Bridging the gap between lattice models and TQFTs

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w/ Kyle Kawagoe

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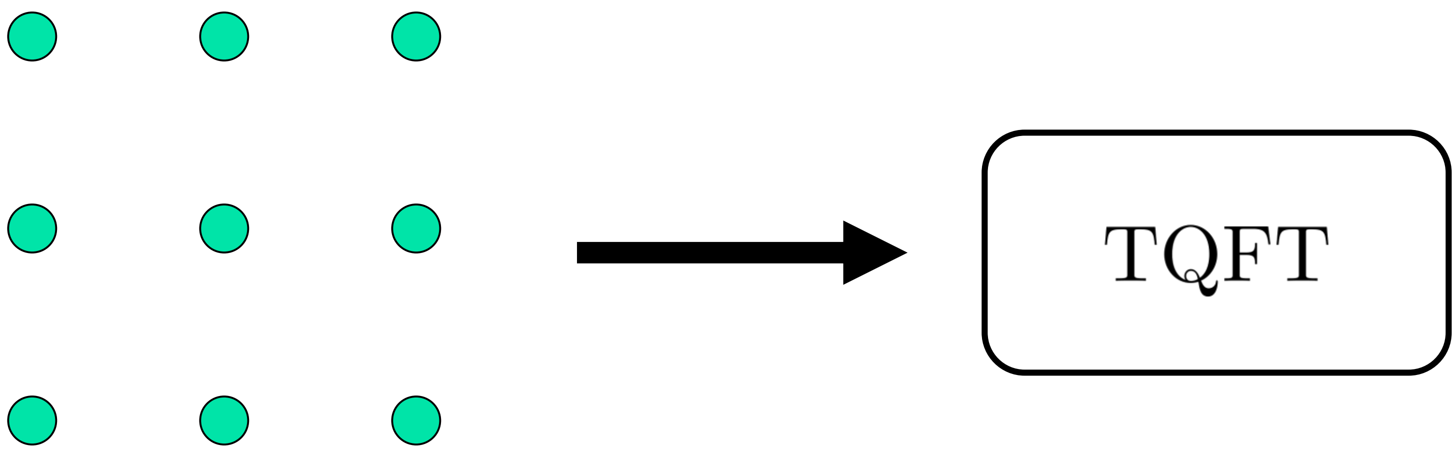
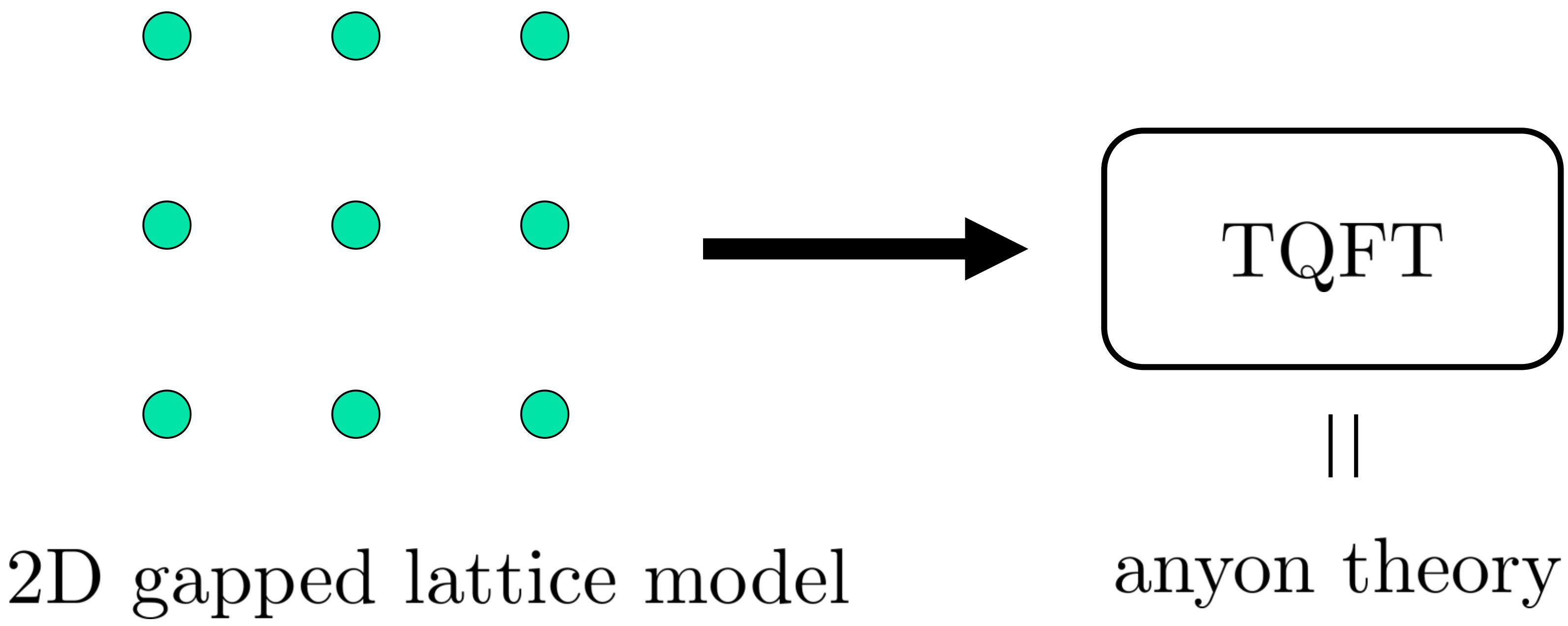
Lattice models and TQFTs



2D gapped lattice model

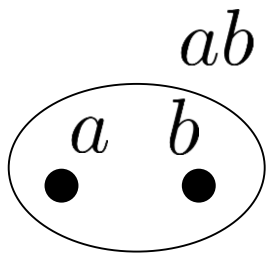
Lattice models and TQFTs



Data for (Abelian) anyon theories

- Anyon types: $\mathcal{C} = \{a, b, c, ...\}$

- Fusion rules: $a \times b = ab$



- Fusion/braiding data: $F(a,b,c),\ R(a,b) \in U(1)$

Main question: What is *microscopic* definition of anyon data?

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Focus of this talk: $F(a, b, c)$ ‘F-symbol’

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Focus of this talk: $F(a, b, c)$ ‘F-symbol’

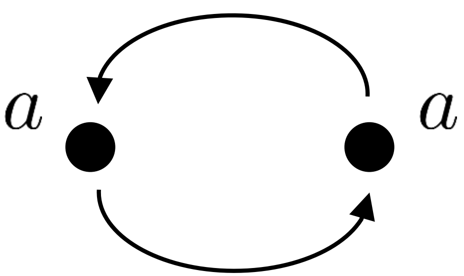
Motivation:

- Poorly understood quantity
- Application to anomaly computation

Warm up: microscopic definition of R(a,a)

$$R(a,a) = \text{exchange statistics of } a$$

Naive definition:



$$|\Psi\rangle \to R(a,a)|\Psi\rangle$$

Warm up: microscopic definition of R(a,a)

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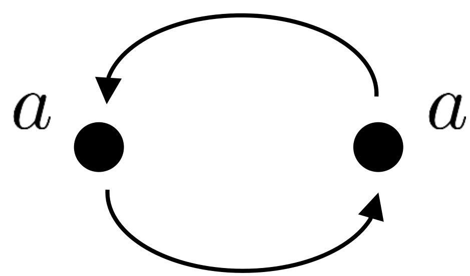
Naive definition:

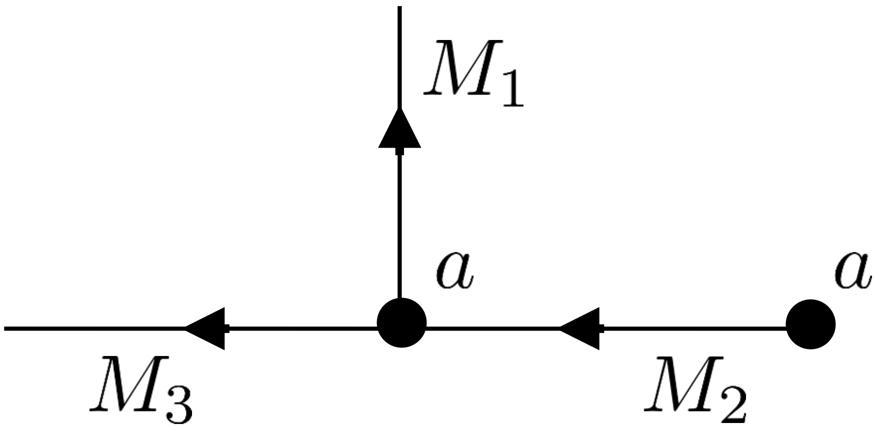


$$|\Psi\rangle \to R(a,a)|\Psi\rangle$$

Non-universal phases do not cancel

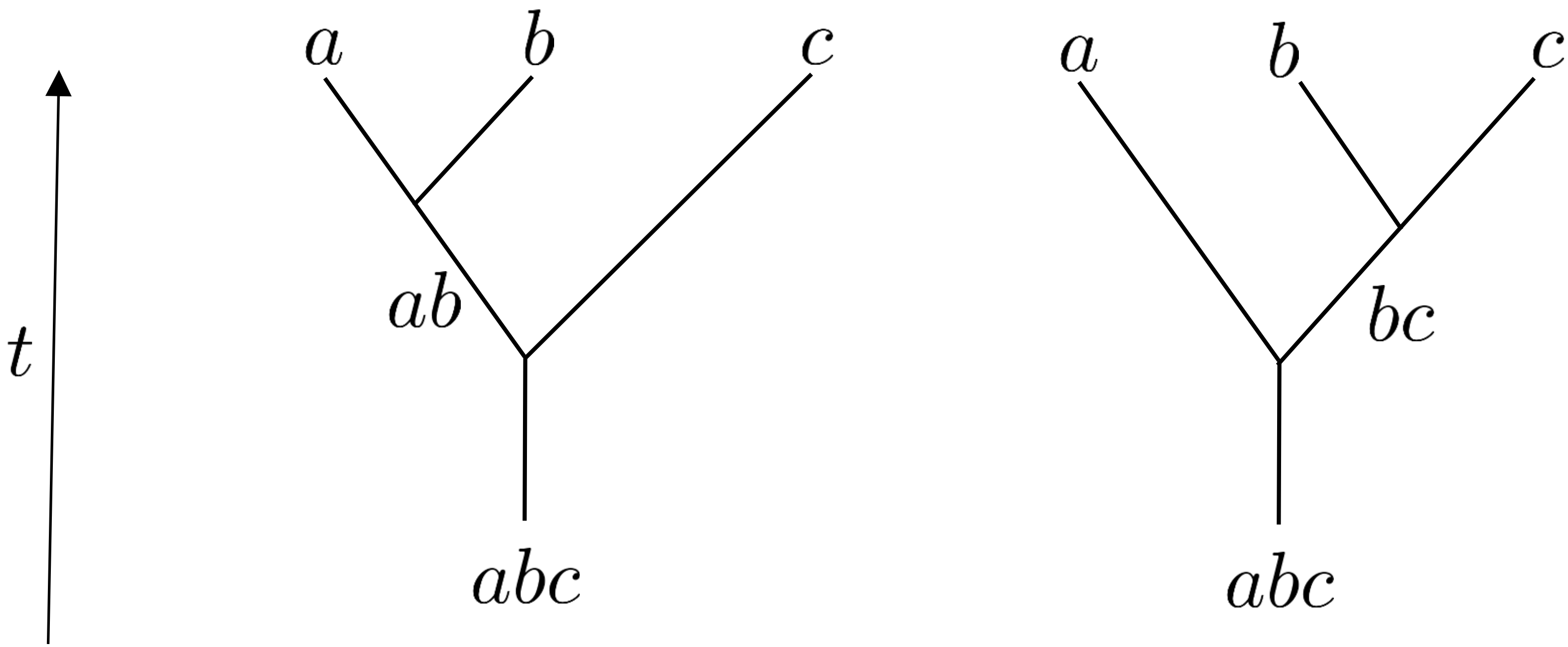
(ML, Wen, 2003)

Better definition:



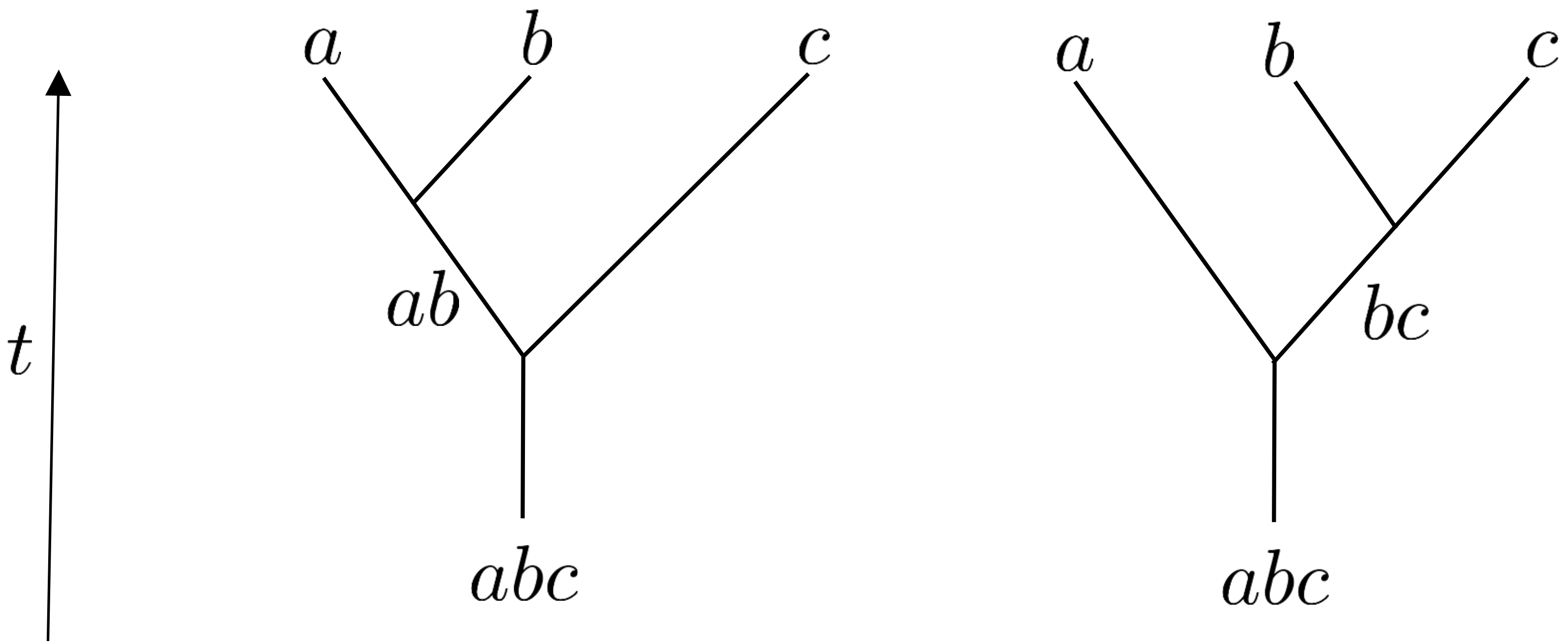
$$M_3 M_2 M_1 |\Psi\rangle = R(a, a) \cdot M_1 M_2 M_3 |\Psi\rangle$$

F-symbol: abstract picture



$|1\rangle$ $|2\rangle$

F-symbol: abstract picture



$$|1\rangle \; = \; F(a,b,c) \cdot \; |2\rangle$$

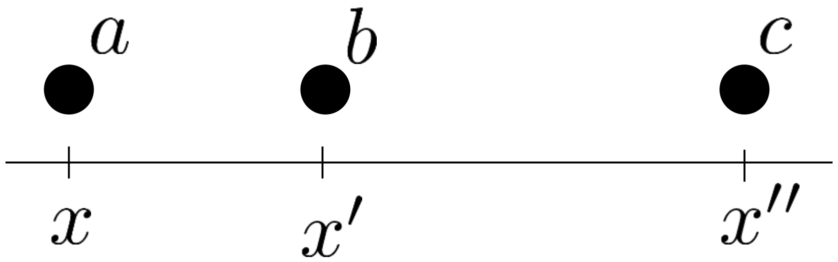
$F(a,b,c)$ well-defined up to:

$$F(a,b,c) \to F(a,b,c) \cdot \frac{e^{i\nu(a,b)}\, e^{i\nu(ab,c)}}{e^{i\nu(b,c)}\, e^{i\nu(a,bc)}}$$

'gauge transformations'

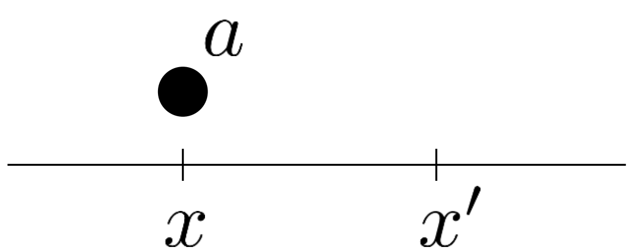
Microscopic definition of F-symbol

Arrange anyons along line:



Label as: $|a_x, b_{x'}, c_{x''}, \ldots\rangle$

Movement operator: $M^a_{x'x}$



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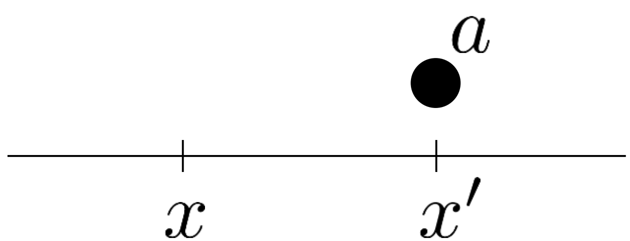

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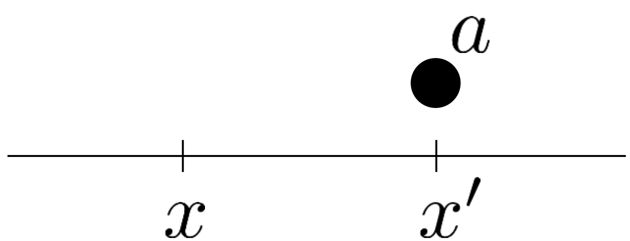

$$M^a_{x'x}|...a_x...\rangle \propto |...a_{x'}...\rangle$$

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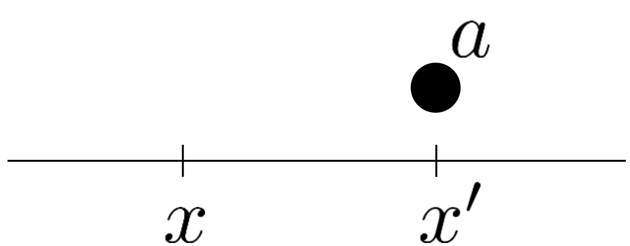
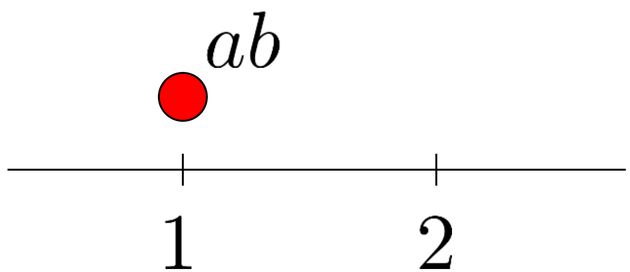


$$M^{a}_{x'x}|\ldots a_x \ldots\rangle \propto |\ldots a_{x'} \ldots\rangle$$

Splitting operator: $S(a,b)$



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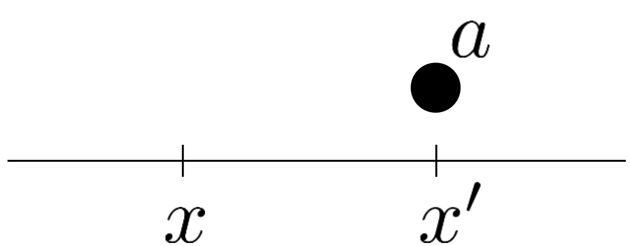

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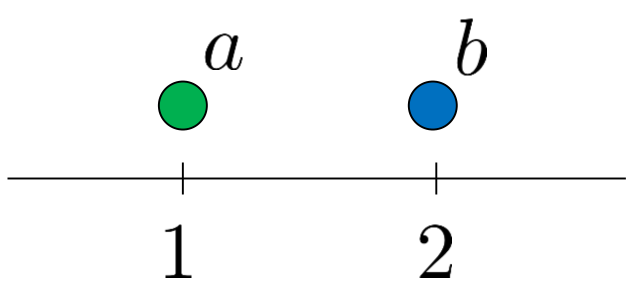

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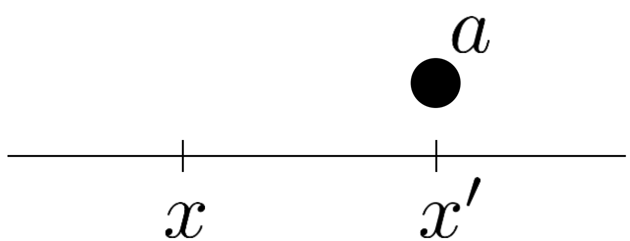

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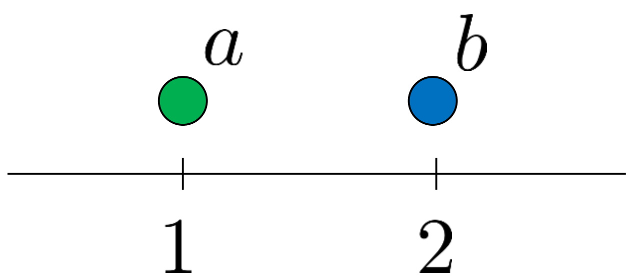
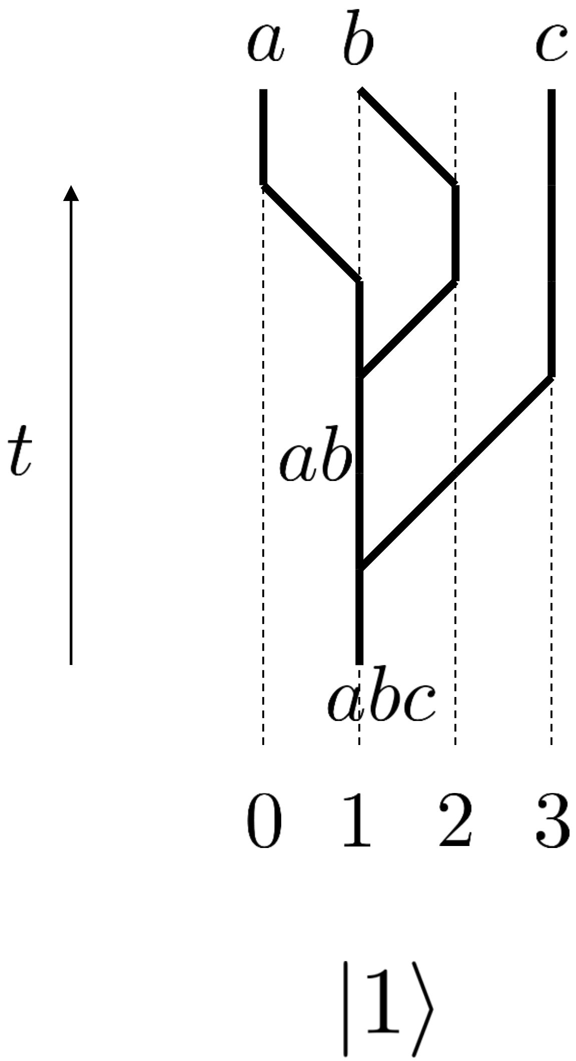
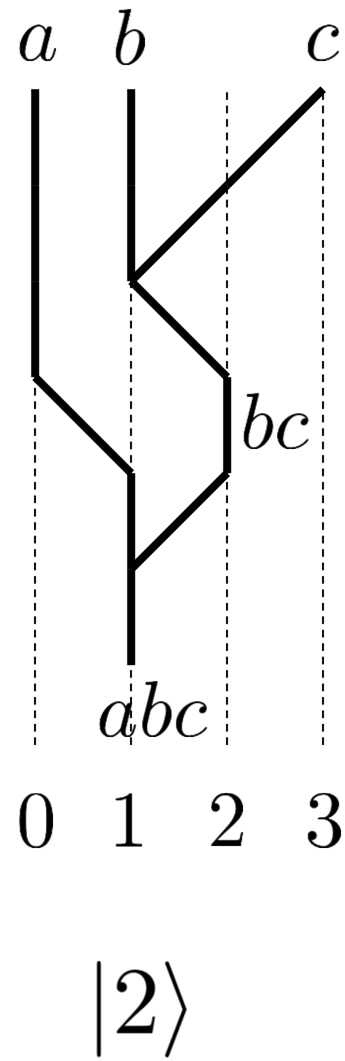


$$S(a,b)|...(ab)_1...\rangle \propto |...a_1, b_2...\rangle$$

(Kawagoe, ML, in prep.)





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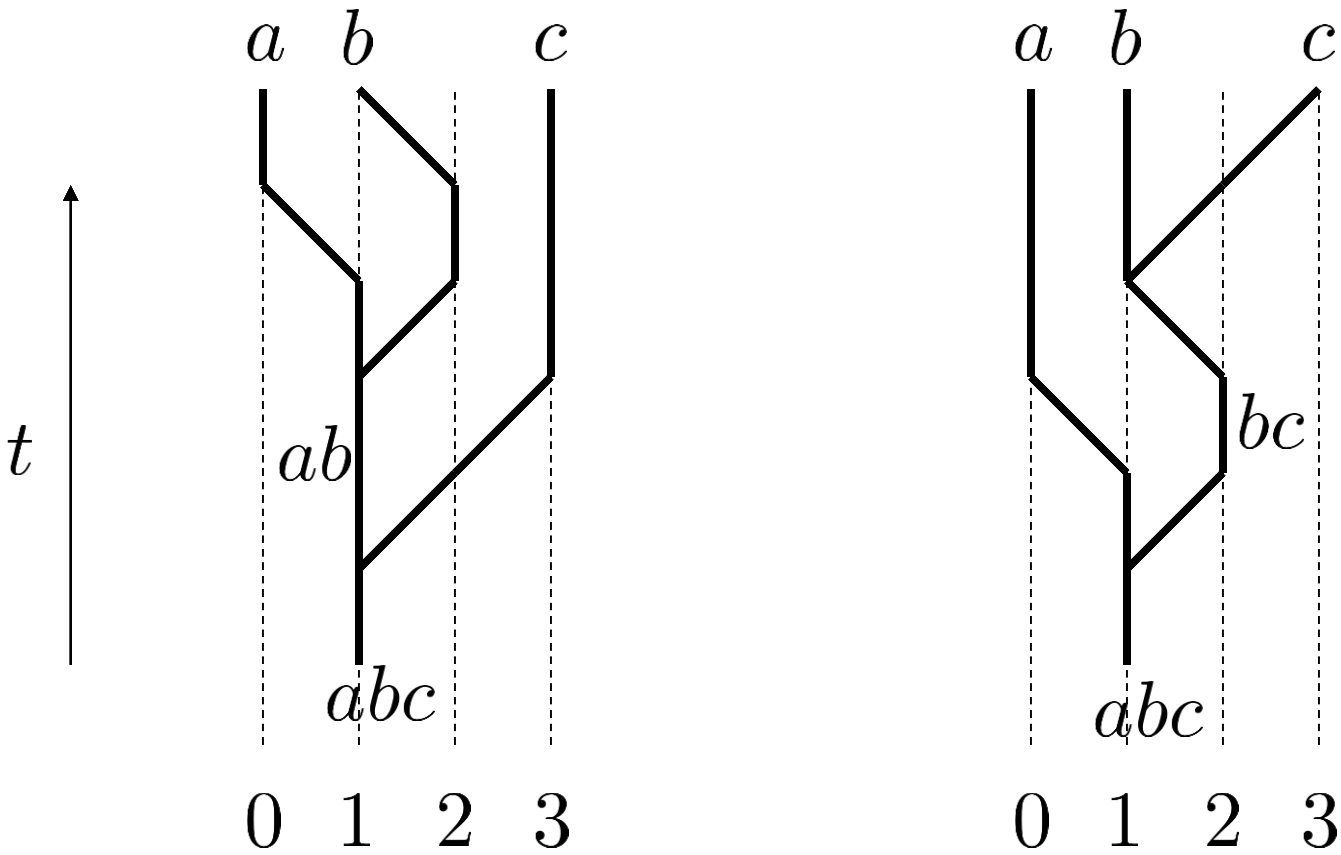

$$|1\rangle \quad = \quad F(a,b,c)\cdot\ |2\rangle$$

$$|1\rangle = M_{12}^{b} \cdot M_{01}^{a} \cdot S(a,b) \cdot M_{32}^{c} \cdot S(ab,c)\ |(abc)_1\rangle$$

$$|2\rangle = M_{32}^{c} \cdot S(b,c) \cdot M_{12}^{bc} \cdot M_{01}^{a} \cdot S(a,bc)\ |(abc)_1\rangle$$

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Phase ambiguity:

$$S(a,b) \to e^{i\phi(a,b)} \cdot S(a,b), \qquad M^a_{x'x} \to e^{i\theta_{x'x}(a)} \cdot M^a_{x'x}$$

$$|1\rangle = M^b_{12} \cdot M^a_{01} \cdot S(a,b) \cdot M^c_{32} \cdot S(ab,c)\ |(abc)_1\rangle$$

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Effect on F:

$$F(a,b,c) \to F(a,b,c) \cdot \frac{e^{i\nu(a,b)} e^{i\nu(ab,c)}}{e^{i\nu(b,c)} e^{i\nu(a,bc)}}$$

$$\nu(a,b) = \phi(a,b) + \theta_{12}(b)$$

$$|1\rangle = M_{12}^{b} \cdot M_{01}^{a} \cdot S(a,b) \cdot M_{32}^{c} \cdot S(ab,c)\ |(abc)_1\rangle$$

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✓

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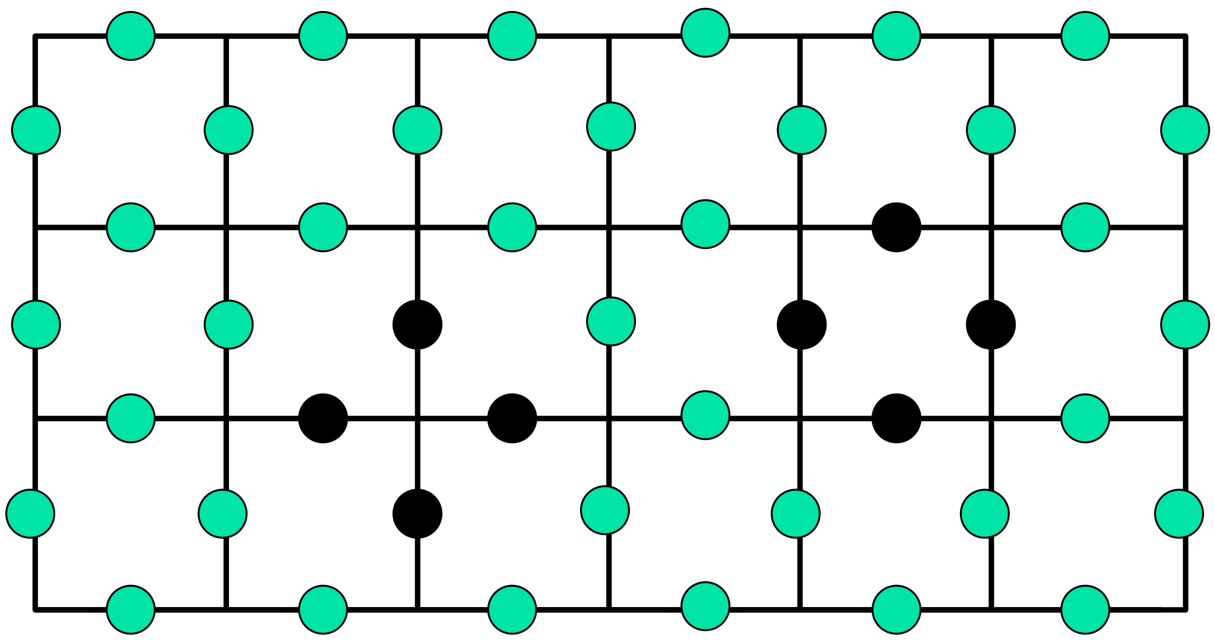
(Kitaev, 2003)

Example: Toric code model



$$H = -\sum_{+} \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x - \sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$$

(Kitaev, 2003)

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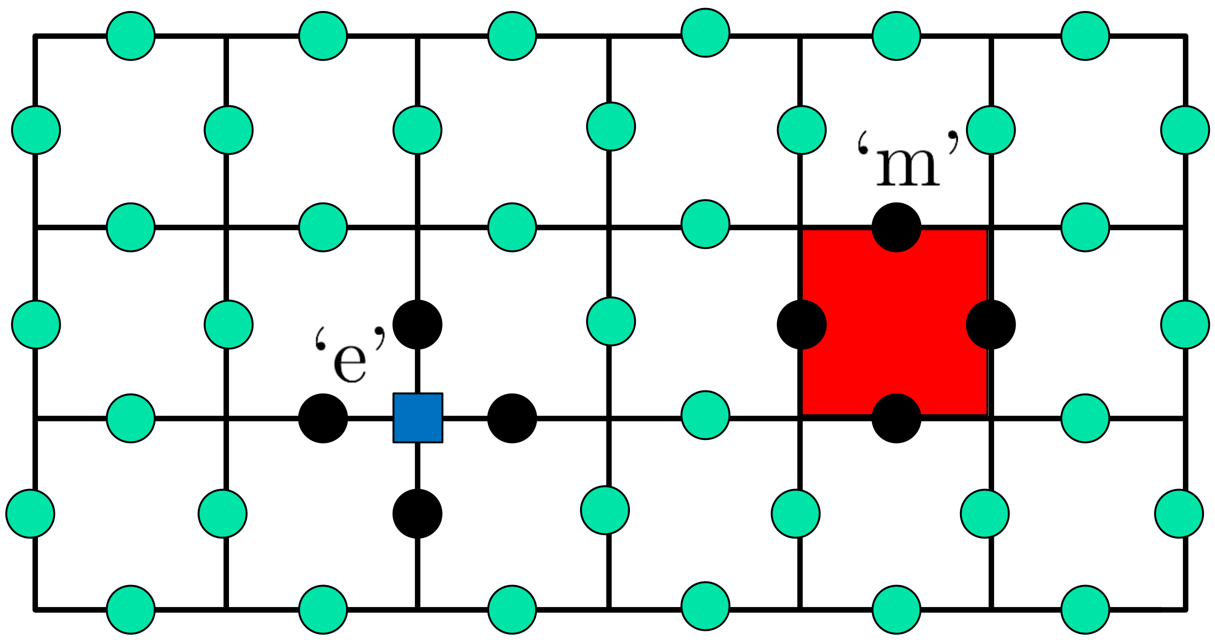

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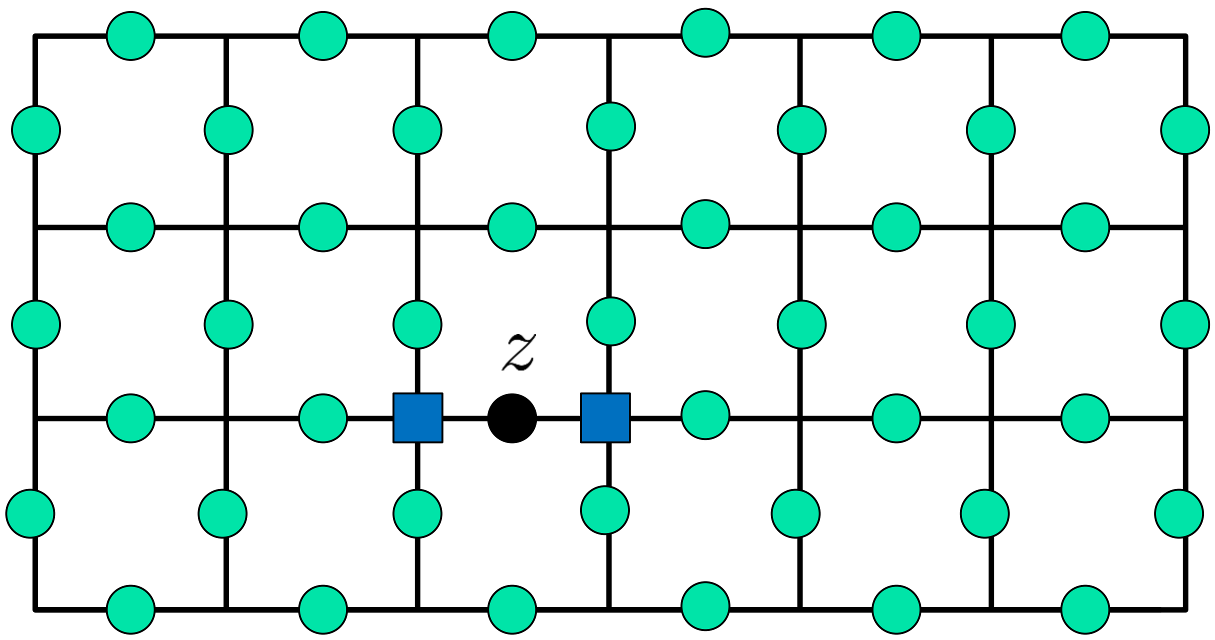

$$M^e = \sigma^z$$
$$S(e,e) = \sigma^z$$
$$\implies F(e,e,e) = 1$$

Remarks

- Can also give microscopic definition of $R(a, b)$

$\Longrightarrow$ can extract **complete** set of anyon data from lattice model

- Application: computing anomalies from SPT edge theories

(see Kyle Kawagoe's poster!)