun() is doing something a bit different than in the logit, so use:

```r
robust <- sandwich(lm.1, meat=crossprod(est.fun)/N)
```
Clustered Standard Errors

First, we have to identify our clusters:

```r
fc <- treaty1$imf_ccode
m <- length(unique(fc))
k <- length(coef(fit))

Then, we sum the u's by cluster

```r
u <- estfun(fit)
u.clust <- matrix(NA, nrow=m, ncol=k)
for(j in 1:k){
u.clust[,j] <- tapply(u[,j], fc, sum)
}
```
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fc \leftarrow \text{treaty1$imf_ccode}
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\[
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Then, we sum the u’s by cluster

\[
u \leftarrow \text{estfun(fit)}
\]

\[
u_{\text{clust}} \leftarrow \begin{bmatrix}
\text{matrix(NA, nrow=m, ncol=k)}
\end{bmatrix}
\]

\[
\text{for(j in 1:k)
} \leftarrow \text{tapply(u[,j], fc, sum)}
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}
```
Last, we can make our cluster robust matrix:

\[
\text{cl.vcov} \leftarrow \text{vcov} \times \left( \frac{m}{m-1} \times (u.clust)^\top \right) \times (u.clust) \times \text{vcov}
\]

And test our coefficients

\[
\text{coeftest(fit, cl.vcov)}
\]
Last, we can make our cluster robust matrix:

```r
cl.vcov <- vcov %*% ((m / (m-1)) * t(u.clust) %*% (u.clust)) %*% + vcov
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cl.vcov <- vcov %*% ((m / (m-1)) * t(u.clust) %*% (u.clust)) %*% + vcov
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And test our coefficients

```r
coeftest(fit, cl.vcov)
```
Last, we can make our cluster robust matrix:

```
cl.vcov <- vcov %*% ((m / (m-1)) * t(u.clust)
    %*% (u.clust)) %*% + vcov
```

And test our coefficients

```
coeftest(fit, cl.vcov)
```
A couple notes

- There are easier ways to do this in R (see for example hccm).
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