

## PROBLEMS

### 5. 1 Turbulent diffusion coefficient

Radon-222 is widely used as a tracer of vertical transport in the troposphere. It is emitted ubiquitously by soils and is removed from the atmosphere solely by radioactive decay ( $k = 2.1 \times 10^{-6} \text{ s}^{-1}$ ). We consider in this problem a continental atmosphere where  $^{222}\text{Rn}$  concentrations are horizontally uniform. Vertical transport is parameterized by a turbulent diffusion coefficient  $K_z$ .

1. Write a one-dimensional continuity equation (Eulerian form) for the  $^{222}\text{Rn}$  concentration in the atmosphere as a function of altitude.
2. An analytical solution to the continuity equation can be obtained by assuming that  $K_z$  is independent of altitude, that  $^{222}\text{Rn}$  is at steady state, and that the mixing ratio  $C$  of  $^{222}\text{Rn}$  decreases exponentially with altitude:

$$C(z) = C(0)e^{-z/h}$$

Observations show that this exponential dependence on altitude, with a scale height  $h = 3 \text{ km}$ , provides a reasonably good fit to average vertical profiles of  $^{222}\text{Rn}$  concentrations. Show that under these conditions

$$K_z = \frac{kh}{\frac{1}{H} + \frac{1}{h}} = 1.3 \times 10^5 \text{ cm}^2 \text{ s}^{-1}$$

where  $H$  is the scale height of the atmosphere.

3. The mean number density of  $^{222}\text{Rn}$  measured in surface air over continents is  $n(0) = 2 \text{ atoms cm}^{-3}$ . Using the assumptions from question 2, calculate the emission flux of  $^{222}\text{Rn}$  from the soil.
4. The mean residence time of water vapor in the atmosphere is 13 days (section 3.1.1). Using the same assumptions as in question 2, estimate a scale height for water vapor in the atmosphere.

[To know more: Liu, S.C., et al., Radon 222 and tropospheric vertical transport, *J. Geophys. Res.*, 89, 7291-7297, 1984.]