Continuum Field Theory With a Vector Global Symmetry

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Introduction

• Motivated by recent advances with fractons, but not sure this talk is relevant to fractons.
• Study global symmetries, whose conserved charge is a vector.
• One example is the momentum $P^i = \int_{\text{space}} T_{0}^i$. Its conserved Noether current is symmetric.
• Oneform symmetry is common in relativistic field theories.
• We’ll present a hierarchical set of three symmetries, starting with the most special one and generalizing it.
• For each symmetry (for simplicity $U(1)$) we will present:
  – the conserved currents
  – a class of field theories with the symmetry
  – a concrete example based on a scalar field theory
Relativistic oneform global symmetry

A lot of earlier work, here we follow [Gaiotto, Kapustin, NS, Willett]

$$\partial_{\mu} J^{[\mu\nu]} = 0$$

Charges

$$Q(C) = \int_{C} J^{[\mu\nu]} n_{[\mu\nu]}$$

$C$ a codimension 2 manifold in spacetime orthogonal to $n_{[\mu\nu]}$. $Q$ is topological – it does not change under small deformations of $C$. Specifically, it does not change when $C$ does not cross another operator.
Relativistic oneform global symmetry

In nonrelativistic terms
\[ \partial_0 J^j_0 - \partial_i J^{ij} = 0 \]
\[ G = \partial_i J^i_0 = 0 \]

Charges at fixed time
\[ Q(\mathcal{C}) = \int_{\mathcal{C}} J^j_0 n_j \]

\( \mathcal{C} \) a codimension 1 manifold in space orthogonal to \( n_j \).

\( Q \) is topological – it does not change under small changes of \( \mathcal{C} \).
Relativistic oneform global symmetry

\[ \partial_0 J^j_0 - \partial_i J^{ij}_0 = 0 \]
\[ G = \partial_i J^i_0 = 0 \]

Example:
Maxwell theory – \( U(1) \) gauge theory (either in the continuum or on the lattice)

\[ J^j_0 = F^j_0 \]
\[ J^{ij}_0 = F^{ij}_0 \]

\( G = 0 \) is Gauss law
\( Q(\mathcal{C}) \) is the electric flux through \( \mathcal{C} \).

The charged operators are Wilson lines.
The continuum theory also has a magnetic symmetry, but we will not discuss it here.
As before,

\[ \partial_0 J_0^j - \partial_i J^{[ij]} = 0 , \]

but now we do not impose \( G = \partial_i J_0^i = 0 \).

The conservation leads to \( \partial_0 G = \partial_i \partial_0 J_0^i = 0 \),

i.e. \( G \) is conserved at every point, but it is not zero.

As before, conserved charges at fixed time

\[ Q(C) = \int_C J_0^j n_j , \]

but now \( Q \) is not topological.

A cruder charge is \( Q^j = \int_{space} J_0^j \).

Correspondingly, point operators can transform under \( Q^j \).
Nonrelativistic oneform global symmetry

\[ \partial_0 J^i_0 - \partial_i J^{[ij]} = 0 \]

Lattice example:

\( U(1) \) lattice gauge theory has an electric relativistic oneform symmetry. Gauss law is imposed.

It is common, following [Kitaev], to relax Gauss law and impose it energetically.

This system has this nonrelativistic oneform symmetry.

\( G = \partial_i J^i_0 \) is nonzero, but it is conserved.

Related discussion in [Hermele, Fisher, Balents; Williamson, Bi, Cheng]
Nonrelativistic oneform global symmetry

A class of examples

\[ \partial_0 J^j_0 - \partial_i J^{[ij]} = 0 \]

Couple the $U(1)$ gauge theory to charged matter fields such that

\[ J^j_0 = F^j_0 \]
\[ J^{[ij]} = F^{[ij]} \]

For that, the spatial components of the $U(1)$ current of the matter theory should vanish

\[ J^i_{\text{matter}} = 0 \]

and hence,

\[ \partial_0 J^i_{\text{matter}} = 0. \]

Then the coupled system

\[ \mathcal{L}_1 = \mathcal{L}_0 + (F^i_0)^2 - (F^{ij})^2 + A_0 J^i_{\text{matter}} \]

with $\mathcal{L}_0$ the matter Lagrangian, is $U(1)$ gauge invariant and has the nonrelativistic oneform symmetry with $G = \partial_i J^i_0 = J^i_{\text{matter}}$. 
Nonrelativistic oneform global symmetry

A class of examples

\[ J_i^{\text{matter}} = 0, \quad \partial_0 J_0^{\text{matter}} = 0 \]

Such a matter theory has infinitely many conserved charges. \( C(x^i)J_0^{\text{matter}} \) is conserved for every \( C(x^i) \).

The charges are not mobile.

A concrete example:

A complex scalar field \( \Phi \) with

\[ \mathcal{L}_0 = i \overline{\Phi} \partial_0 \Phi - \partial_i (\overline{\Phi} \Phi) \partial^i (\overline{\Phi} \Phi) - |\Phi|^4 + \ldots \]

No \( \partial_i \overline{\Phi} \partial^i \Phi \) term. Highly nonstandard. Similar to fractons.

Invariance under \( \Phi \to e^{iC(x^i)} \Phi \)

The charge density \( J_0^{\text{matter}} = |\Phi|^2 \) at a point is conserved.
A more general vector symmetry

Previous case
\[ \partial_0 J^j_0 - \partial_i J^{ij}_i = 0 \]

With antisymmetric \( J^{ij} \).

Generalize to
\[ J^{ij} = J^{[ij]} + \delta^{ij} J \]
i.e. it is a sum of an antisymmetric tensor and a scalar.

Now
\[ \partial_0 G = \partial_i \partial_0 J^i_0 = \partial_i \partial^i J \neq 0 . \]

Therefore, the charge operators \( Q(\mathcal{C}) = \int_{\mathcal{C}} J^j_0 n_j \) are no longer conserved, but the cruder charge
\[ Q^j = \int_{\text{space}} J^j_0 \]
is conserved.
A more general vector symmetry

A class of examples

\[ \partial_0 J^j_0 - \partial_i J^{ij} = 0 \]

\[ J^{ij} = J[ij] + \delta^{ij} J \]

For \( J = 0 \), we coupled charged matter with \( J^\text{matter}_i = 0 \) (and hence \( \partial_0 J^\text{matter}_0 = 0 \)) to a \( U(1) \) gauge field.

Now, we take a matter theory with

\[ \partial_0 J^\text{matter}_0 - \partial_i \partial^i J^\text{matter} = 0 \]

i.e. \( J^\text{matter}_i = \partial_i J^\text{matter} \), and couple it to the \( U(1) \) gauge field

\[ \mathcal{L}_1 = \mathcal{L}_0 + (F^i_0)^2 - (F^{ij})^2 + A_0 J^\text{matter}_0 - A_i \partial^i J^\text{matter} \]

The conserved currents of the global symmetry are

\[ J^j_0 = F^j_0 \]

\[ J[ij] = F[ij] - \delta^{ij} J^\text{matter} \]
A more general vector symmetry

A class of examples

\[ \partial_0 J^\text{matter}_0 - \partial_i \partial^i J^\text{matter}_0 = 0 \]

Such a matter theory has the conserved charges

\[ Q = \int_{\text{space}} J^\text{matter}_0 \]

\[ Q^j = \int_{\text{space}} x^j J^\text{matter}_0 \]

They can be interpreted as:

• a global \( U(1) \) symmetry (which we gauge)

• a vector symmetry, dipole symmetry, with charge \( Q^j \)

After the gauging we are left only with the vector symmetry
A more general vector symmetry

A concrete example [Pretko]

\[ \partial_0 J_0^{\text{matter}} - \partial_i \partial^i J^{\text{matter}} = 0 \]

A complex scalar field \( \Phi \) with

\[ \mathcal{L}_0 = i \overline{\Phi} \partial_0 \Phi - \partial_i (\overline{\Phi} \Phi) \partial^i (\overline{\Phi} \Phi) - |\Phi|^4 \\
+ i (\overline{\Phi}^2 \partial_i \Phi \partial^i \Phi - \Phi^2 \partial_i \overline{\Phi} \partial^i \overline{\Phi}) + \ldots \]

Again, No \( \partial_i \overline{\Phi} \partial^i \Phi \) term.

The new term \( i (\overline{\Phi}^2 \partial_i \Phi \partial^i \Phi - \Phi^2 \partial_i \overline{\Phi} \partial^i \overline{\Phi}) \) breaks the symmetry \( \Phi \rightarrow e^{i \alpha} \Phi \) to \( \Phi \rightarrow e^{i \alpha + ic_i x^i} \Phi \).

Here

\[ J_0^{\text{matter}} = |\Phi|^2 \]
\[ J^{\text{matter}} = |\Phi|^4 \]
A possible further generalization

\[ \partial_0 J_0^j - \partial_i J^{ij} = 0 \]

\( J^{ij} \) includes an antisymmetric tensor, a scalar, and a symmetric tensor.

We have not analyzed it in detail.
Summary of the symmetries

\[ \partial_0 J^j_0 - \partial_i J^{ij} = 0 \]

• The general vector symmetry:
  \[ J^{ij} = J^{[ij]} + \delta^{ij} J \]

The charges
  \[ Q^j = \int_{space} J^j_0 \]

• Nonrelativistic oneform symmetry: impose also
  \[ J = 0 \]

\[ Q(C) = \int_C J^j_0 n_j \] is associated with a nontopological manifold \( C \)

• As in the relativistic symmetry: impose also
  \[ \partial_j J^j_0 = 0 \]

\[ Q(C) = \int_C J^j_0 n_j \] is associated with a topological manifold \( C \)
Conclusions

• A hierarchy of global symmetries, whose charges $Q^i$ carry a spatial vector index.

• For every one of these we presented a large class of continuum theories exhibiting them.
  – All these examples are based on a $U(1)$ gauge theory coupled to a special matter theory
  – We showed concrete examples of these matter theories

• We can gauge these new symmetries. The needed gauge field is an antisymmetric tensor $B_{[\mu\nu]}$ and in the general symmetry we also need a scalar (that couples to $J$).