Continuum Field Theory With a Vector Global Symmetry

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Introduction

- Motivated by recent advances with fractons, but not sure this talk is relevant to fractons.
- Study global symmetries, whose conserved charge is a vector.
- One example is the momentum $P^i = \int_{space} T_0^i$. Its conserved Noether current is symmetric.
- Oneform symmetry is common in relativistic field theories.
- We'll present a hierarchical set of three symmetries, starting with the most special one and generalizing it.
- For each symmetry (for simplicity $U(1)$) we will present:
  - the conserved currents
  - a class of field theories with the symmetry
  - a concrete example based on a scalar field theory

Relativistic oneform global symmetry

A lot of earlier work, here we follow [Gaiotto, Kapustin, NS, Willett]

$$\partial_\mu J^{[\mu\nu]} = 0$$

Charges

$$\mathcal{Q}(\mathcal{C}) = \int_{\mathcal{C}} J^{[\mu\nu]} n_{[\mu\nu]}$$

$\mathcal{C}$ a codimension 2 manifold in spacetime orthogonal to $n_{[\mu\nu]}$.

$\mathcal{Q}$ is topological – it does not change under small deformations of $\mathcal{C}$. Specifically, it does not change when $\mathcal{C}$ does not cross another operator.

Relativistic oneform global symmetry

In nonrelativistic terms

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0$$
$$G = \partial_i J_0^i = 0$$

Charges at fixed time

$$\mathcal{Q}(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$$

$\mathcal{C}$ a codimension 1 manifold in space orthogonal to n_j.

$\mathcal{Q}$ is topological – it does not change under small changes of $\mathcal{C}$

Relativistic oneform global symmetry

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0$$
$$G = \partial_i J_0^i = 0$$

Example:

Maxwell theory – $U(1)$ gauge theory (either in the continuum or on the lattice)

$$J_0^j = F_0^j$$
$$J^{[ij]} = F^{[ij]}$$

$G = 0$ is Gauss law

$Q(\mathcal{C})$ is the electric flux through $\mathcal{C}$.

The charged operators are Wilson lines.

The continuum theory also has a magnetic symmetry, but we will not discuss it here.

Nonrelativistic oneform global symmetry

As before,

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0 \,,$$

but now we do not impose $G = \partial_i J_0^i = 0$.

The conservation leads to $\partial_0 G = \partial_i \partial_0 J_0^i = 0$,

i.e. G is conserved at every point, but it is not zero.

As before, conserved charges at fixed time

$$\mathcal{Q}(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j \,,$$

but now $\mathcal{Q}$ is not topological.

A cruder charge is $Q^j = \int_{space} J_0^j$.

Correspondingly, point operators can transform under Q^j.

Nonrelativistic oneform global symmetry

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0$$

Lattice example:

$U(1)$ lattice gauge theory has an electric relativistic oneform symmetry. Gauss law is imposed.

It is common, following [Kitaev], to relax Gauss law and impose it energetically.

This system has this nonrelativistic oneform symmetry.

$G = \partial_i J_0^i$ is nonzero, but it is conserved.

Related discussion in [Hermele, Fisher, Balents; Williamson, Bi, Cheng]

Nonrelativistic oneform global symmetry

A class of examples

$$\partial_0 J_0^j - \partial_i J^{[ij]} = 0$$

Couple the $U(1)$ gauge theory to charged matter fields such that

$$J_0^j = F_0^j$$
$$J^{[ij]} = F^{[ij]}$$

For that, the spatial components of the $U(1)$ current of the matter theory should vanish

$$J_i^{matter} = 0 \quad \text{and hence,} \qquad \partial_0 J_0^{matter} = 0\,.$$

Then the coupled system

$$\mathcal{L}_1 = \mathcal{L}_0 + \left(F_0^i\right)^2 - \left(F^{ij}\right)^2 + A_0 J_0^{matter}$$

with $\mathcal{L}_0$ the matter Lagrangian, is $U(1)$ gauge invariant and has the nonrelativistic oneform symmetry with $G = \partial_i J_0^i = J_0^{matter}$.

Nonrelativistic oneform global symmetry

A class of examples

$$J_i^{matter} = 0 \quad , \qquad \partial_0 J_0^{matter} = 0$$

Such a matter theory has infinitely many conserved charges.

$C(x^i)J_0^{matter}$ is conserved for every $C(x^i)$.

The charges are not mobile.

A concrete example:

A complex scalar field Φ with

$$\mathcal{L}_0 = i\overline{\Phi}\partial_0\Phi - \partial_i(\overline{\Phi}\Phi)\partial^i(\overline{\Phi}\Phi) \; - |\Phi|^4 + \cdots$$

No $\partial_i\overline{\Phi}\partial^i\Phi$ term. Highly nonstandard. Similar to fractons.

Invariance under $\Phi \to e^{iC(x^i)}\Phi$

The charge density $J_0^{matter} = |\Phi|^2$ at a point is conserved.

A more general vector symmetry

Previous case $$\partial_0 J_0^j - \partial_i J^{ij} = 0$$

With antisymmetric J^{ij}.

Generalize to

$$J^{ij} = \mathcal{J}^{[ij]} + \delta^{ij}\mathcal{J}$$

i.e. it is a sum of an antisymmetric tensor and a scalar.

Now $$\partial_0 G = \partial_i \partial_0 J_0^i = \partial_i \partial^i \mathcal{J} \neq 0\,.$$

Therefore, the charge operators $Q(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$ are no longer conserved, but the cruder charge

$$Q^j = \int_{space} J_0^j$$

is conserved.

A more general vector symmetry

A class of examples

$$\partial_0 J_0^j - \partial_i J^{ij} = 0$$
$$J^{ij} = \mathcal{J}^{[ij]} + \delta^{ij}\mathcal{J}$$

For $\mathcal{J} = 0$, we coupled charged matter with $J_i^{matter} = 0$ (and hence $\partial_0 J_0^{matter} = 0$) to a $U(1)$ gauge field.

Now, we take a matter theory with

$$\partial_0 J_0^{matter} - \partial_i \partial^i J^{matter} = 0$$

i.e. $J_i^{matter} = \partial_i J^{matter}$, and couple it to the $U(1)$ gauge field

$$\mathcal{L}_1 = \mathcal{L}_0 + \left(F_0^i\right)^2 - \left(F^{ij}\right)^2 + A_0 J_0^{matter} - A_i \partial^i J^{matter}$$

The conserved currents of the global symmetry are

$$J_0^j = F_0^j$$
$$J^{[ij]} = F^{[ij]} - \delta^{ij} J^{matter}$$

A more general vector symmetry

A class of examples

$$\partial_0 J_0^{matter} - \partial_i \partial^i J^{matter} = 0$$

Such a matter theory has the conserved charges

$$Q = \int_{space} J_0^{matter}$$

$$Q^j = \int_{space} x^j J_0^{matter}$$

They can be interpreted as:

- a global $U(1)$ symmetry (which we gauge)
- a vector symmetry, dipole symmetry, with charge Q^j

After the gauging we are left only with the vector symmetry

A more general vector symmetry

A concrete example [Pretko]

$$\partial_0 J_0^{matter} - \partial_i \partial^i J^{matter} = 0$$

A complex scalar field Φ with

$$\mathcal{L}_0 = i\bar{\Phi}\partial_0\Phi - \partial_i(\bar{\Phi}\Phi)\partial^i(\bar{\Phi}\Phi) - |\Phi|^4$$
$$+i\left(\bar{\Phi}^2\partial_i\Phi\partial^i\Phi - \Phi^2\partial_i\bar{\Phi}\partial^i\bar{\Phi}\right) + \cdots$$

Again, No $\partial_i\bar{\Phi}\partial^i\Phi$ term.

The new term $i\left(\bar{\Phi}^2\partial_i\Phi\partial^i\Phi - \Phi^2\partial_i\bar{\Phi}\partial^i\bar{\Phi}\right)$ breaks the symmetry

$\Phi \to e^{iC(x^i)}\Phi$ to $\Phi \to e^{i\alpha + ic_i x^i}\Phi$.

Here

$$J_0^{matter} = |\Phi|^2$$
$$J^{matter} = |\Phi|^4$$

A possible further generalization

$$\partial_0 J_0^j - \partial_i J^{ij} = 0$$

J^{ij} includes an antisymmetric tensor, a scalar, and a symmetric tensor.

We have not analyzed it in detail.

Summary of the symmetries

$$\partial_0 J_0^j - \partial_i J^{ij} = 0$$

- The general vector symmetry:

$$J^{ij} = \mathcal{J}^{[ij]} + \delta^{ij}\mathcal{J}$$

The charges $$Q^j = \int_{space} J_0^j$$

- Nonrelativistic oneform symmetry: impose also

$$\mathcal{J} = 0$$

$\mathcal{Q}(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$ is associated with a nontopological manifold $\mathcal{C}$

- As in the relativistic symmetry: impose also

$$\partial_j J_0^j = 0$$

$\mathcal{Q}(\mathcal{C}) = \int_{\mathcal{C}} J_0^j n_j$ is associated with a topological manifold $\mathcal{C}$

Conclusions

- A hierarchy of global symmetries, whose charges Q^i carry a spatial vector index.
- For every one of these we presented a large class of continuum theories exhibiting them.
  - All these examples are based on a $U(1)$ gauge theory coupled to a special matter theory
  - We showed concrete examples of these matter theories
- We can gauge these new symmetries. The needed gauge field is an antisymmetric tensor $B_{[\mu\nu]}$ and in the general symmetry we also need a scalar (that couples to $\mathcal{J}$).