Mimicking the edge of 2d topological superconductors and insulators in 1d lattice models

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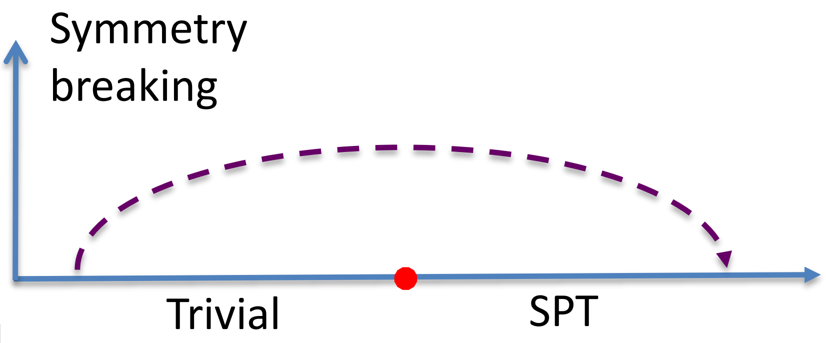
Simons Collaboration on Ultra Quantum Matter: Kickoff Meeting.
Harvard, September 12, 2019

Robert Jones,
MIT

R. A. Jones and MM, arXiv:1902.05957
MM, arXiv:1908.08958

Symmetry protected topological phases (SPTs)



- Protected by symmetry G
- Gapped bulk

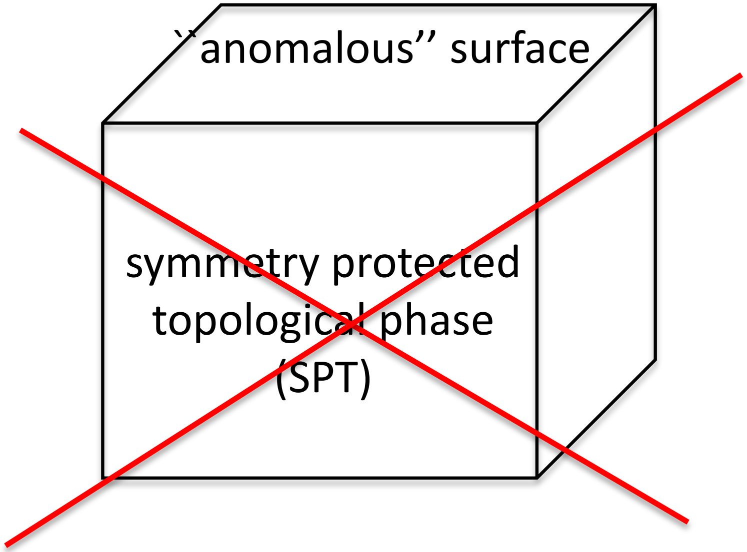
  - unique ground state on a closed spatial manifold

  - no anyon exciations

- Surface must be non-trivial

  -t'Hooft anomaly (can't couple to a background gauge-field).

Q: Can we realize/mimick the boundary in a lattice model without the bulk?



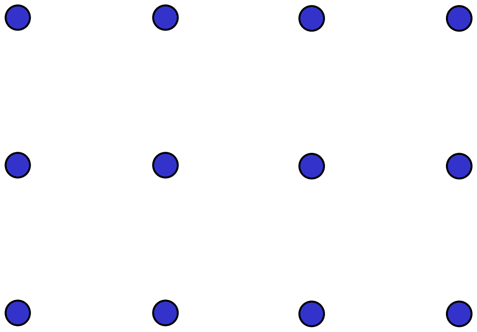
Motivation:

i) Simulating edge of SPT without the bulk overhead

ii) A more lattice based understanding of anomalies

Q: Can we realize/mimick the boundary in a lattice model without the bulk?

A: No, subject to



i) $V = \otimes_i V_i$

ii) $U(g) = \prod_i U_i(g)$

$$U_i(g)U_i(h) = U_i(gh)$$

(On-site symmetry)
- Can always be gauged!

Cohomology phases

i) $V = \otimes_i V_i$

ii) $\xcancel{U(g) = \prod_i U_i(g)}$

- Class of boson SPT phases X. Chen, Z.-C. Gu, Z.-X, Liu, X.G.-Wen (2012)

- Classified by $H^{d+1}(G, U(1))$
  (when G is gauged – Dijkgraaf-Witten theories)

- Edge can be mimicked if ii) is dropped (discrete G)

- 1+1D boundary: extract anomaly $H^3(G, U(1))$

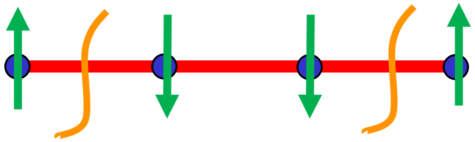
Chen, Liu, Wen (2011); D. Else, C. Nayak (2014)

Example

- $G = Z_2$

- $H^3(G, U(1)) = Z_2$

- 1+1D boundary



S = 1/2

$$U = (-1)^{N_{dw}/2} \prod_i \sigma_i^x$$

X. Chen, Z. X. Liu, X.-G. Wen (2012)
M. Levin, Z. C. Gu (2012)

Cohomology phases

i) $V = \otimes_i V_i$

ii) $\xcancel{U(g) = \prod_i U_i(g)}$

- Extends to “super-cohomology” phases of fermions

  (explicit construction for 1+1D boundary and discrete G in MM, arXiv:1908.08958)

This talk

- Beyond (super)cohomology phases:
  - 2+1D fermion topological superconductors

$$\xcancel{V = \otimes_i V_i}$$

$$\xcancel{U(g) = \prod_i U_i(g)}$$

R. A. Jones and MM, arXiv:1902.05957

This talk

- (Super)cohomology phases with a continuous symmetry group G

  - infinite dimensional site Hilbert space?

- 2+1D fermion topological insulators (quantum spin Hall)

$$V = \otimes_i V_i$$

$$\xcancel{U(g) = \prod_i U_i(g)}$$

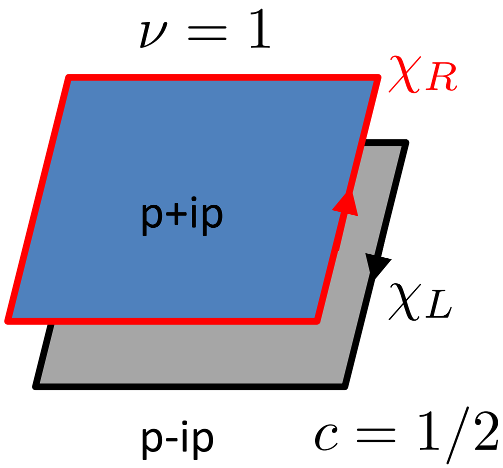
MM, arXiv:1908.08958

See also J. H. Son and J. Alicea, 1906.11846

Beyond (super)-cohomology

- Ex: 2+1D, fermions, $G = Z_2 \times Z_2^f$, $\nu \in Z_8$

Gu, Levin (14)



$$Z_2 : \chi_R \to -\chi_R, \quad \chi_L \to \chi_L$$

$$\delta L = im\chi_R\chi_L$$

$$m \to -m$$

Kramers-Wannier duality:

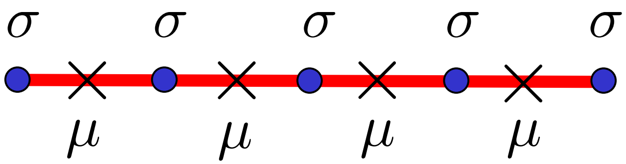
High T ⟷ low T

- Mimick in 1+1D?

Kramers-Wannier

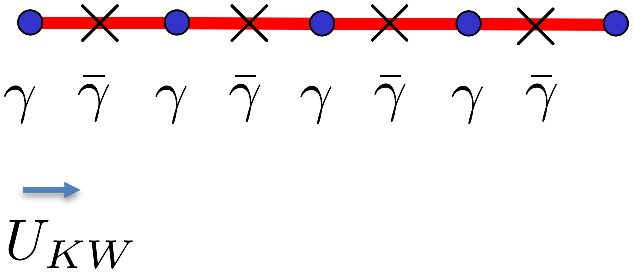
$$H_{TFIM} = -J \sum_i \sigma_i^x \sigma_{i+1}^x + h \sum_i \sigma_i^z$$

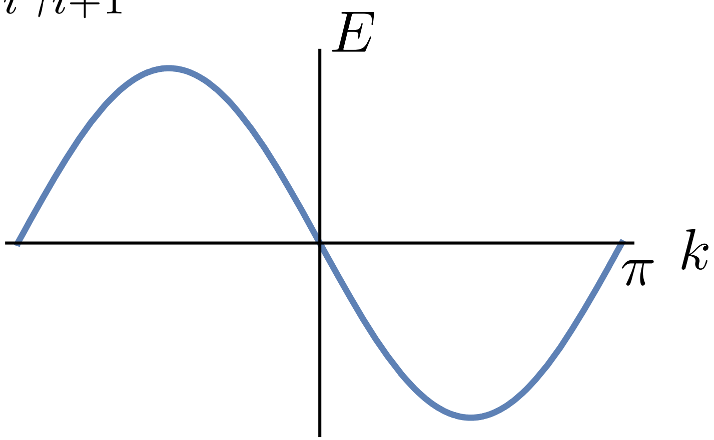
$$\mu_{i+1/2}^z = -\sigma_i^x \sigma_{i+1}^x, \quad \mu_{i-1/2}^x \mu_{i+1/2}^x = -\sigma_i^z \qquad J \leftrightarrow h$$



$$U_{KW}^2 = T_x$$

- Fermionize: $H = ih \sum_i \gamma_i \bar{\gamma}_i + iJ \sum_i \bar{\gamma}_i \gamma_{i+1}$





i) $V = \otimes_i V_i$ $\qquad V_{phys} \subset V$

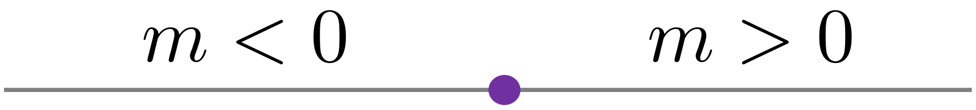
ii) $U(g) = \prod_i U_i(g)$

- not enough to drop this if $U^2 = 1$

- Starting point:
  Exactly solvable bulk model of Tarantino and Fidkowski (2016)

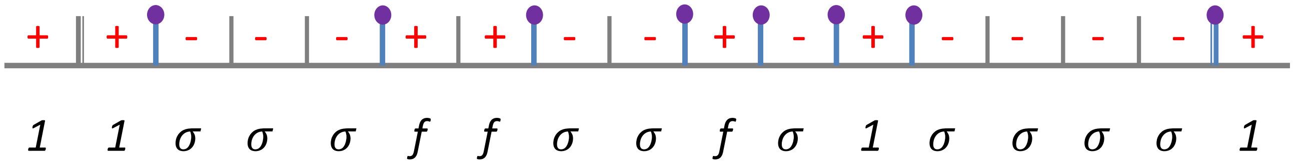
Intuition

$$\delta L = im\chi_R\chi_L$$



- Restore symmetry – Majorana liquid

$$\dim V_{Maj} = 2^{N_{dw}/2}$$

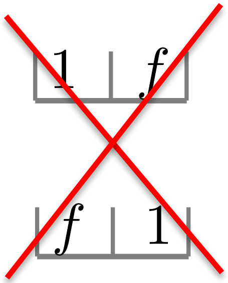


Link labels: $\{1, \sigma, f\}$,

$$\sigma \times \sigma = 1 + f$$

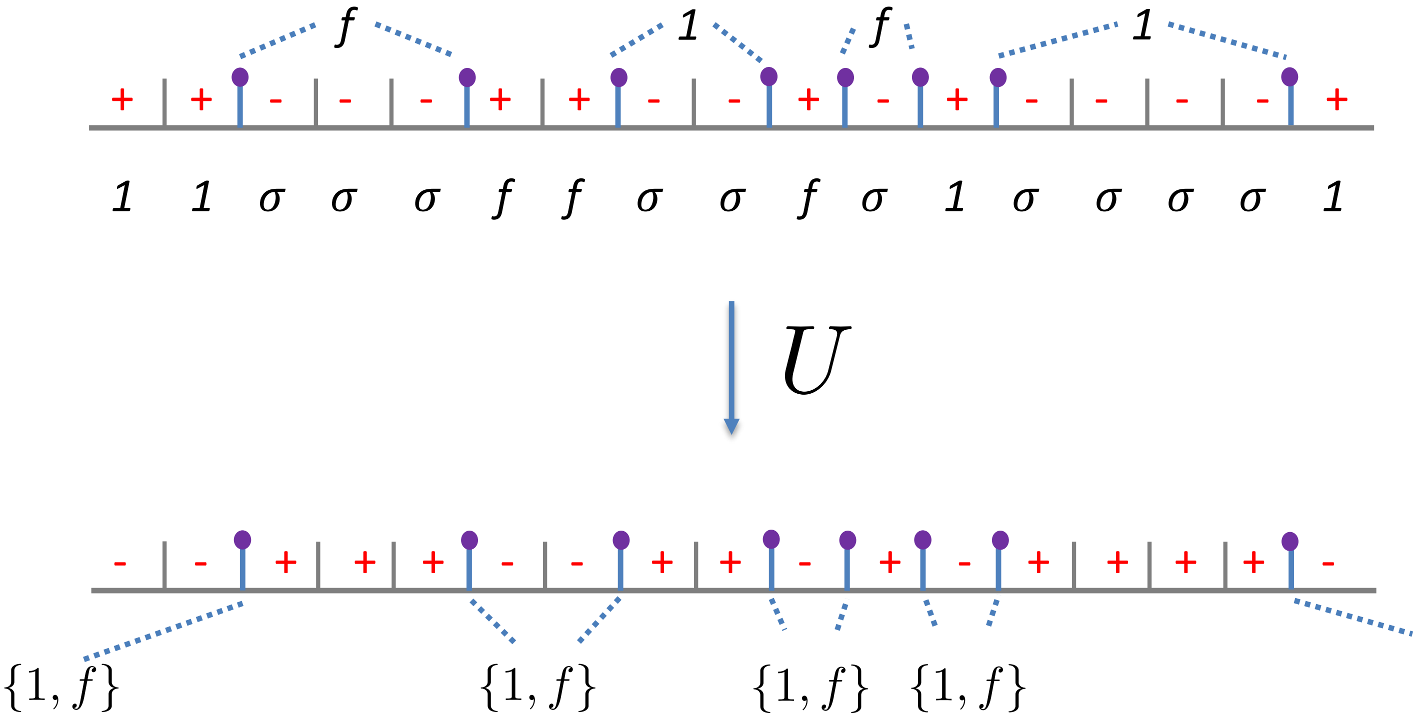
- σ

+ 1 or f



$$\dim(V_{phys}) \sim (\sqrt{2}+1)^N$$

Symmetry action



- **Strings:** $\{1, f\} \to \sigma$

  $\sigma \to \{1, f\}$ -superposition

Hamiltonian

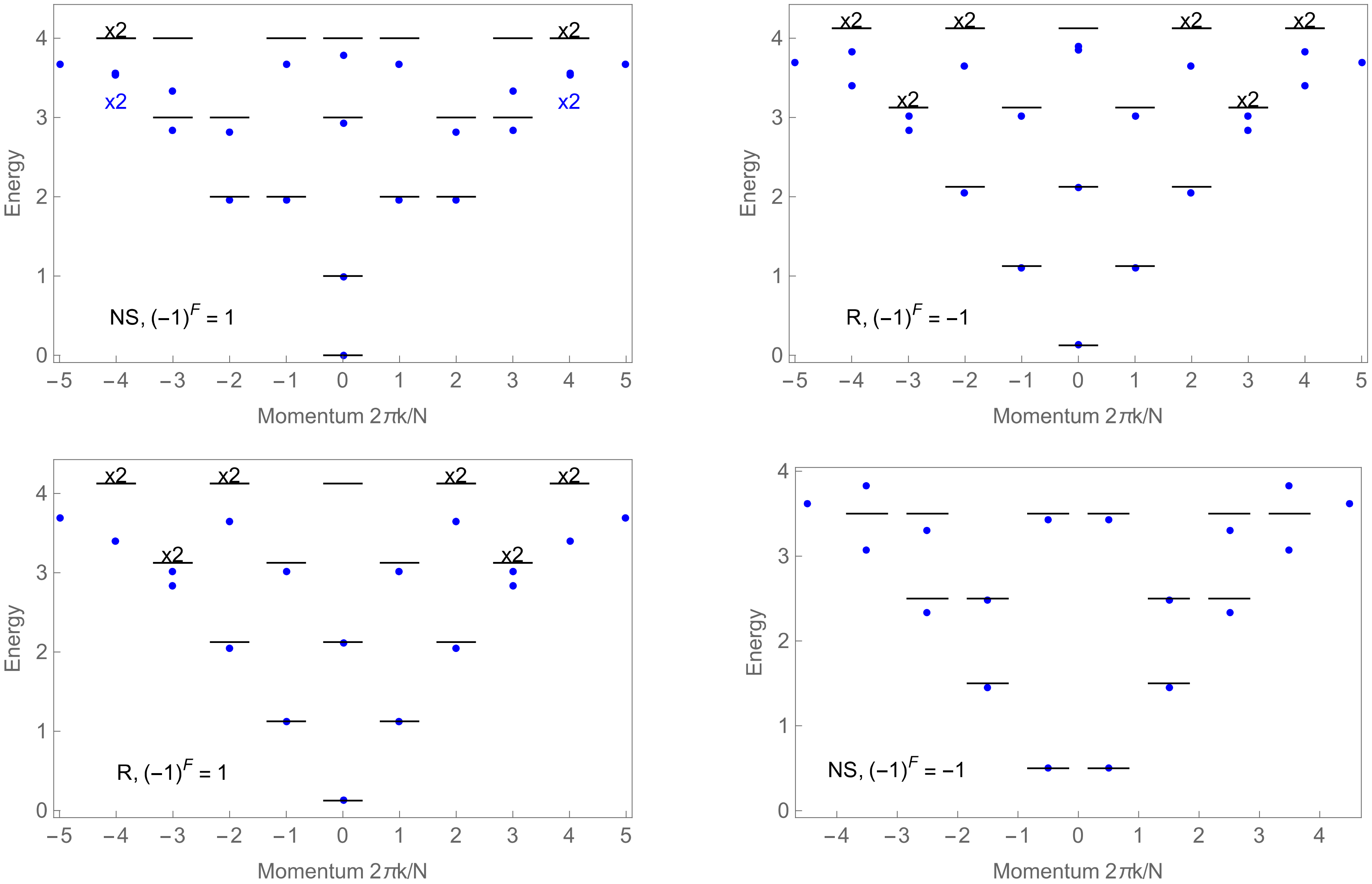
c_1

c_2

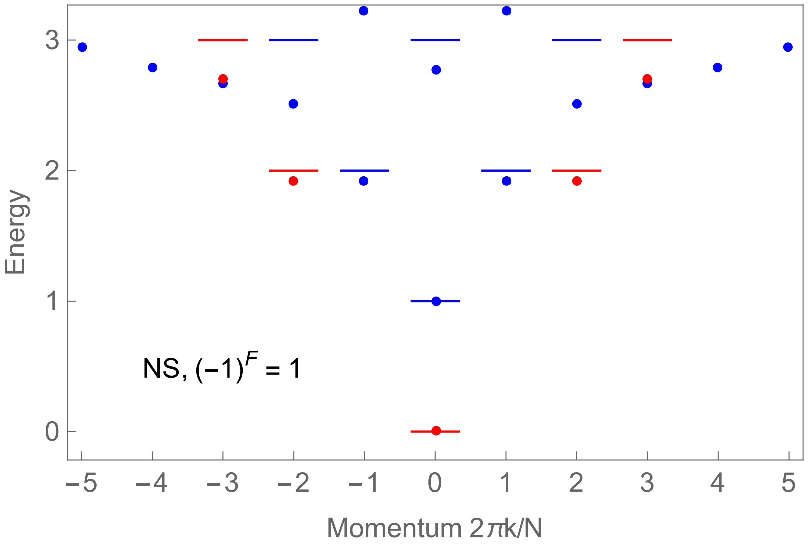
- $c_1 \approx c_2$ - solid numerical evidence (ED & DMRG) for Ising CFT

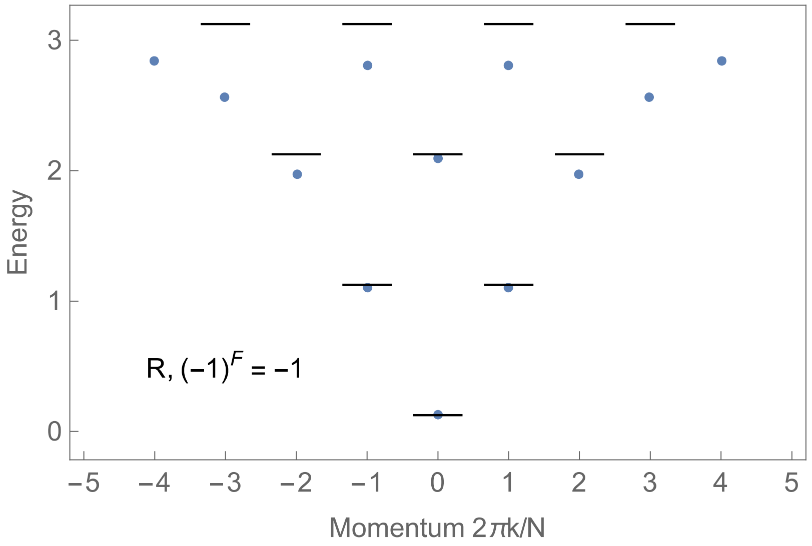
$$Z_2 : \chi_R \to -\chi_R, \quad \chi_L \to \chi_L$$

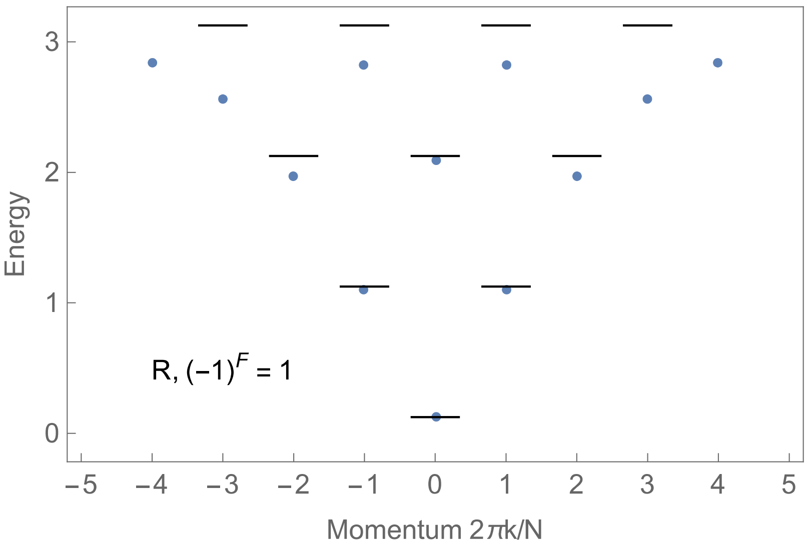
Exact diagonalization

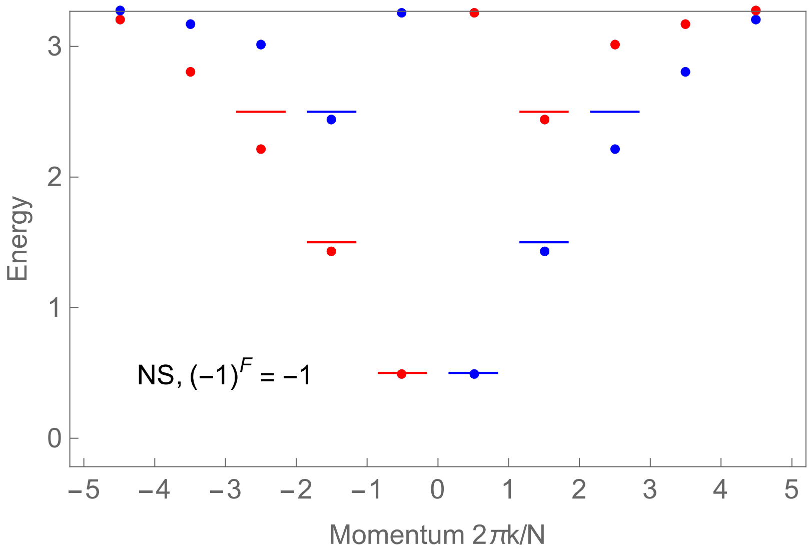


Exact diagonalization

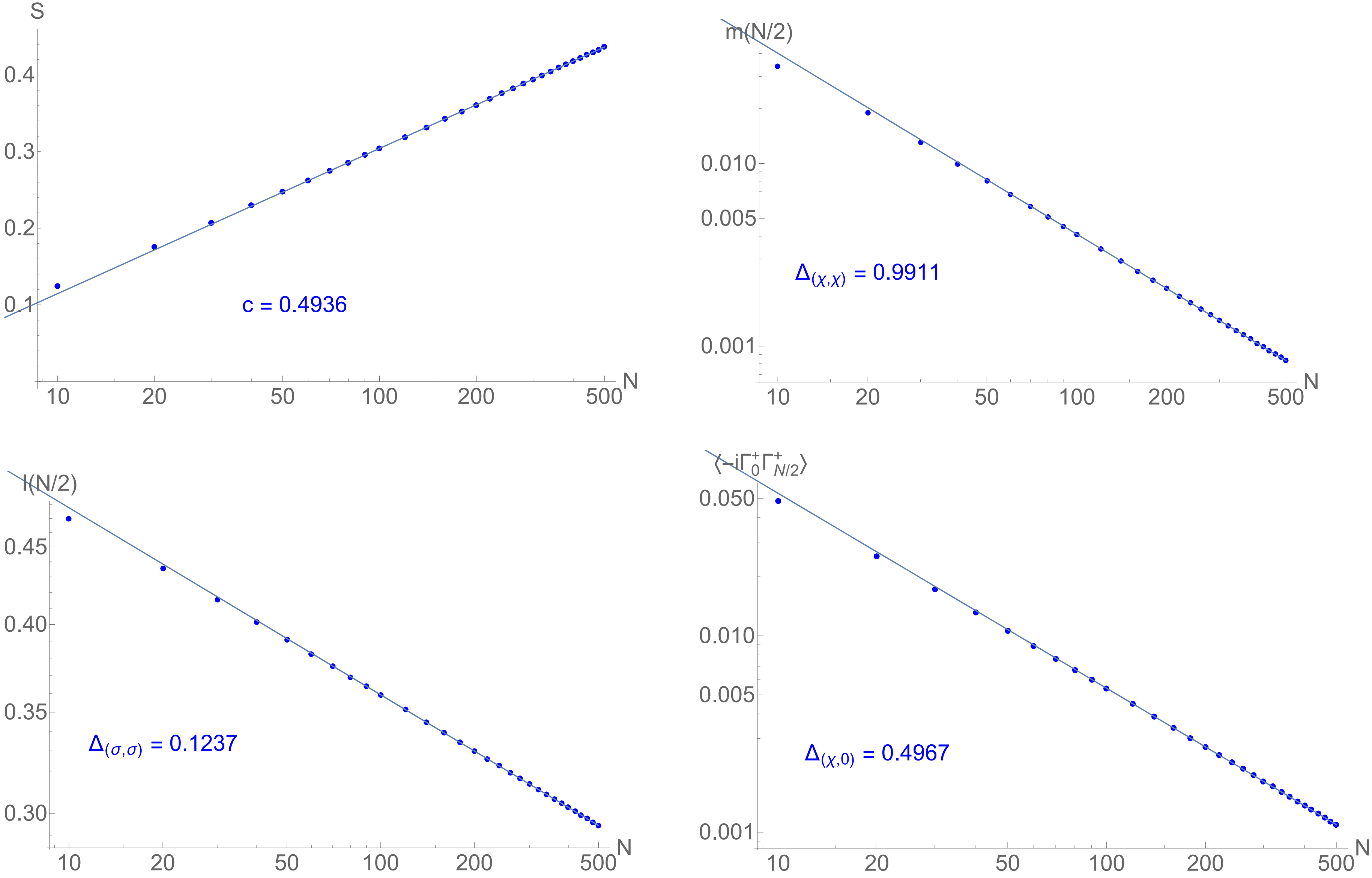








DMRG


S
0.4
0.3
0.2
0.1
N
10
20
50
100
200
500
c = 0.4936
m(N/2)
0.010
0.005
0.001
N
10
20
50
100
200
500
$\Delta_{(\chi,\chi)}$ = 0.9911
I(N/2)
0.45
0.40
0.35
0.30
N
10
20
50
100
200
500
$\Delta_{(\sigma,\sigma)}$ = 0.1237
$\langle -i\Gamma_0^+\Gamma_{N/2}^+ \rangle$
0.050
0.010
0.005
0.001
N
10
20
50
100
200
500
$\Delta_{(\chi,0)}$ = 0.4967

Lessons & open questions

- Mimick edge of 2+1D beyond super-cohomology fermion SPT

    - constrained Hilbert space

    - similar to 3+1D, U(1)xT fermion SPTs

    - generalizes to 2+1D fSPTs with $G \times Z_2^f$

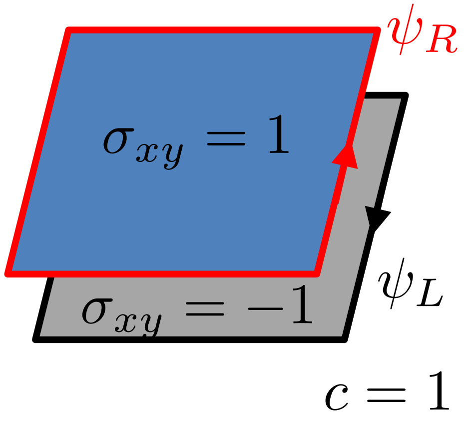
    - obstruction?

    - other dimensions?

$\nu = 1/2$

Quantum spin-Hall

- 2+1D, fermions



$$U : \ \psi_R \to -\psi_R, \quad \psi_L \to \psi_L$$

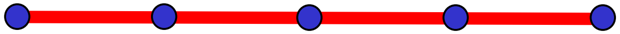
$$U(1) : \ N = N_R + N_L$$

$$\mathcal{T}_{NK} : \ \psi_R \leftrightarrow \psi_L, \quad i \to -i. \quad \mathcal{T}_{NK}^2 = 1$$

$$\mathcal{T} = U\mathcal{T}_{NK}, \qquad \mathcal{T}^2 = (-1)^F$$

QSH edge

$$c_i = \frac{1}{2}(\gamma_i + i\bar{\gamma}_i)$$



$$g_{i,i+1} \in \{0,1\}$$

$$\tau_i^z = (-1)^{g_i}$$

$$U = \left(\prod_{j=1}^{L} \tau^x_{j,j+1}\right)\left(\prod_{j=1}^{L} \gamma_j^{g_{j-1,j}+g_{j,j+1}}\right)(-i)^{N_{dw}/2}$$

- inspired by Tantivasadakarn and Vishwanath (18)

$$N = \sum_i n_i, \quad n_i = (-1)^{g_i(g_{i+1}+1)} c_i^\dagger c_i$$

$$\mathcal{T}_{NK} = \left(\prod_{j=1}^{L} (-1)^{g_{j,j+1}\, c_j^\dagger c_j}\right) \mathcal{T}_0$$

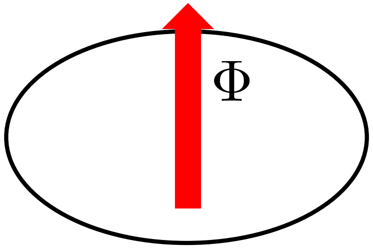
MM, arXiv:1908.08958

Properties

- $\mathcal{T}$ domain wall:

$$N \in 2\mathbb{Z}+1$$

- Kramers parity switching



$$\Phi = 0, \quad \mathcal{T}^2 = (-1)^F$$

$$\Phi = \pi, \quad \mathcal{T}^2 = -(-1)^F$$

- Symmetric Hamiltonians
  - analyze by Jordan-Wigner
  - Luttinger liquid of QSH edge

- Can extract anomaly data: $G_b = G_f / Z_2^f$

$$\lambda \in H^2(G_b, Z_2), \quad \sigma \in H^2(G_b, Z_2), \quad w_3 \in C^3(G_b, Z_2)$$

$$d_T w_3 = (-1)^{\sigma \cup \sigma + \lambda \cup \sigma}$$

Open questions

- Mimick edge of 2+1D quantum spin-Hall

  - mimick other SPTs with continuous symmetry groups?

  - continous vs discrete anomalies?

Thank you!