Economics 2010c: Lecture 6
Quasi-hyperbolic discounting

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Outline:

1. Hyperbolic Discounting

2. Hyperbolic Euler Equation

3. Lifecycle simulations

4. Method of Simulated Moments (MSM)

1 Hyperbolic Discounting

Check the item below that you would most prefer.

1. 15 minute free massage now

2. 20 minute free massage in an hour

Check the item below that you would most prefer.

1. 15 minute free massage in 168 hours

2. 20 minute free massage in 169 hours

Read and van Leeuwen (1998): Food experiment.

- Choose for Next Week: Fruit (74%) or Chocolate (26%)
- Choose for Today: Fruit (30%) or Chocolate (70%).

Read, Loewenstein & Kalyanaraman (1999): Video experiment

- Choose for Next Week: Low-brow (37%) or High-brow (63%)
- Choose for Today: Low-brow (66%) or High-brow (34%).

Evidence from gyms (Della Vigna and Malmendier 2004).

- Average cost of gym membership: \$75 per month.

- Average number of visits per month: 4.

- Average cost per visit: \$19.

- Cost of “pay-per-visit:” \$10.

- Quasi-hyperbolic discounting (Phelps and Pollak 1968, Laibson 1997): 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ...

$$U_t = u(c_t) + \beta\delta u(c_{t+1}) + \beta\delta^2 u(c_{t+2}) + \beta\delta^3 u(c_{t+3}) + ...$$

- For exponentials: $\beta = 1$

$$U_t = u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + ...$$

- For "quasi-hyperbolics": $\beta < 1$

$$U_t = u(c_t) + \beta \left[\delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + ...\right]$$

- To build intution, assume that $\beta \simeq \frac{1}{2}$ and $\delta \simeq 1$

$$\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...\}$$

$$U_t = u(c_t) + \frac{1}{2}\left[u(c_{t+1}) + u(c_{t+2}) + u(c_{t+3}) + ...\right]$$

- Relative to the current period, all future periods are worth less (weight $\frac{1}{2}$).

- Most (for this example, *all*) of the discounting takes place between the current period and the immediate future.

- There is little (for this example, *no*) additional discounting between future periods.

$$U_t = u(c_t) + \frac{1}{2}\left[u(c_{t+1}) + u(c_{t+2}) + u(c_{t+3}) + ...\right]$$

- Preferences are dynamically inconsistent.

- At date t we prefer to be patient between $t+1$ and $t+2$.

- At date $t+1$ we want immediate gratification at $t+1$.

$$U_{t+1} = u(c_{t+1}) + \frac{1}{2}\left[u(c_{t+2}) + u(c_{t+3}) + u(c_{t+4}) + ...\right]$$

Akerlof (1992) and O'Donoghue and Rabin (1999) on procrastination:

- Assume $\beta = \frac{1}{2}$ and $\delta = 1$.

- Suppose exercise (cost 6) generates delayed benefits (value 8).

- Exercise Today? $\quad -6 + \frac{1}{2}[8] = -2 < 0$

- Exercise Tomorrow? $\quad 0 + \frac{1}{2}[-6 + 8] = 1 > 0$

- Agent would like to exercise tomorrow.

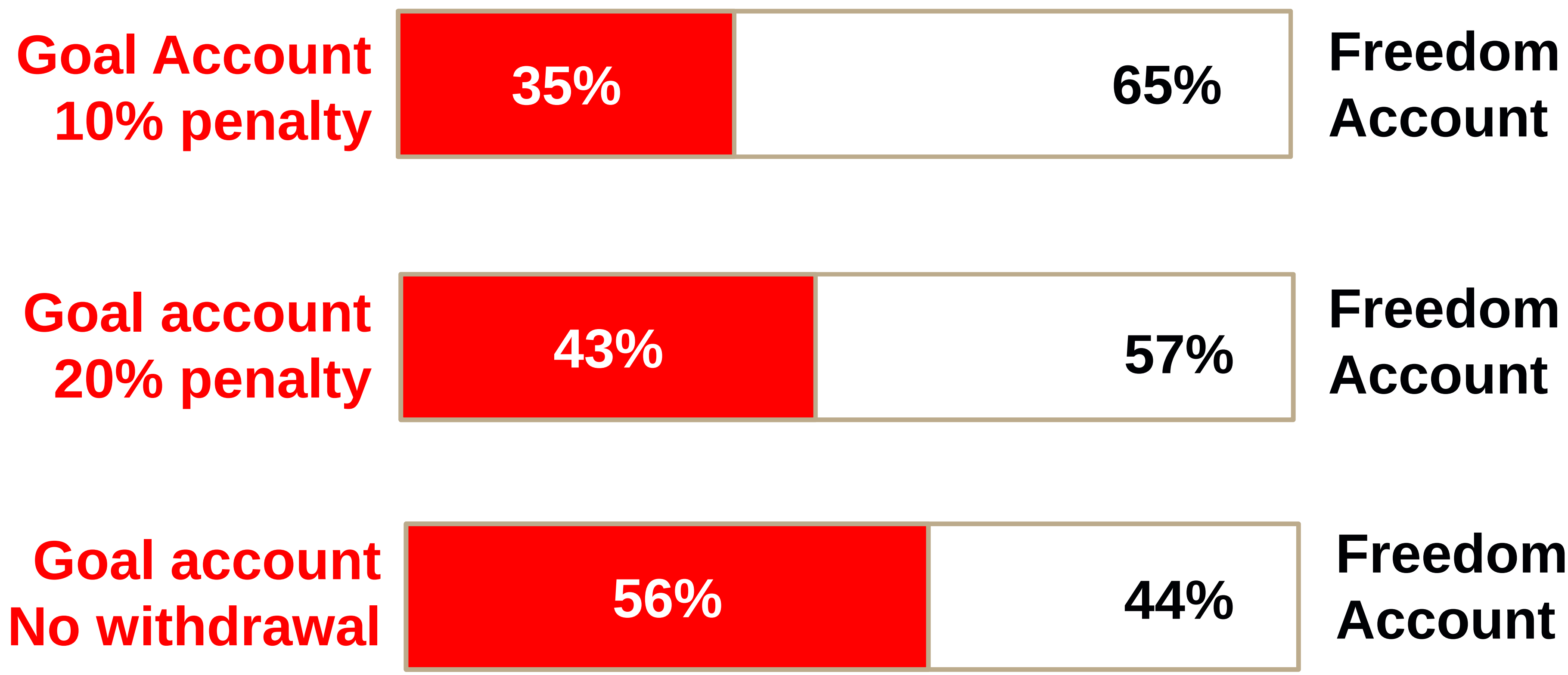
Predictions:

- Procrastination when costs precede benefits (Della Vigna and Malmendier 2004, 2006; Ariely and Wertenbroch 2002; Augenblick, Niederle, and Sprenger 2013)

- Downward sloping consumption paths within pay-cycle (e.g., Shapiro 2005, Mastrobuoni and Weinberg 2009)

- Willingness to use commitment: savings (Ashraf, Karlan, and Yin, 2006; Beshears et al 2013), student productivity (Ariely and Wertenbroch, 2002; Houser et al., 2010, Chow 2011; Augenblick, Niederle, and Sprenger, 2013), cigarette smoking (Gine, Karlan, and Zinman, 2010), workplace productivity (Kaur, Kremer, and Mullainathan, 2010), and exercise (Milkman, Minson, and Volpp, 2012; Royer, Stehr, and Sydnor, 2012).

Goal account usage
(Beshears et al 2013)



2 Hyperbolic Euler Equation

- Let c represent consumption.

- Let x represent cash-on-hand.

- Let $\tilde{y}$ represent iid stochastic income.

- Let R represent gross interest rate.

- So $x_{t+1} = R(x_t - c_t) + \tilde{y}_{t+1}$.

- A (Markov) strategy is a map from state x to control c.

- Let V be the continuation-value function, W be the current-value function and C be the consumption function. Then:

$$\begin{aligned} V(x) &= U(C(x)) + \delta \, \mathsf{E}[V(R(x - C(x)) + y)] \\ W(x) &= U(C(x)) + \beta\delta \, \mathsf{E}[V(R(x - C(x)) + y)] \\ C(x) &= \operatorname*{argmax}_c U(c) + \beta\delta \, \mathsf{E}[V(R(x - c) + y)] \end{aligned}$$

- Note that V accumulates utils exponentially.

- Note that W accumulates utils quasi-hyperbolically.

- Envelope Theorem.

$$W'(x) = U'(C(x))$$

- First-order-condition.

$$U'(C(x)) = R\beta\delta \, \mathsf{E}\Big[V'(R(x - C(x)) + y)\Big]$$

- Identity linking V and W.

$$\beta V(x) = W(x) - (1 - \beta)U(C(x))$$

2.1 Problem is recursive

- Start with V.

- Find C:

$$C(x) = \underset{c}{\text{argmax}}\, U(c) + \beta\delta\, \mathsf{E}[V(R(x-c)+y)]\,.$$

- Find $\hat{V}$:

$$\hat{V}(x) = U(C(x)) + \delta\, \mathsf{E}[V(R(x-C(x))+y)]$$

- In this way, generate an operator $T : V \mapsto \hat{V}$.

2.2 Can also derive an Euler Equation

We have

$$\begin{aligned} u'(c_t) &= R\beta\delta \, \mathsf{E}_t \Big[V'(x_{t+1}) \Big] \\ &= R\delta \, \mathsf{E}_t \left[W'(x_{t+1}) - (1-\beta) u'(c_{t+1}) \frac{dC_{t+1}}{dx_{t+1}} \right] \\ &= R\delta \, \mathsf{E}_t \left[u'(c_{t+1}) - (1-\beta) u'(c_{t+1}) \frac{dC_{t+1}}{dx_{t+1}} \right]. \end{aligned}$$

So,

$$u'(c_t) = R \, \mathsf{E}_t \left[\beta\delta \left(\frac{dC_{t+1}}{dX_{t+1}} \right) + \delta \left(1 - \frac{dC_{t+1}}{dX_{t+1}} \right) \right] u'(c_{t+1}).$$

See Harris and Laibson (2003).

3 Lifecycle simulations (Angeletos et al 2001)

3.1 Demographic Assumptions

- Mortality
- Retirement (PSID)
- Dependents (PSID)
- Three educational groups: NHS, HS, COLL
- Stochastic labor income (PSID)

3.2 Dynamic Budget Constraints

- Credit limit: $(.30)(\bar{Y}_n)$. (Calibrate from SCF.)

- Real after-tax rate of return: 3.75%

- Real rate of return on illiquid investment: 5.00%

- Real credit card interest rate: 11.75%

- State variables: liquid wealth, illiquid wealth, autocorrelated income.

- Choice variables: liquid wealth investment, illiquid wealth investment.

3.3 Preferences

- Instantaneous utility function: CRRA=2.

- Quasi-hyperbolic discounting: 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ...

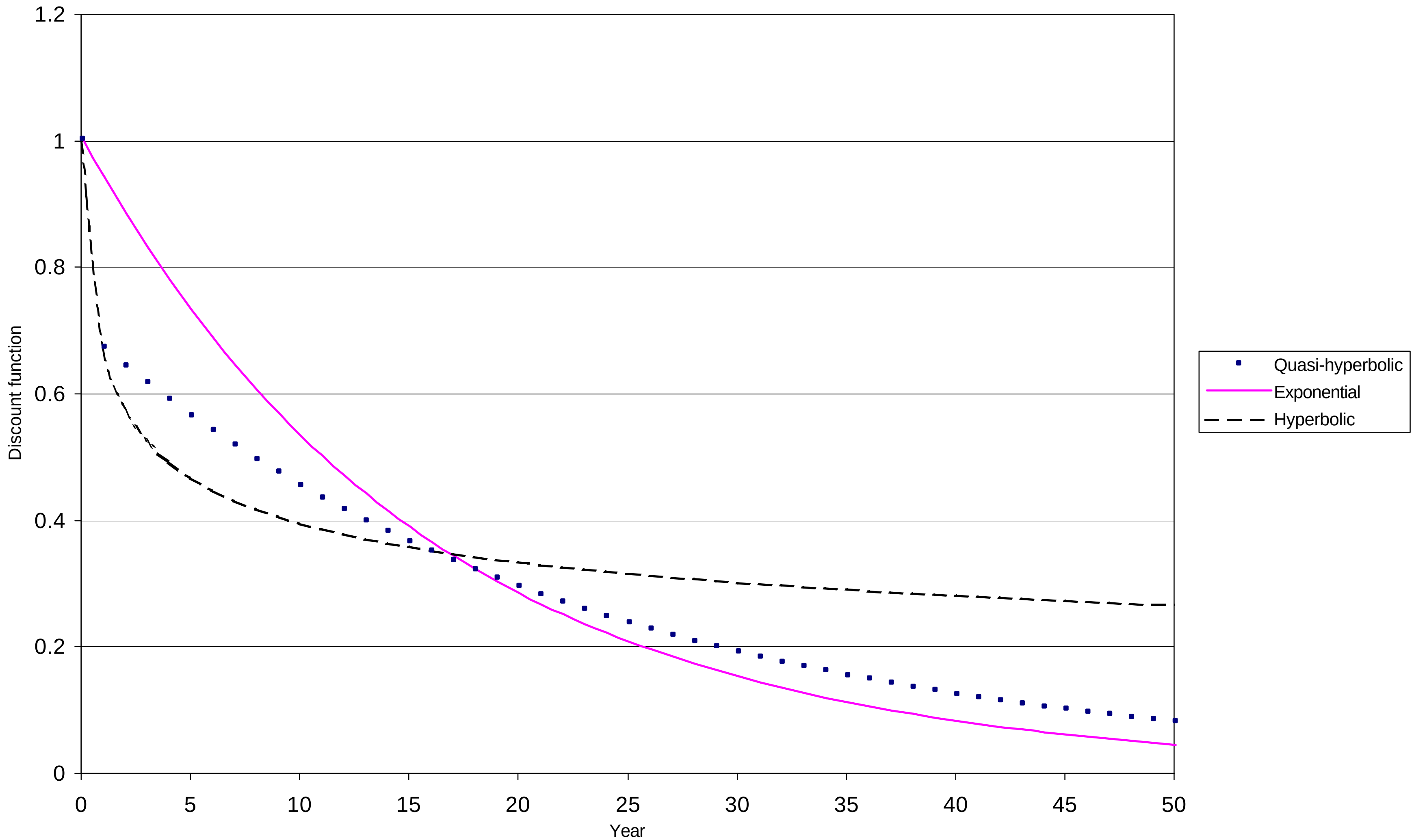
- For exponentials: $\beta = 1$. For hyperbolics: $\beta = 0.7$.

- For exponentials: $\delta = \delta^{\text{Exponential}}$. For hyperbolics: $\delta = \delta^{\text{Hyperbolic}}$.

- Calibration: Pick value of $\delta^{\text{Exponential}}$ (0.94) that matches empirical retirement wealth: median 'wealth to income ratio' ages 50-59.

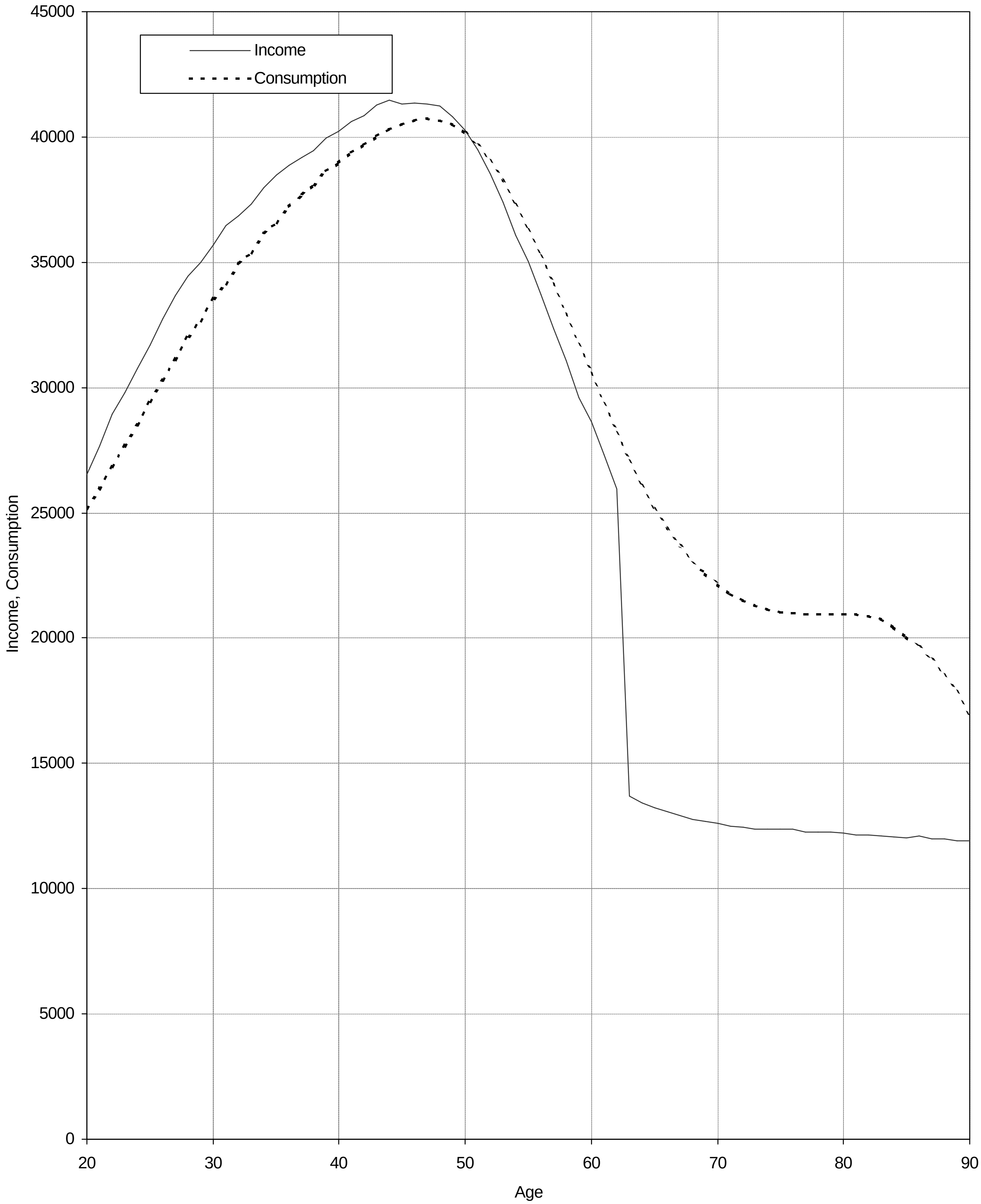
- Pick $\delta^{\text{Hyperbolic}}$ the same way (0.96).

Figure1: Discount functions



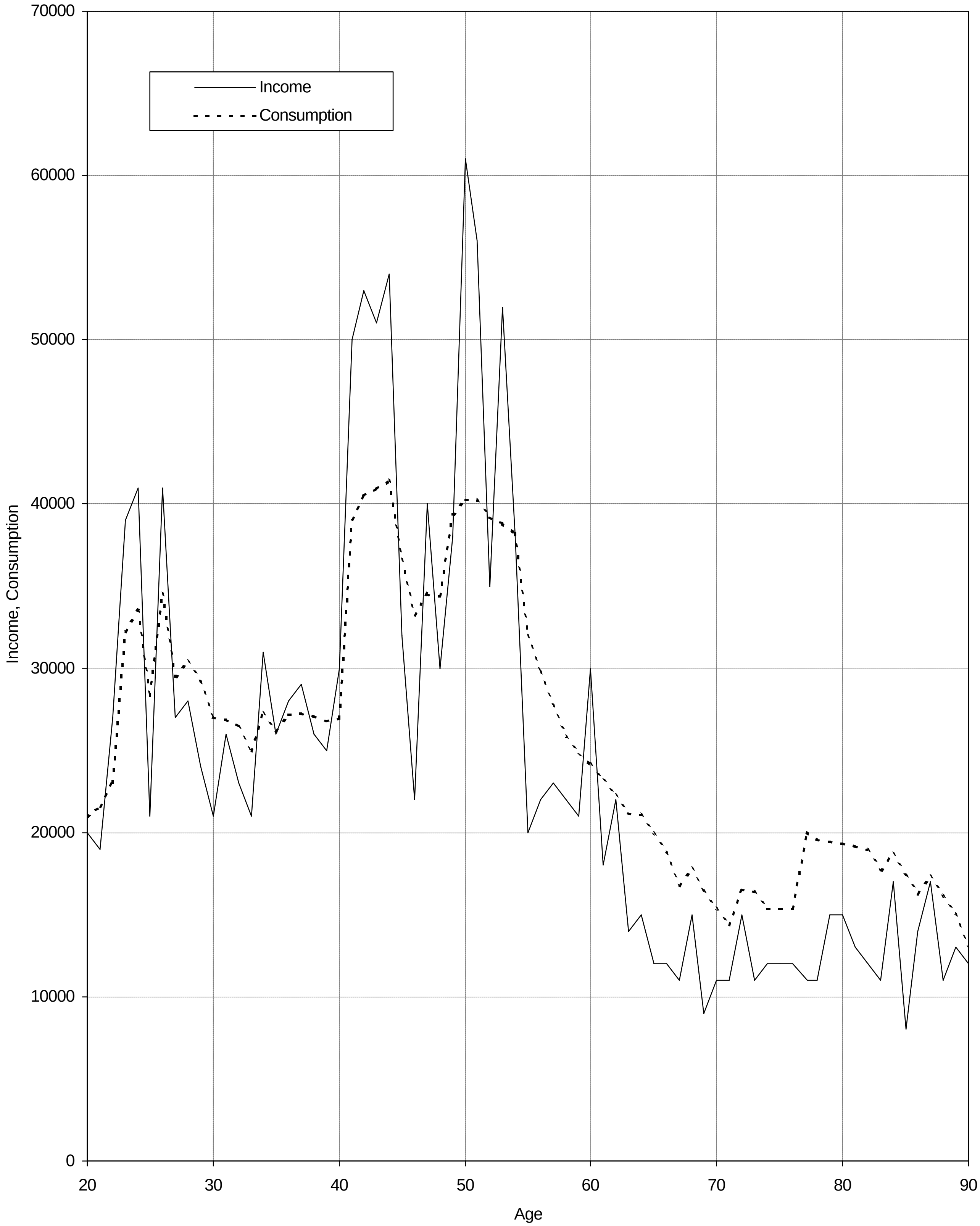
Source: Authors' calculations. Exponential: δ^t, with δ=0.939; hyperbolic: $(1+\alpha t)^{-\gamma/\alpha}$, with α=4 and γ=1; and quasi-hyperbolic: $\{1,\beta\delta,\beta\delta^2,\beta\delta^3,...\}$, with β=0.7 and δ=0.957.

Figure 2: Simulated Mean Income and Consumption of Exponential Households



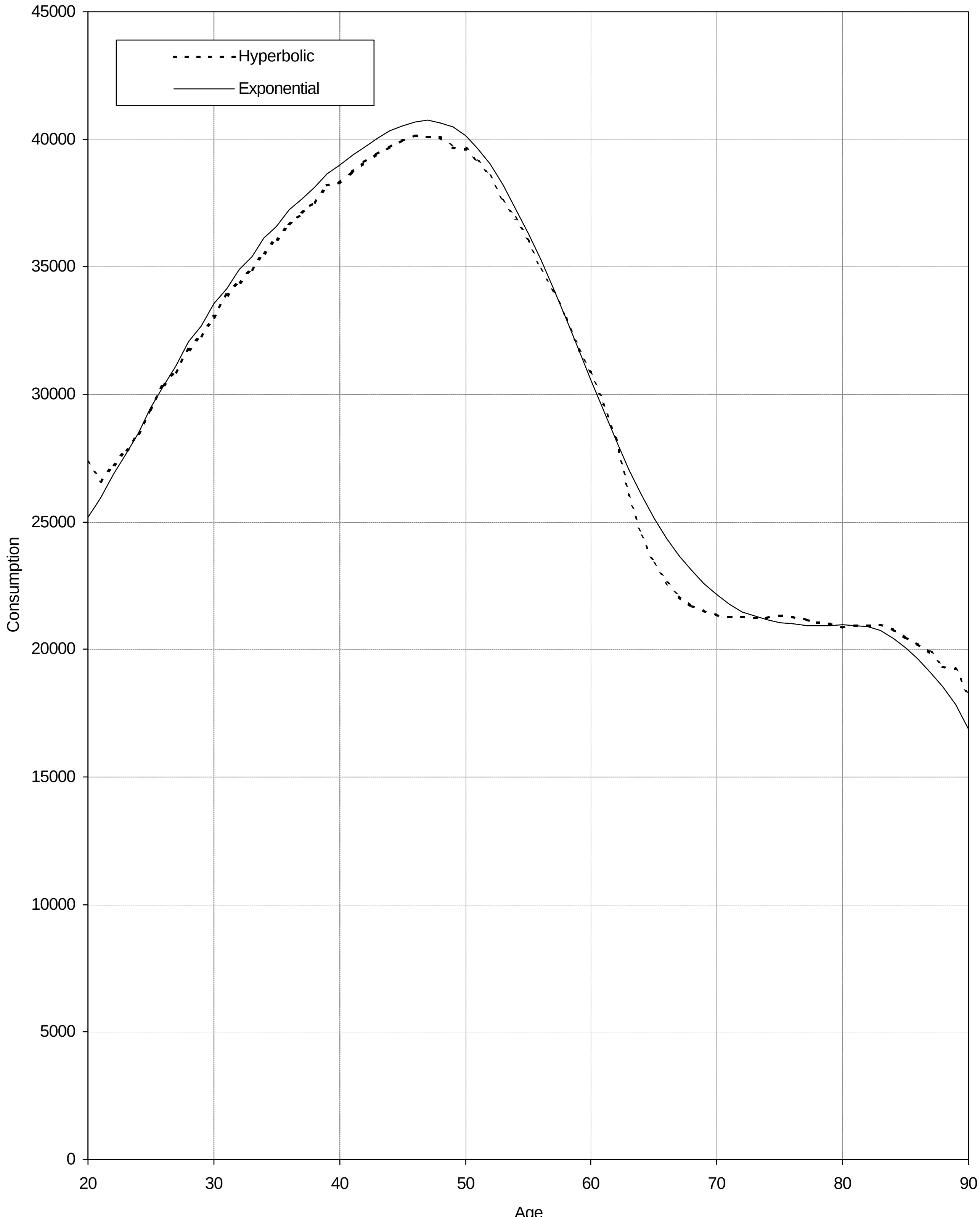
Source: Authors' simulations.
The figure plots the simulated average values of consumption and income for households with high school graduate heads.

Figure 3: Simulated Income and Consumption of a Typical Exponential Household



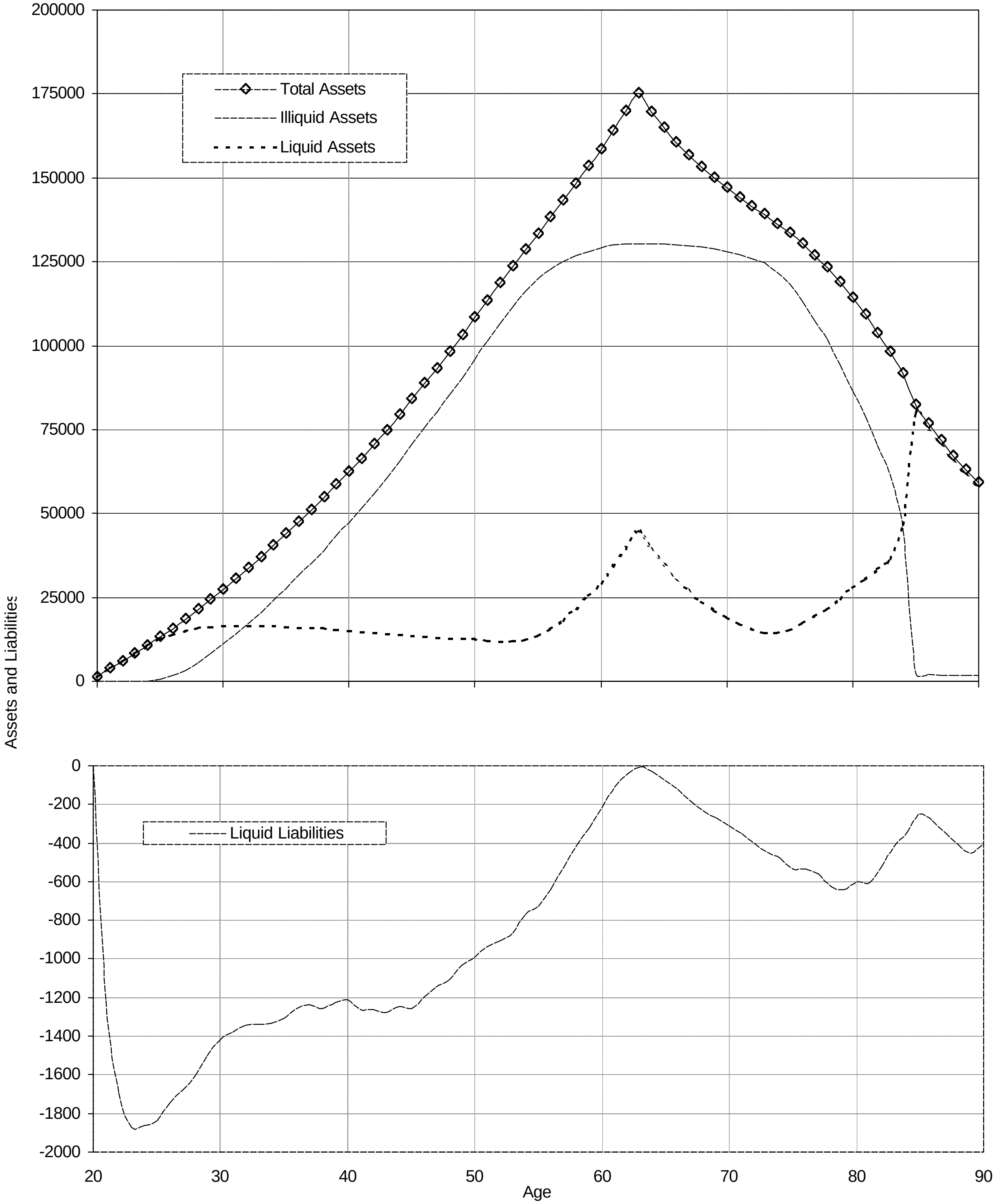
Source: Authors' simulations.
The figure plost the simulated life-cycle profiles of consumption and income for a typical household with a high school graduate head.

Figure 4: Mean Consumption of Exponential and Hyperbolic Households



Source: Author's simulations.
The figure plots average consumption over the life-cycle for simulated exponential and hyperbolic households with high-school graduate heads.

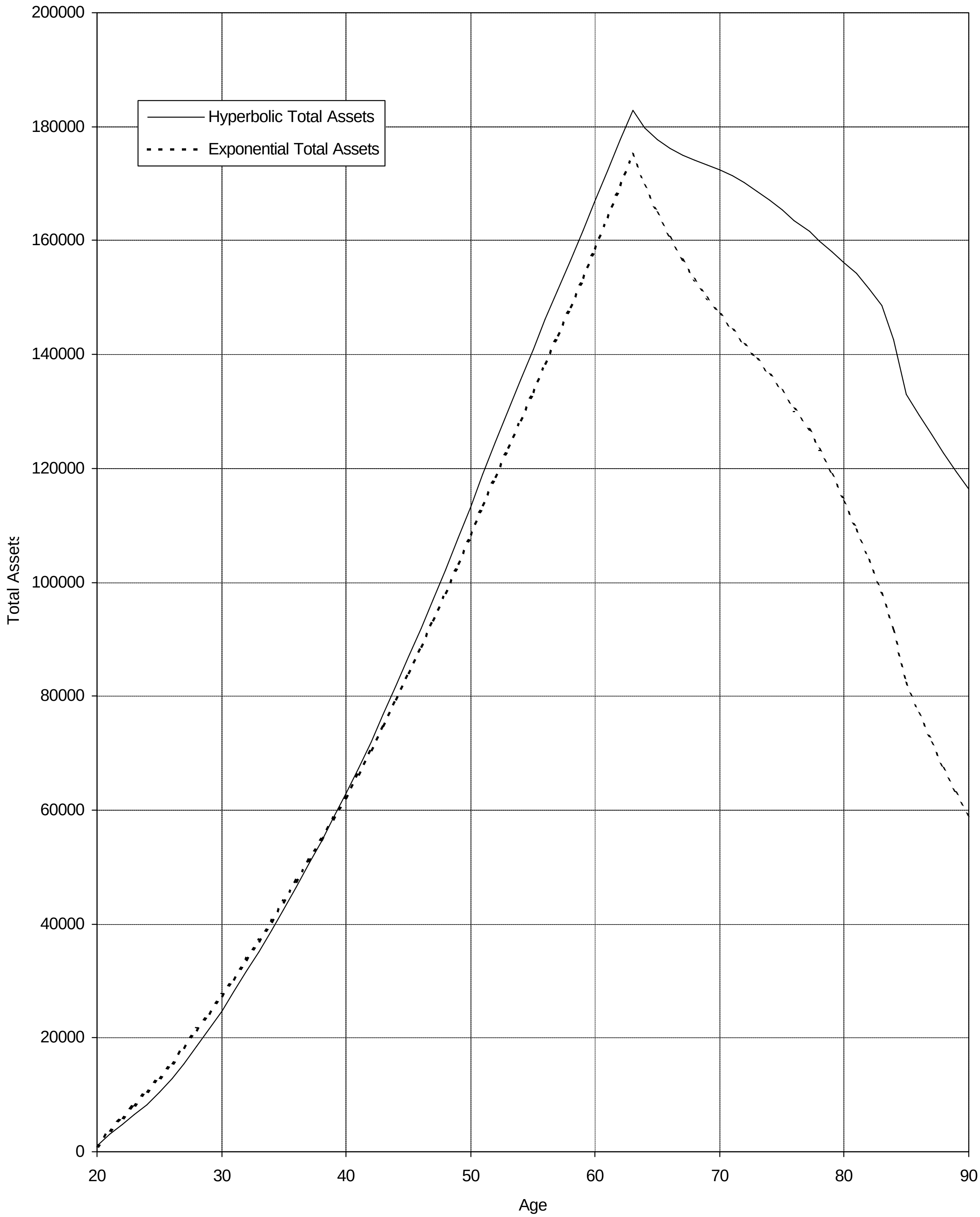
Figure 5: Simulated Total Assets, Illiquid Assets, Liquid Assets, and Liquid Liabilities for Exponential Consumers



Source: Authors' simulations.

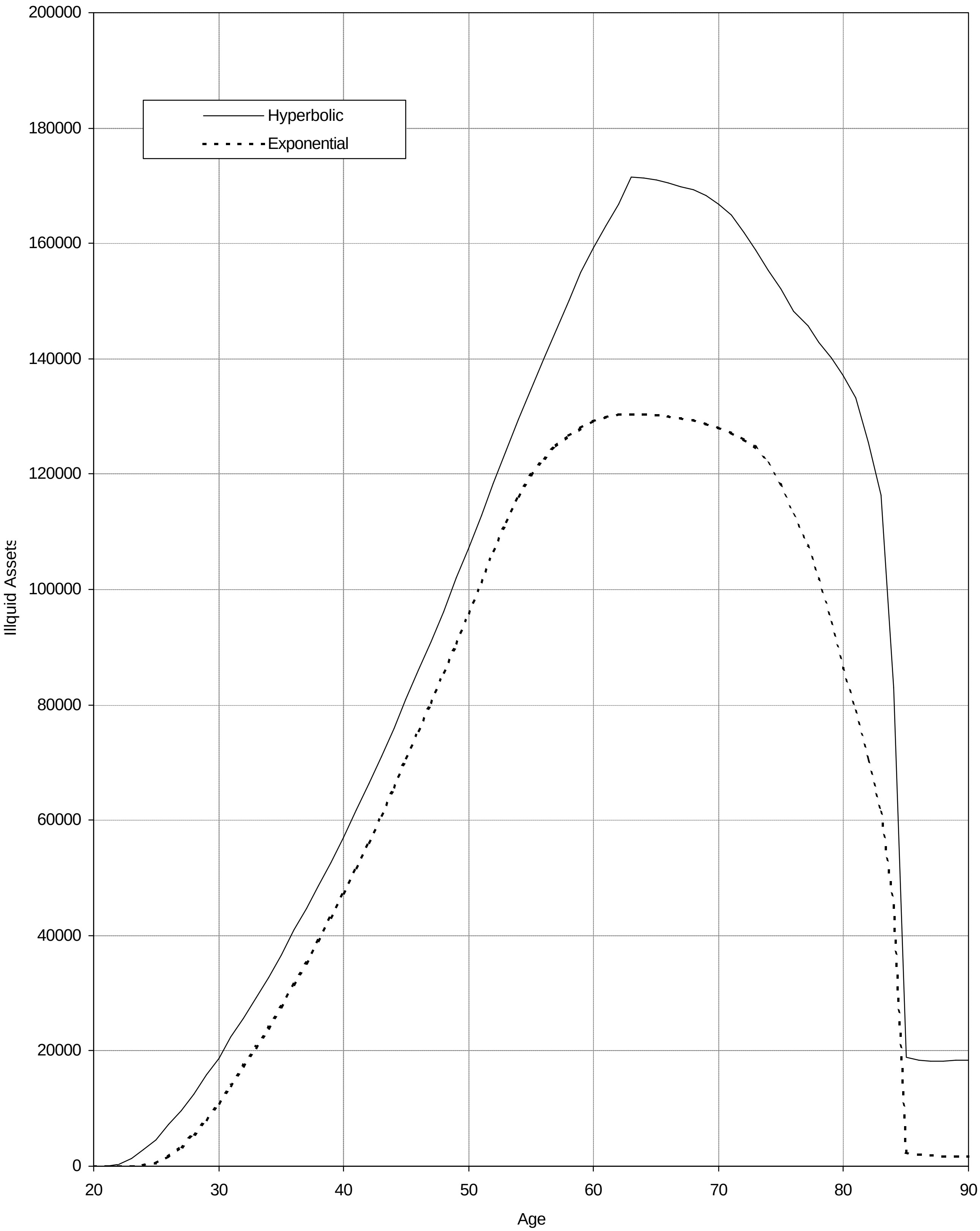
The figure plots the simulated mean level of liquid assets (excluding credit card debt), illiquid assets. total assets, and liquid liabilities for households with high school graduate heads.

Figure 6: Mean Total Assets of Exponential and Hyperbolic Households



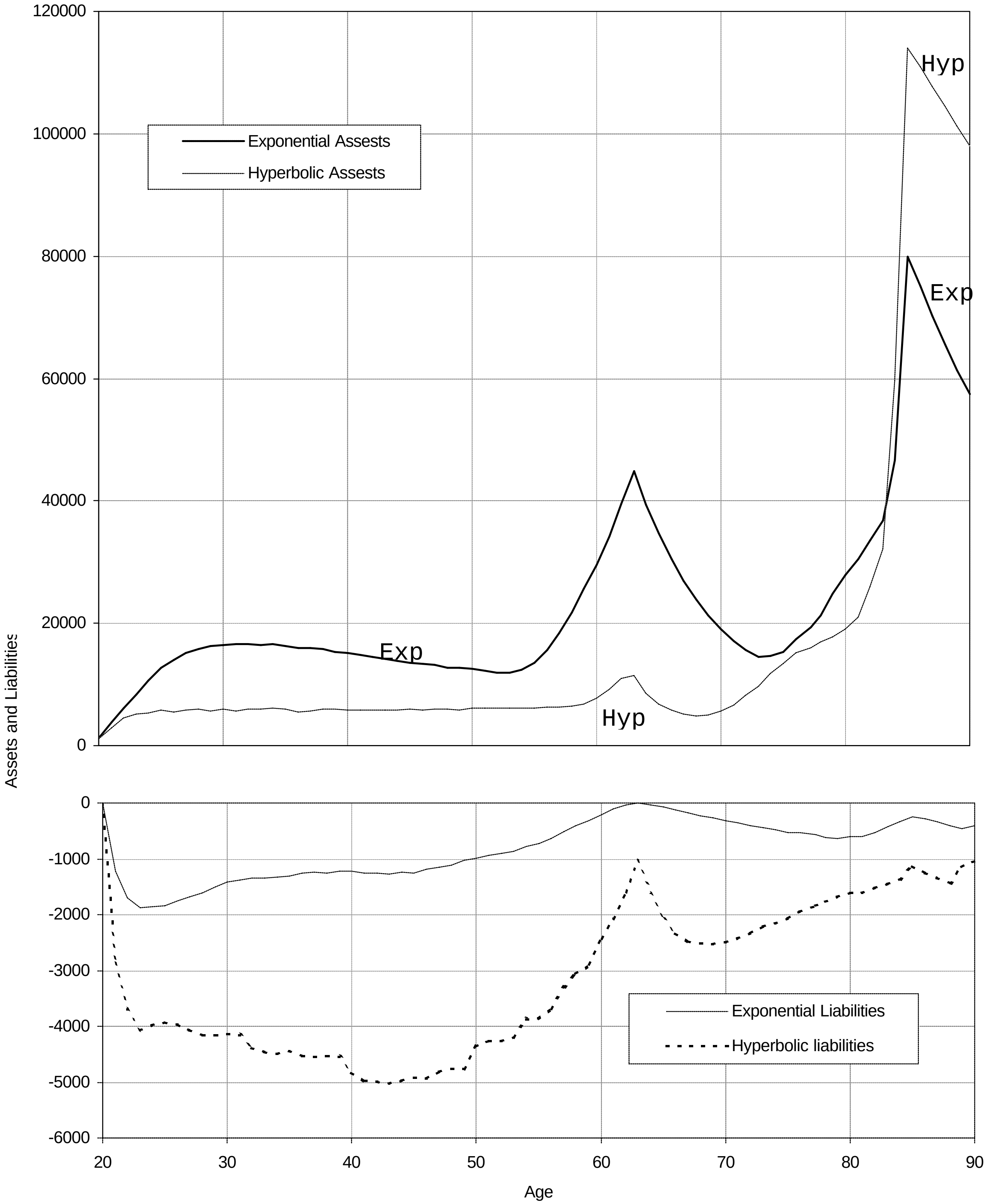
Source: Author's simulations.
The figure plots mean total assets, excluding credit card debt, over the life-cycle for simulated exponential and hyperbolic households with high school graduate heads.

Figure 7: Mean Illiquid Wealth of Exponential and Hyperbolic Households



Source: Authors' simulations.
The figure plots average illiquid wealth over the life-cycle for simulated exponential and hyperbolic households with high school graduate heads.

Figure 8: Mean Liquid Assets and Liabilities of Exponential and Hyperbolic Households



Source: Authors' simulations.
The figure plots average liquid assets (liquid wealth excluding credit card debt) and liabilities (credit card debt) over the life-cycle for simulated exponential and hyperbolic households with high school graduate heads.

	Exponential	Hyperbolic	Empirical Data
% with $\frac{\text{liquid}}{Y} > \frac{1}{12}$	73%	40%	42%
mean $\frac{\text{liquid assets}}{\text{liquid + illiquid assets}}$	0.50	0.39	0.08
% borrowing on "Visa"	19%	51%	70%
mean borrowing	\$900	\$3408	\$5000+
C-Y comovement	0.03	0.17	0.23

$$\Delta \ln(C_{it}) = \alpha E_{t-1} \Delta \ln(Y_{it}) + X_{it}\beta + \varepsilon_{it}$$

4 MSM Estimation (Laibson, Repetto, Tobacman 2008):

- Use the Method of Simulated Moments (Pakes and Pollard 1989).

- Pick parameter values to minimize the gap between simulated data and empirical data.

  - Substantial retirement wealth accumulation (SCF)

  - Extensive credit card borrowing (SCF, Fed, Gross and Souleles 2000, Laibson, Repetto, and Tobacman 2000)

  - Consumption-income comovement (Hall and Mishkin 1982, many others)

4.1 Data

Statistic	m_e	se_{m_e}
% borrowing on 'Visa'? (*% Visa*)	0.68	0.015
borrowing / mean income ($mean$ *Visa*)	0.13	0.01
C-Y comovement (*CY*)	0.23	0.11
median 50-59 $\frac{wealth}{income}$	3.88	0.25
weighted mean 50-59 $\frac{wealth}{income}$ (*wealth*)	2.60	0.13

4.2 Estimator

Estimate parameter vector θ and evaluate models wrt data.

- $m_e =$ N empirical moments
- $m_s(\theta) =$ analogous simulated moments
- $q(\theta) \equiv (m_s(\theta) - m_e) W^{-1} (m_s(\theta) - m_e)'$, a (scalar) loss function
- Minimize loss function: $\hat{\theta} = \arg\min_{\theta} q(\theta)$

- $\hat{\theta}$ is the MSM estimator.

- Pakes and Pollard (1989) prove asymptotic consistency and normality.

- Specification tests: $q(\hat{\theta}) \sim \chi^2(N - \#parameters)$

4.3 Results

- Exponential ($\beta = 1$) case:

$$\hat{\delta} = 0.846 \;\; (0.025); \qquad q\left(\hat{\delta}, 1\right) = 217$$

- Hyperbolic case:

$$\begin{cases} \hat{\beta} = 0.7031 \;\; (0.109) \\ \hat{\delta} = 0.958 \;\; (0.007) \end{cases} \qquad q\left(\hat{\delta}, \hat{\beta}\right) = 3.01$$

(Benchmark case: $\left[R^X, \gamma, R^{CC}\right] = [1.0375,\; 0.05,\; 1.1152]$)

Punchlines:

- β estimated significantly below 1.
- Reject $\beta = 1$ null hypothesis with a t-stat of 3.
- Specification tests reject only the exponential model.

Benchmark Model	Exponential	Hyperbolic	Data	Std err
Statistic:	$m_s(1,\hat{\delta})$ $\hat{\delta} = .846$	$m_s(\hat{\beta},\hat{\delta})$ $\hat{\beta} = .703$ $\hat{\delta} = .958$	m_e	se_{m_e}
% $Visa$	0.67	0.634	0.68	0.015
$mean\ Visa$	0.15	0.167	0.13	0.01
CY	0.293	0.314	0.23	0.11
$wealth$	-0.05	2.69	2.60	0.13
$q(\hat{\theta})$	217	3		