Problem 1: Solve the Merton Consumption problem assuming that

\[ u(c) = \ln(c). \]

Do not set \( \gamma = 1 \), as we did in class. Instead, solve the problem from scratch assuming \( u(c) = \ln(c) \). When you guess the value function, try this functional form:

\[ V(x) = \psi \ln(x) + \text{constant}. \]

Problem 2: True/false/uncertain. Quality of explanation determines grade.

1. The variance of \( x(t) - x(0) \) increases linearly with \( t \) if \( x \) is any Ito Process.
2. An Ito Process can’t be mean-reverting.
3. An Ito Process is not differentiable since it is not continuous.
Problem 3 [you’ll need the material from the lecture on October 7]: Recall the stopping problem from Lecture 10. The lecture notes show that the optimal threshold rule is

\[ x^* = -\frac{b^2}{a + \sqrt{a^2 + 2b^2 \gamma}} - \frac{a}{\gamma} \]

Show that \( x^* < 0 \). Show that the following limit conditions apply. Provide intuitive (economic) explanations for all of these results.

\[ x^*_{(a=0)} = -\frac{b}{\sqrt{2} \rho} < 0 \]
\[ \lim_{a \to \infty} x^* = -\infty \]
\[ \lim_{a \to -\infty} x^* = 0 \]
\[ \lim_{b \to 0} x^* = \begin{cases} -\frac{a}{\rho} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases} \]
\[ \lim_{b \to \infty} x^* = -\infty \]
\[ \lim_{\rho \to 0} x^* = \begin{cases} -\infty & \text{if } a \geq 0 \\ \frac{b^2}{2a} & \text{if } a < 0 \end{cases} \]
\[ \lim_{\rho \to \infty} x^* = 0 \]

Hint: you may need to apply L’Hopital’s rule for some of these results.
Problem 4 [you’ll need the material from the lecture on October 7] (Stopping problem with geometric Brownian motion): Consider a new variant of the stopping problem from lecture 10. Now assume that the price of the good evolves according to geometric Brownian motion.

\[ dx = axdt + bxdz. \]

In addition, assume that the instantaneous profitability of production is

\[ w(x, t) = x - c \]

where \( c \) is the (fixed) cost of production.

\[ \Omega(x, t) = 0 \]

Solve for the value function and the optimal threshold rule \( x^* \).