

# Quantifying the Impact of Model Errors on Top-down CO Emission Estimates

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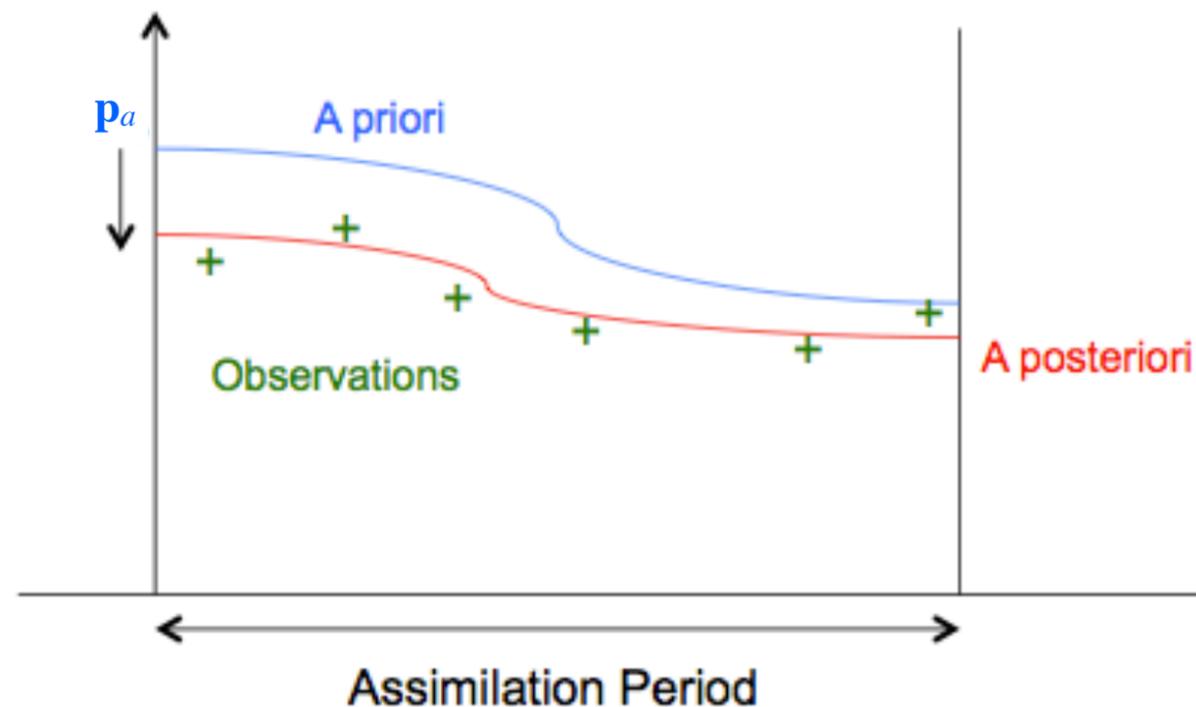
# Strong Constraint 4-Dimensional Variational (4D-Var) Data Assimilation

$$J(\mathbf{p}) = (\mathbf{p} - \mathbf{p}_a)^T \mathbf{B}^{-1} (\mathbf{p} - \mathbf{p}_a) + \sum_{n=1}^N (\mathbf{y}_n - H\mathbf{x}_n)^T \mathbf{R}^{-1} (\mathbf{y}_n - H\mathbf{x}_n)$$

$\mathbf{y}_n$  = observations;  $\mathbf{x}_n$  = modeled tracer distribution;  $\mathbf{p}$  = model parameter (e.g. emissions) to be optimized

$$\mathbf{x}_{n+1} = M(\mathbf{x}_n, \mathbf{p})$$

We optimize the model parameter  $\mathbf{p}$  to minimize the mismatch between the model and the observations over the assimilation period, subject to the constraint that the model forecast is perfect.



# Weak Constraint 4-Dimensional Variational (4D-Var) Data Assimilation

If the model has systematic errors, we can account for them in the forecast equation as a model forcing term  $\mathbf{u}_n$

$$\mathbf{x}_{n+1} = M(\mathbf{x}_n, \mathbf{p}) + \mathbf{u}_n$$

Here  $\mathbf{u}_n$  is a correction to the CO distribution in the model.

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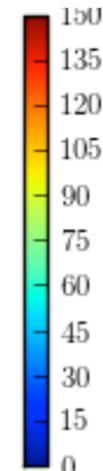
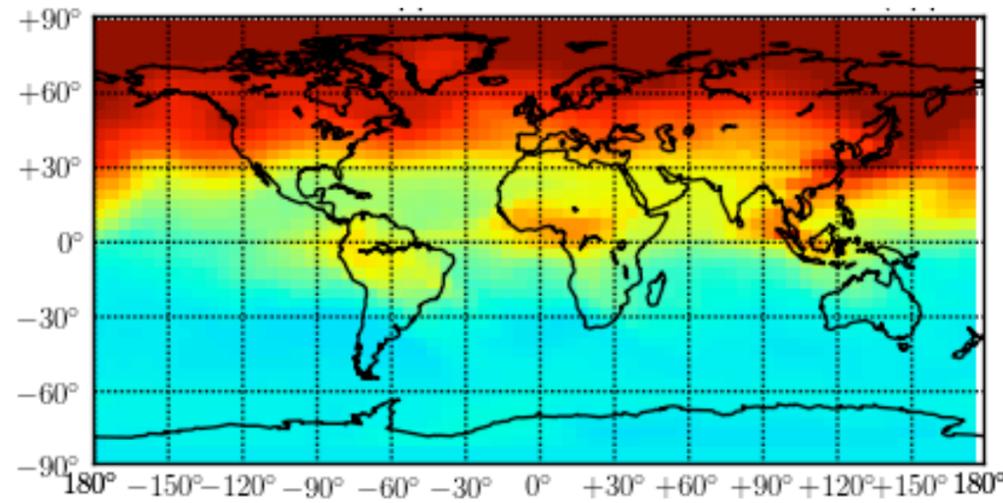
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## Inversion configuration

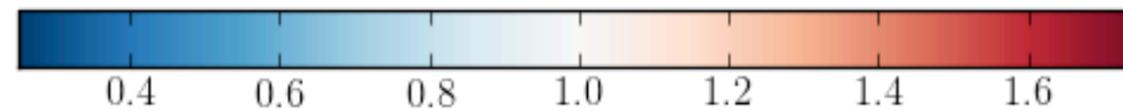
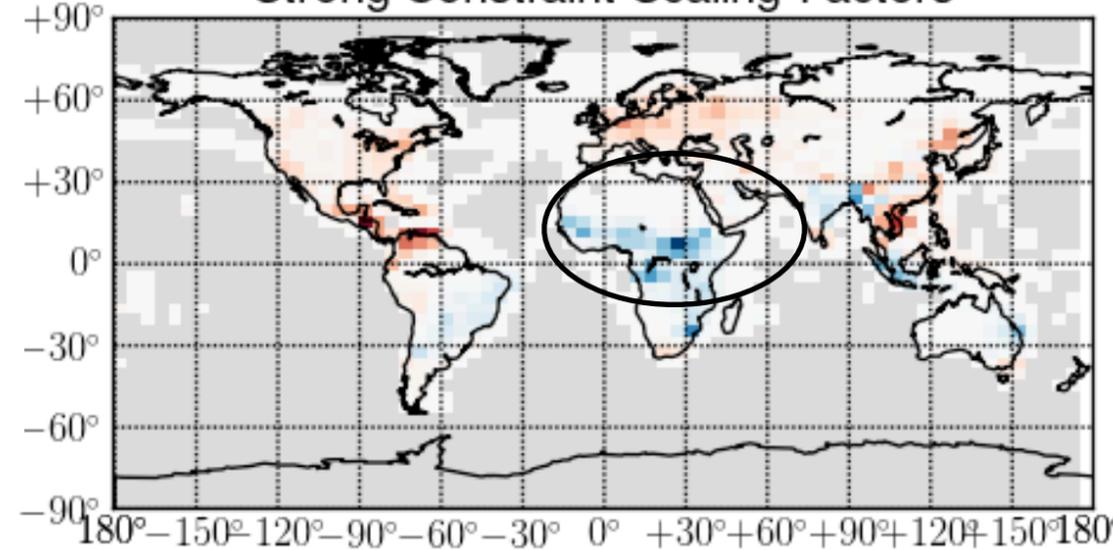
- Use MOPITT V5 data for March 2006
- The model forcing terms ( $\mathbf{u}_n$ ) are estimated only in the tropics and subtropics, 30°S-30°N
- The forcing is kept constant over a time window of 5 days and is applied only in the free troposphere
- We assume  $\mathbf{Q}$  is diagonal with  $\sigma = 0.5$  ppbv, that increases toward the PBL and the tropopause
- Assume that  $\mathbf{B}$  is diagonal, whereas  $\mathbf{R}$  accounts for vertical correlations in the MOPITT retrievals

# GEOS-5/GEOS-4 Simulation Experiment

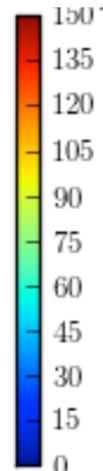
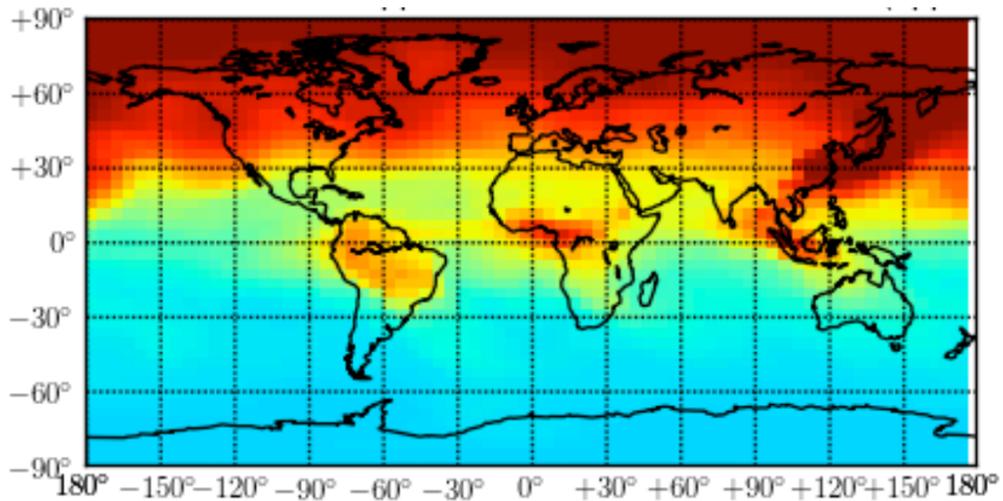
GEOS-4 CO, March 2006 on level 7 (approx 4 km)



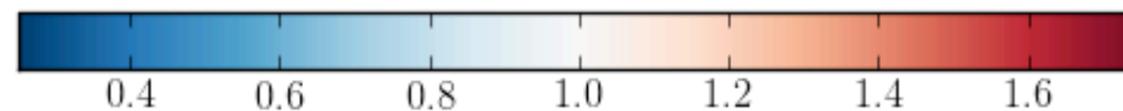
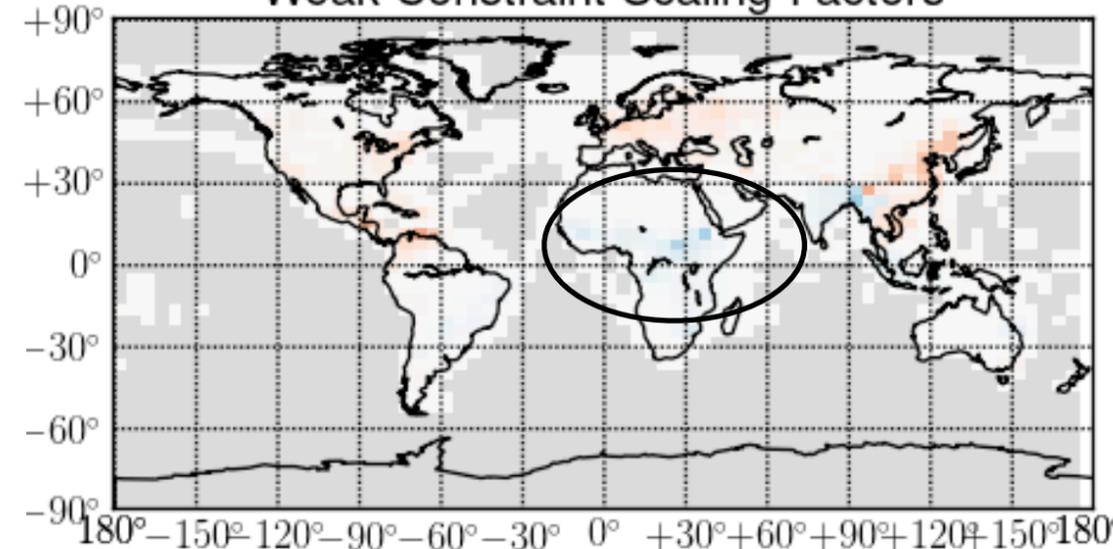
Strong Constraint Scaling Factors



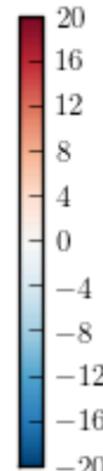
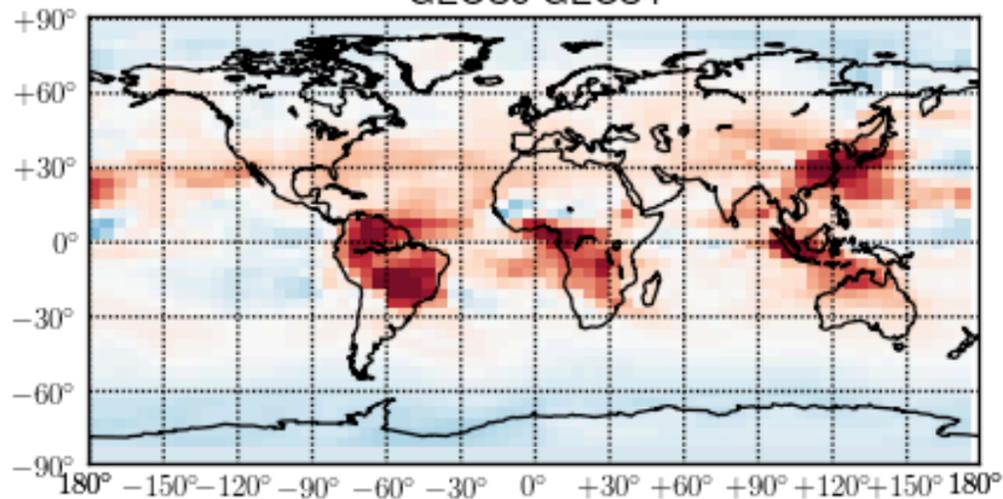
GEOS-5 CO, March 2006 on level 20 (approx 4 km)



Weak Constraint Scaling Factors



GEOS5-GEOS4



- Emissions are the same in both models.
- Sample GEOS-4 along MOPITT orbit to make pseudo-data
- Use GEOS-5 for inversion of the “MOPITT” data to recover CO emissions, starting from an a priori equal to the truth.

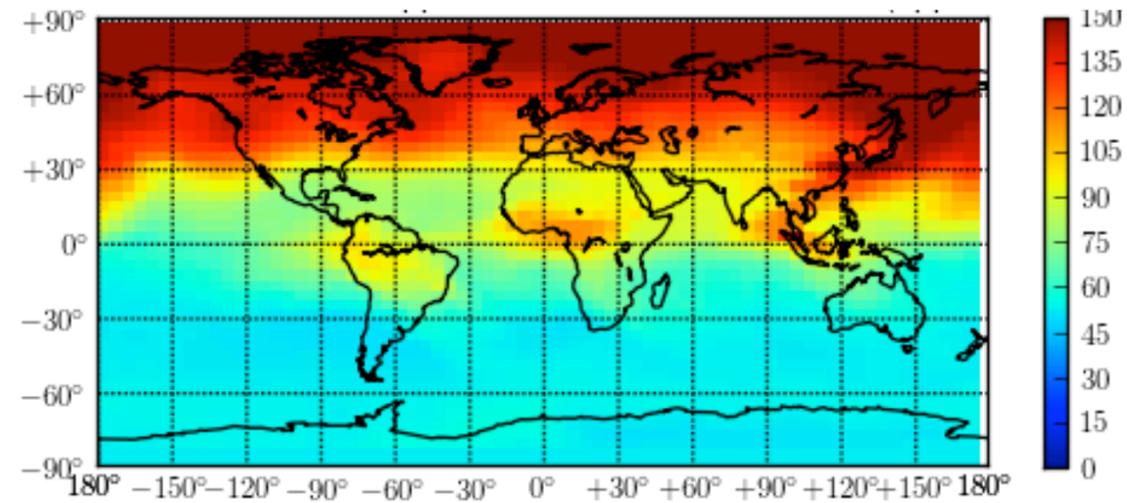
Since the emissions are the same, the a posteriori scaling factors should be 1.

Strong constraint 4D-var:

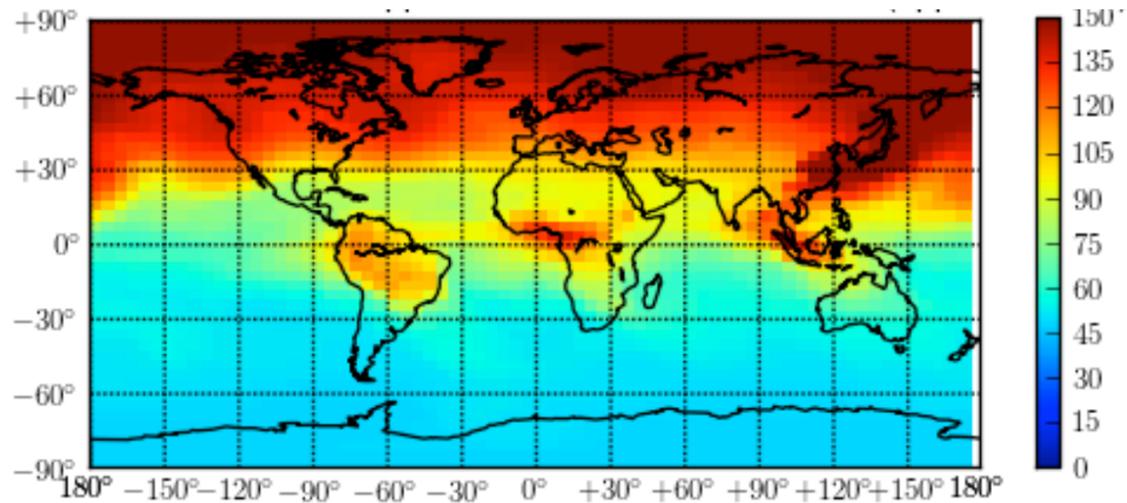
- underestimates emissions in Africa, south Asia, and eastern South America
- overestimates emissions in North America, Europe, east Asia, and tropical South America

# GEOS-5/GEOS-4 Simulation Experiment

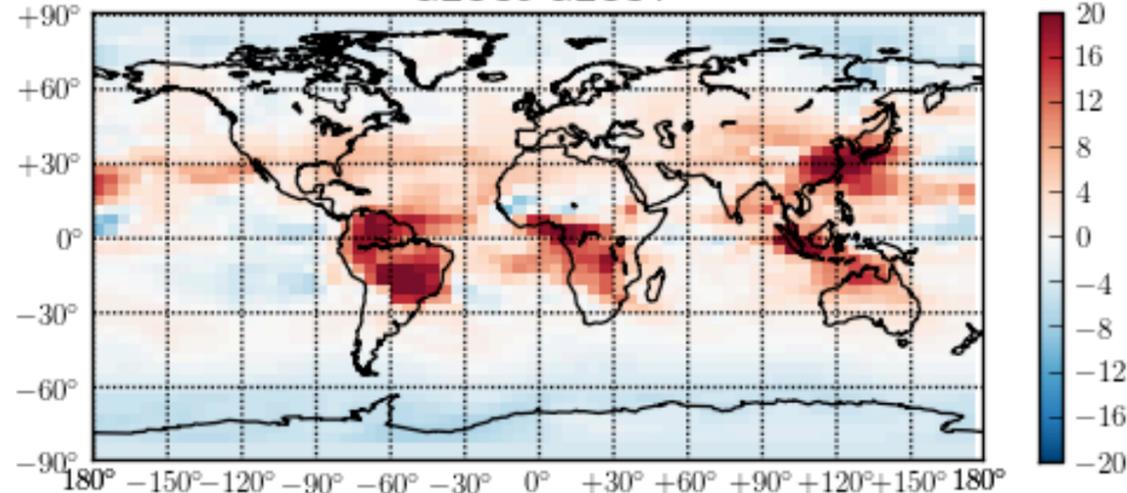
GEOS-4 CO, March 2006 on level 7 (approx 4 km)



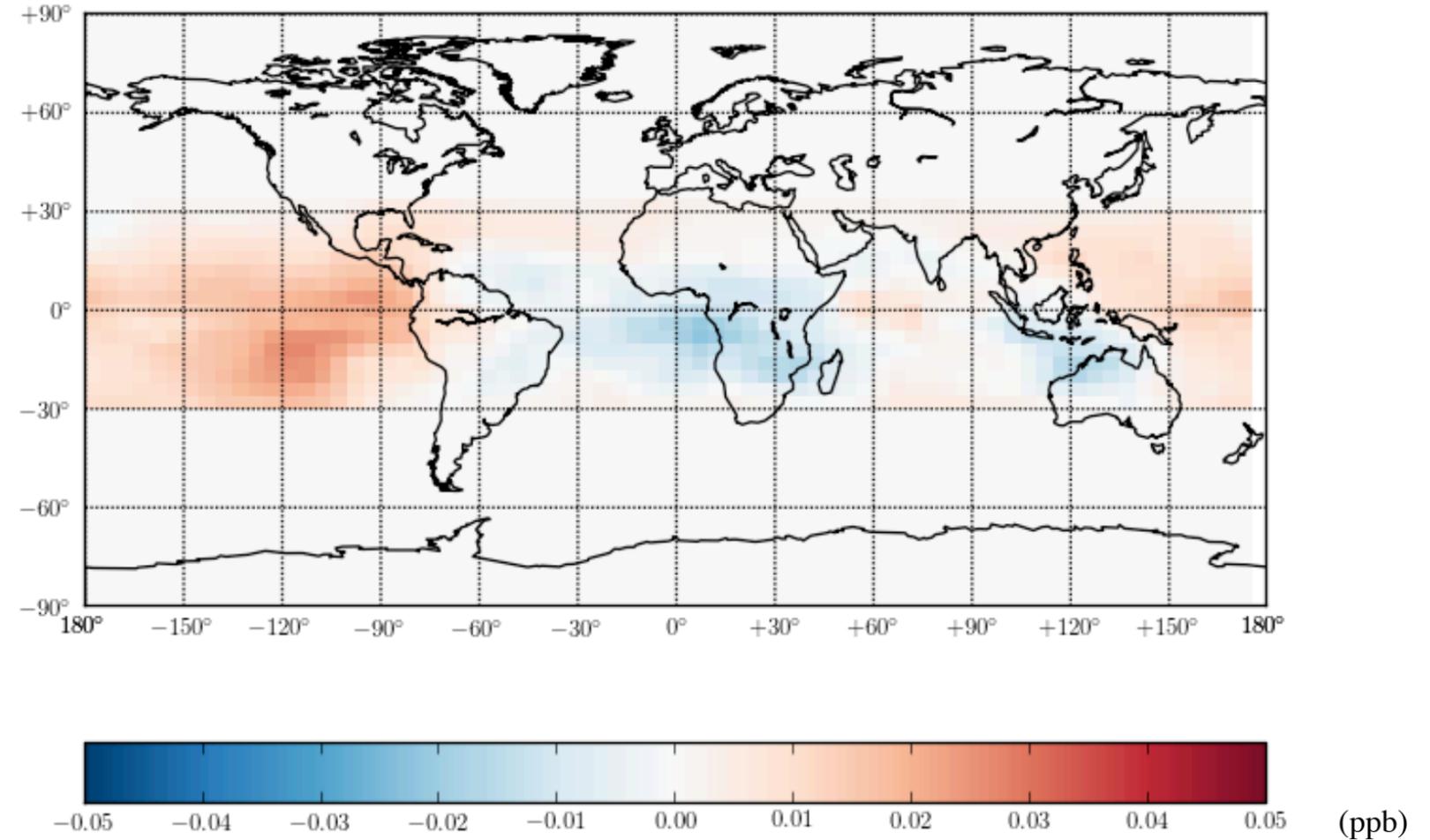
GEOS-5 CO, March 2006 on level 20 (approx 4 km)



GEOS5-GEOS4



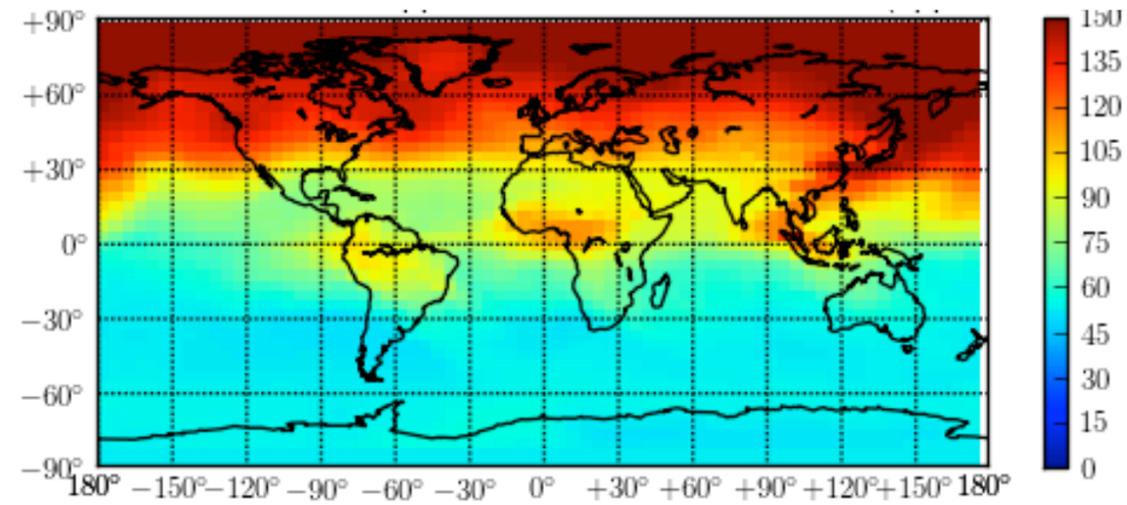
Monthly mean (in ppb) in the mid-troposphere



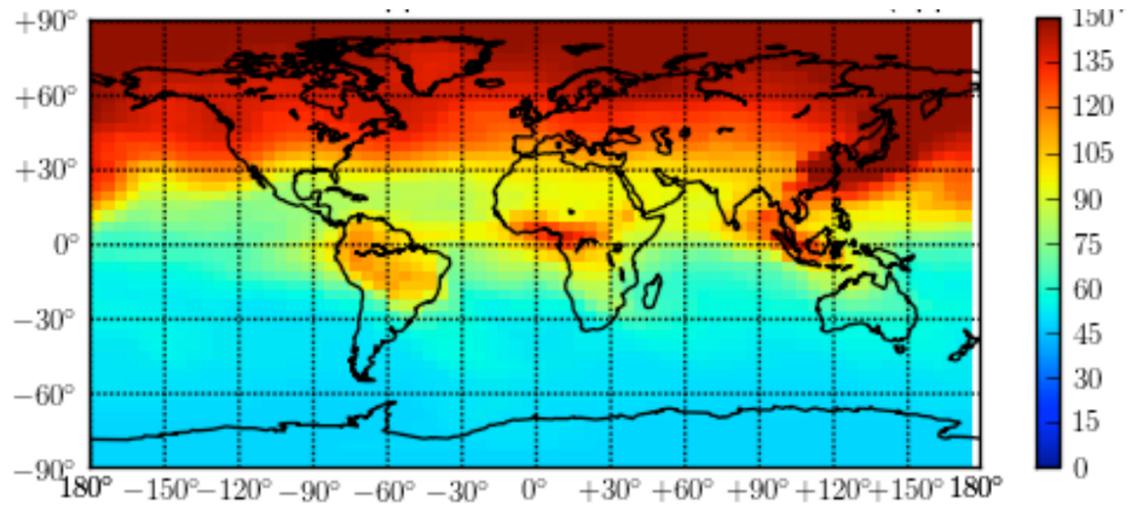
- Forcing (in ppb) applied each time step acts to reduce the bias in the CO abundances so that those biases are not projected onto the emissions.
- The forcing pattern is consistent with the CO biases in the OSSE
- Forcing is weak over South America, despite large the bias in CO

# GEOS-5/GEOS-4 Simulation Experiment

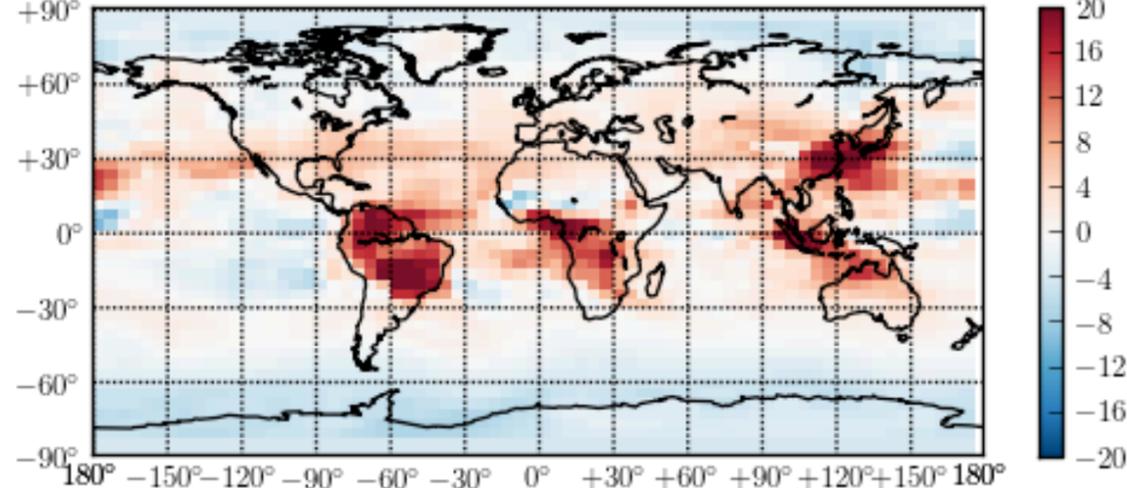
GEOS-4 CO, March 2006 on level 7 (approx 4 km)



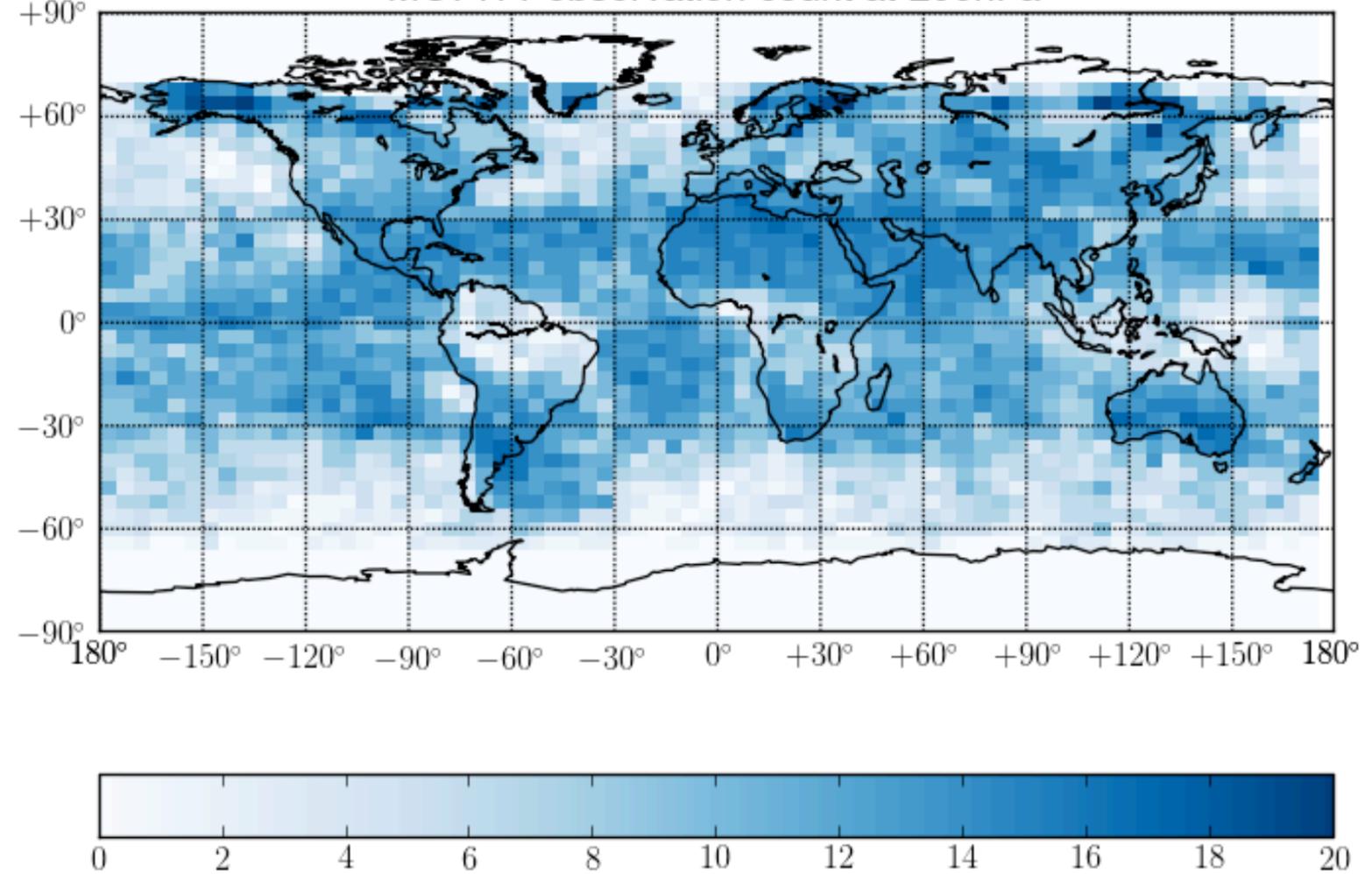
GEOS-5 CO, March 2006 on level 20 (approx 4 km)



GEOS5-GEOS4

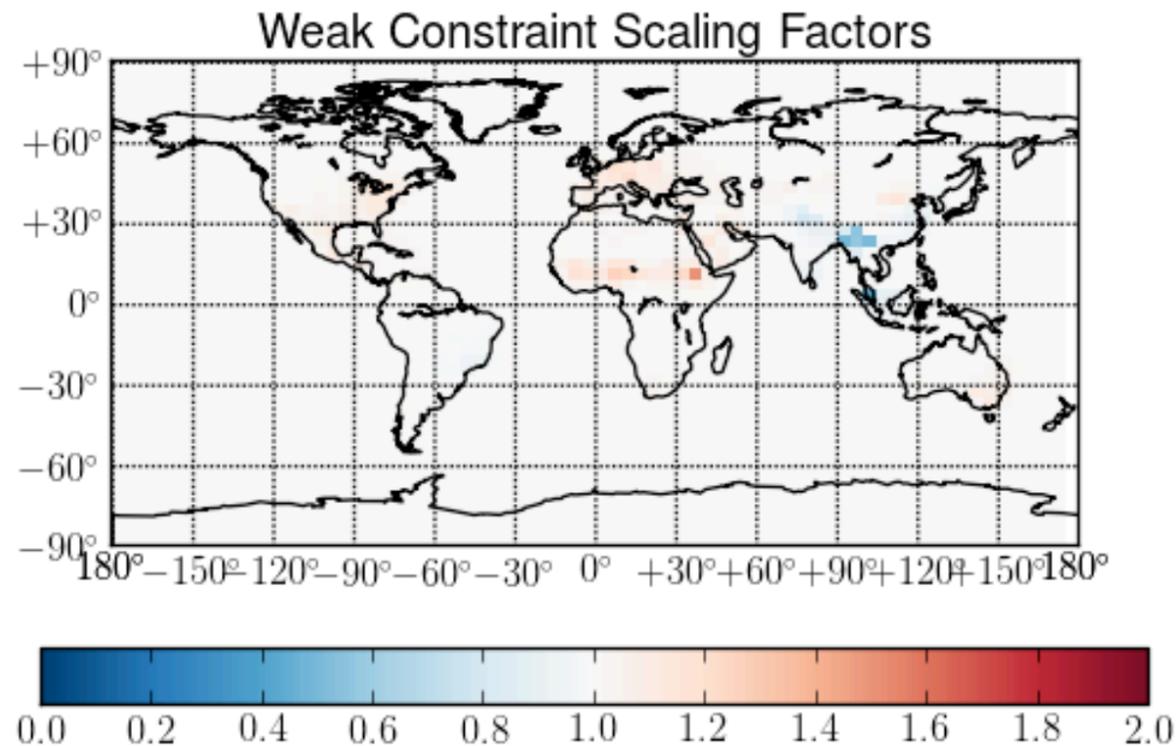
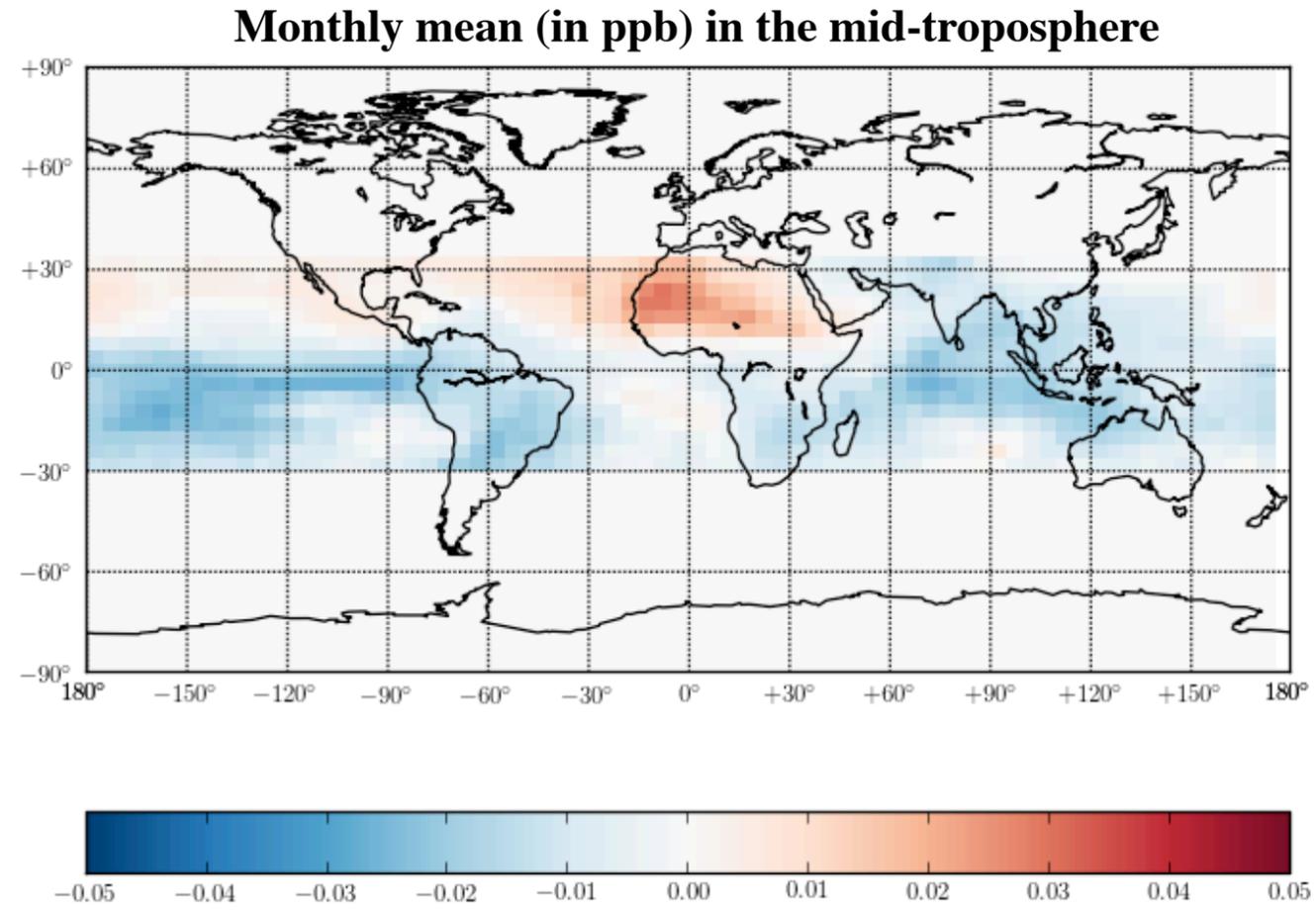
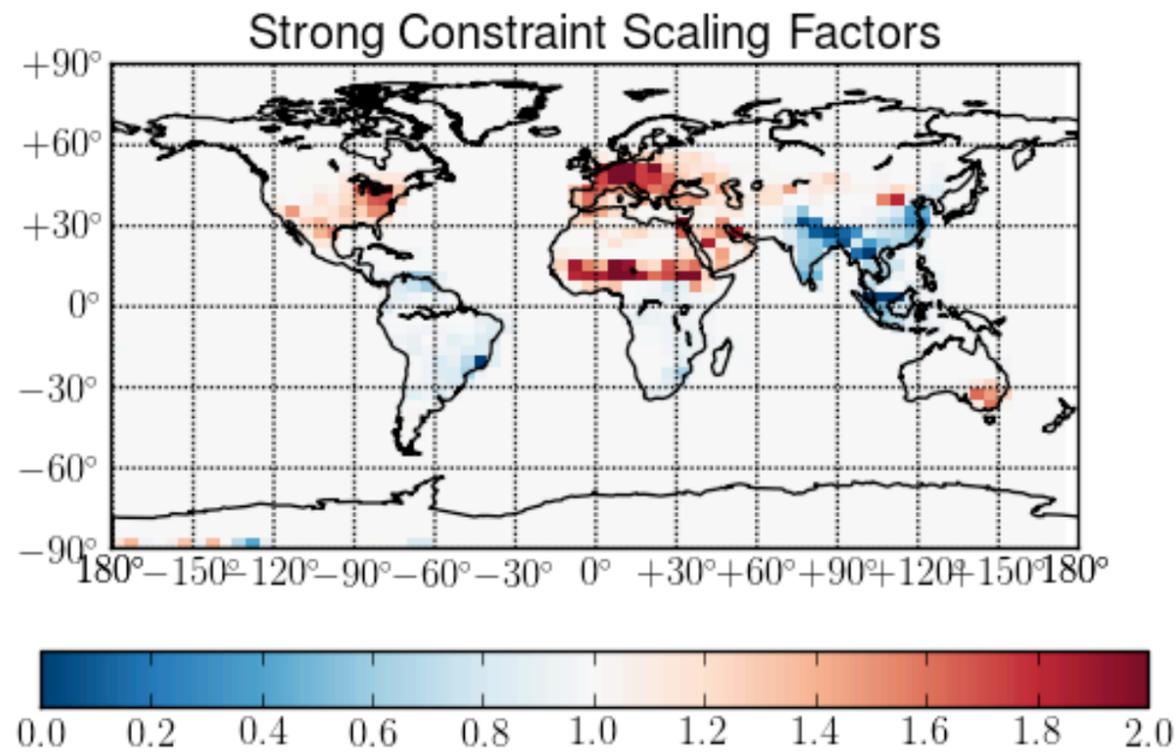


MOPITT observation count at 200hPa



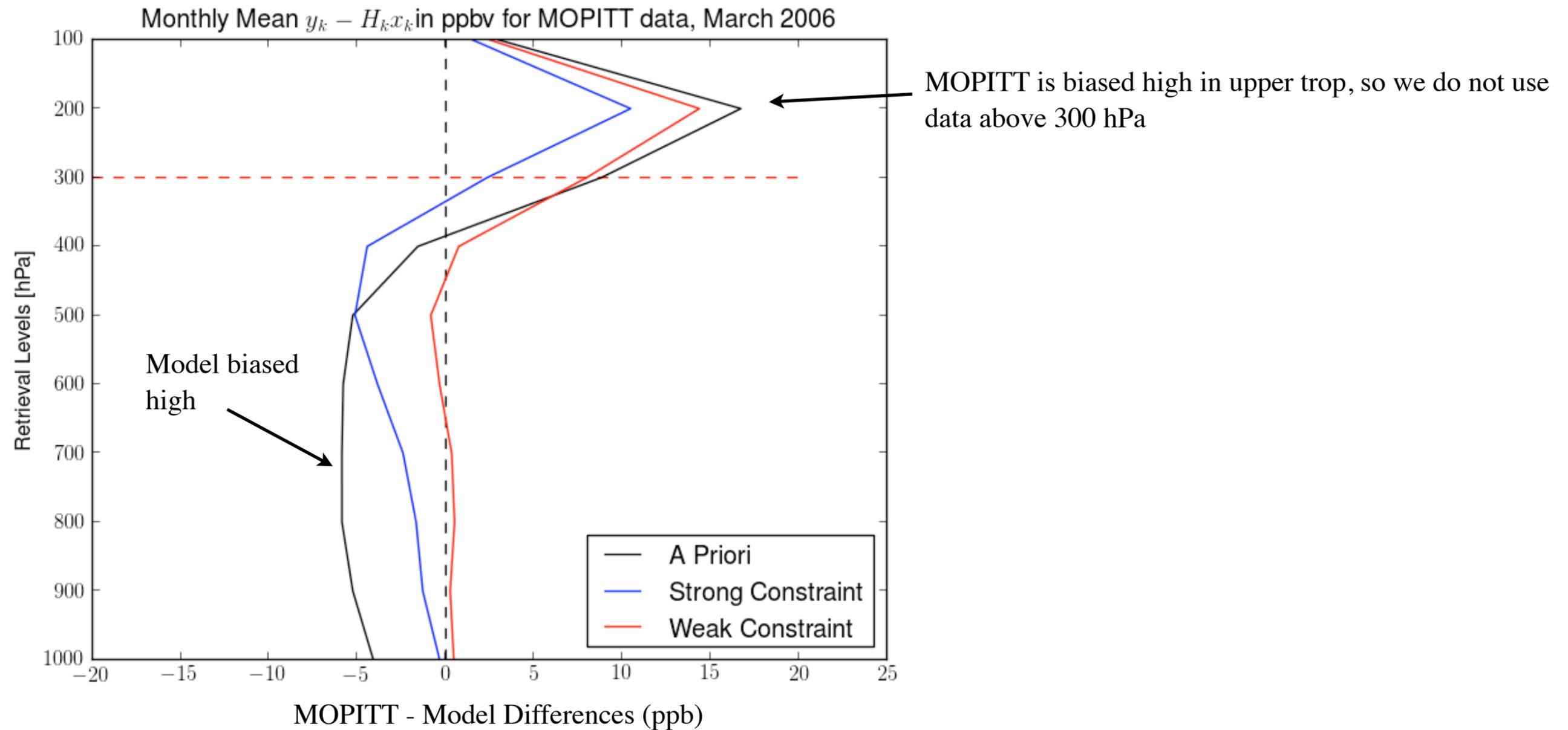
There are few MOPITT “observations” over tropical South America in March 2006, which would explain the weak forcing over South America to correct the CO bias there.

# Inversion Analysis of MOPITT Data in March 2006



- Strong constraint 4D-var suggests large increases in CO emissions in North America, Europe, and northern Africa, and large decreases in Asia.
- Weak constraint 4D-var produced more modest changes in the emissions.
- The model forcing is negative across much of tropics, which is difficult to interpret.

# Inversion Analysis of MOPITT Data: Residual bias in the tropics



- Strong constraint 4D-var significantly reduced the bias near the surface, but slightly enhanced the bias near 400 hPa.
- Weak constraint 4D-var significantly reduced the bias throughout the lower and middle troposphere.
- A concern is that the inversion could be overfitting the data.