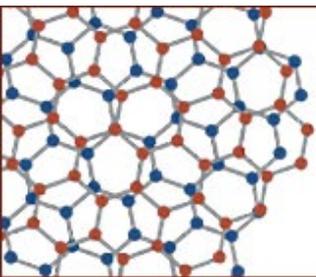


# Continuum Field Theory With a Vector Global Symmetry

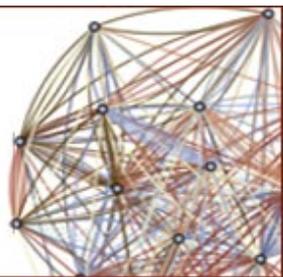
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Simons Collaboration on  
Ultra-Quantum Matter



# Introduction

- Motivated by recent advances with fractons, but not sure this talk is relevant to fractons.
- Study global symmetries, whose conserved charge is a vector.
- One example is the momentum  $P^i = \int_{space} T_0^i$ . Its conserved Noether current is symmetric.
- Oneform symmetry is common in relativistic field theories.
- We'll present a hierarchical set of three symmetries, starting with the most special one and generalizing it.
- For each symmetry (for simplicity  $U(1)$ ) we will present:
  - the conserved currents
  - a class of field theories with the symmetry
  - a concrete example based on a scalar field theory

# Relativistic oneform global symmetry

A lot of earlier work, here we follow [Gaiotto, Kapustin, NS, Willett]

$$\partial_\mu J^{[\mu\nu]} = 0$$

Charges

$$Q(\mathcal{C}) = \int_{\mathcal{C}} J^{[\mu\nu]} n_{[\mu\nu]}$$

$\mathcal{C}$  a codimension 2 manifold in spacetime orthogonal to  $n_{[\mu\nu]}$ .

$Q$  is topological – it does not change under small deformations of  $\mathcal{C}$ . Specifically, it does not change when  $\mathcal{C}$  does not cross another operator.

# Relativistic oneform global symmetry

In nonrelativistic terms

$$\begin{aligned}\partial_0 J_0^j - \partial_i J^{[ij]} &= 0 \\ G = \partial_i J_0^i &= 0\end{aligned}$$

Charges at fixed time

$$Q(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$$

$\mathcal{C}$  a codimension 1 manifold in space orthogonal to  $n_j$ .

$Q$  is topological – it does not change under small changes of  $\mathcal{C}$

# Relativistic oneform global symmetry

$$\begin{aligned}\partial_0 J_0^j - \partial_i J^{[ij]} &= 0 \\ G = \partial_i J_0^i &= 0\end{aligned}$$

Example:

Maxwell theory –  $U(1)$  gauge theory (either in the continuum or on the lattice)

$$\begin{aligned}J_0^j &= F_0^j \\ J^{[ij]} &= F^{[ij]}\end{aligned}$$

$G = 0$  is Gauss law

$Q(\mathcal{C})$  is the electric flux through  $\mathcal{C}$ .

The charged operators are Wilson lines.

The continuum theory also has a magnetic symmetry, but we will not discuss it here.

# Nonrelativistic oneform global symmetry

As before,

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0 ,$$

but now we do not impose  $G = \partial_i J_0^i = 0$ .

The conservation leads to  $\partial_0 G = \partial_i \partial_0 J_0^i = 0$ ,

i.e.  $G$  is conserved at every point, but it is not zero.

As before, conserved charges at fixed time

$$Q(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j ,$$

but now  $Q$  is not topological.

A cruder charge is  $Q^j = \int_{space} J_0^j$ .

Correspondingly, point operators can transform under  $Q^j$ .

# Nonrelativistic oneform global symmetry

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0$$

Lattice example:

$U(1)$  lattice gauge theory has an electric relativistic oneform symmetry. Gauss law is imposed.

It is common, following [\[Kitaev\]](#), to relax Gauss law and impose it energetically.

This system has this nonrelativistic oneform symmetry.

$G = \partial_i J_0^i$  is nonzero, but it is conserved.

Related discussion in [\[Hermele, Fisher, Balents; Williamson, Bi, Cheng\]](#)

# Nonrelativistic oneform global symmetry

## A class of examples

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0$$

Couple the  $U(1)$  gauge theory to charged matter fields such that

$$\begin{aligned} J_0^j &= F_0^j \\ J^{[ij]} &= F^{[ij]} \end{aligned}$$

For that, the spatial components of the  $U(1)$  current of the matter theory should vanish

$$J_i^{matter} = 0 \quad \text{and hence,} \quad \partial_0 J_0^{matter} = 0.$$

Then the coupled system

$$\mathcal{L}_1 = \mathcal{L}_0 + (F_0^i)^2 - (F^{ij})^2 + A_0 J_0^{matter}$$

with  $\mathcal{L}_0$  the matter Lagrangian, is  $U(1)$  gauge invariant and has the nonrelativistic oneform symmetry with  $G = \partial_i J_0^i = J_0^{matter}$ .

# Nonrelativistic oneform global symmetry

## A class of examples

$$J_i^{matter} = 0 \quad , \quad \partial_0 J_0^{matter} = 0$$

Such a matter theory has infinitely many conserved charges.

$C(x^i)J_0^{matter}$  is conserved for every  $C(x^i)$ .

The charges are not mobile.

A concrete example:

A complex scalar field  $\Phi$  with

$$\mathcal{L}_0 = i\bar{\Phi}\partial_0\Phi - \partial_i(\bar{\Phi}\Phi)\partial^i(\bar{\Phi}\Phi) - |\Phi|^4 + \dots$$

No  $\partial_i\bar{\Phi}\partial^i\Phi$  term. Highly nonstandard. Similar to fractons.

Invariance under  $\Phi \rightarrow e^{iC(x^i)}\Phi$

The charge density  $J_0^{matter} = |\Phi|^2$  at a point is conserved.

# A more general vector symmetry

Previous case  $\partial_0 J_0^j - \partial_i J^{ij} = 0$

With antisymmetric  $J^{ij}$ .

Generalize to

$$J^{ij} = \mathcal{J}^{[ij]} + \delta^{ij} \mathcal{J}$$

i.e. it is a sum of an antisymmetric tensor and a scalar.

Now  $\partial_0 G = \partial_i \partial_0 J_0^i = \partial_i \partial^i \mathcal{J} \neq 0$ .

Therefore, the charge operators  $Q(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$  are no longer conserved, but the cruder charge

$$Q^j = \int_{space} J_0^j$$

is conserved.

# A more general vector symmetry

## A class of examples

$$\begin{aligned}\partial_0 J_0^j - \partial_i J^{ij} &= 0 \\ J^{ij} &= \mathcal{J}^{[ij]} + \delta^{ij} \mathcal{J}\end{aligned}$$

For  $\mathcal{J} = 0$ , we coupled charged matter with  $J_i^{matter} = 0$  (and hence  $\partial_0 J_0^{matter} = 0$ ) to a  $U(1)$  gauge field.

Now, we take a matter theory with

$$\partial_0 J_0^{matter} - \partial_i \partial^i J^{matter} = 0$$

i.e.  $J_i^{matter} = \partial_i J^{matter}$ , and couple it to the  $U(1)$  gauge field

$$\mathcal{L}_1 = \mathcal{L}_0 + (F_0^i)^2 - (F^{ij})^2 + A_0 J_0^{matter} - A_i \partial^i J^{matter}$$

The conserved currents of the global symmetry are

$$\begin{aligned}J_0^j &= F_0^j \\ J^{[ij]} &= F^{[ij]} - \delta^{ij} J^{matter}\end{aligned}$$

# A more general vector symmetry

## A class of examples

$$\partial_0 J_0^{matter} - \partial_i \partial^i J^{matter} = 0$$

Such a matter theory has the conserved charges

$$Q = \int_{space} J_0^{matter}$$
$$Q^j = \int_{space} x^j J_0^{matter}$$

They can be interpreted as:

- a global  $U(1)$  symmetry (which we gauge)
- a vector symmetry, dipole symmetry, with charge  $Q^j$

After the gauging we are left only with the vector symmetry

# A more general vector symmetry

## A concrete example [Pretko]

$$\partial_0 J_0^{matter} - \partial_i \partial^i J^{matter} = 0$$

A complex scalar field  $\Phi$  with

$$\begin{aligned} \mathcal{L}_0 = & i\bar{\Phi}\partial_0\Phi - \partial_i(\bar{\Phi}\Phi)\partial^i(\bar{\Phi}\Phi) - |\Phi|^4 \\ & + i(\bar{\Phi}^2\partial_i\Phi\partial^i\Phi - \Phi^2\partial_i\bar{\Phi}\partial^i\bar{\Phi}) + \dots \end{aligned}$$

Again, No  $\partial_i\bar{\Phi}\partial^i\Phi$  term.

The new term  $i(\bar{\Phi}^2\partial_i\Phi\partial^i\Phi - \Phi^2\partial_i\bar{\Phi}\partial^i\bar{\Phi})$  breaks the symmetry

$\Phi \rightarrow e^{iC(x^i)}\Phi$  to  $\Phi \rightarrow e^{i\alpha+ic_ix^i}\Phi$ .

Here

$$\begin{aligned} J_0^{matter} &= |\Phi|^2 \\ J^{matter} &= |\Phi|^4 \end{aligned}$$

# A possible further generalization

$$\partial_0 J_0^j - \partial_i J^{ij} = 0$$

$J^{ij}$  includes an antisymmetric tensor, a scalar, and a symmetric tensor.

We have not analyzed it in detail.

# Summary of the symmetries

$$\partial_0 J_0^j - \partial_i J^{ij} = 0$$

- The general vector symmetry:

$$J^{ij} = \mathcal{J}^{[ij]} + \delta^{ij} \mathcal{J}$$

The charges

$$Q^j = \int_{space} J_0^j$$

- Nonrelativistic oneform symmetry: impose also

$$\mathcal{J} = 0$$

$Q(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$  is associated with a nontopological manifold  $\mathcal{C}$

- As in the relativistic symmetry: impose also

$$\partial_j J_0^j = 0$$

$Q(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$  is associated with a topological manifold  $\mathcal{C}$

# Conclusions

- A hierarchy of global symmetries, whose charges  $Q^i$  carry a spatial vector index.
- For every one of these we presented a large class of continuum theories exhibiting them.
  - All these examples are based on a  $U(1)$  gauge theory coupled to a special matter theory
  - We showed concrete examples of these matter theories
- We can gauge these new symmetries. The needed gauge field is an antisymmetric tensor  $B_{[\mu\nu]}$  and in the general symmetry we also need a scalar (that couples to  $\mathcal{J}$ ).