Gov 50 Week 9

Scatterplots, Correlation, & Linear Regression (Answers)

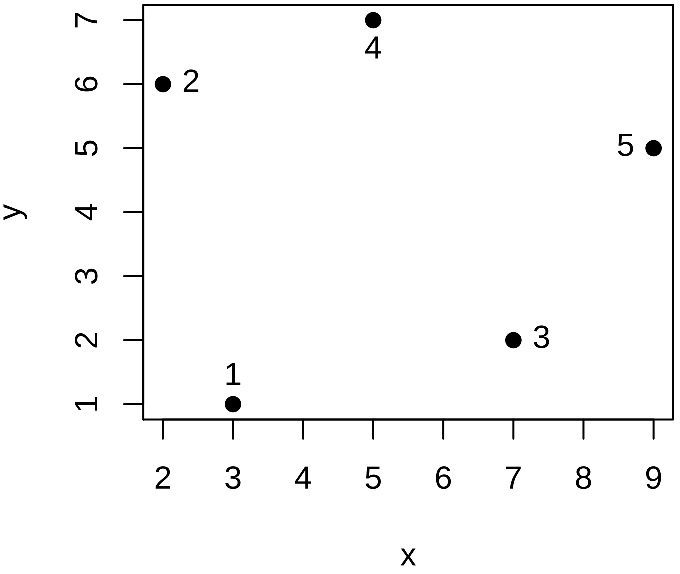
Recitation Handout *TF: Michael Gill*

1 Scatterplot

A scatterplot is a way to visually display the relationship between two interval-level variables.

Quickly fill in the data table on the left using the scatter plot below:

Obs.	x	y
1	3	1
2	2	6
3	7	2
4	5	7
5	9	5

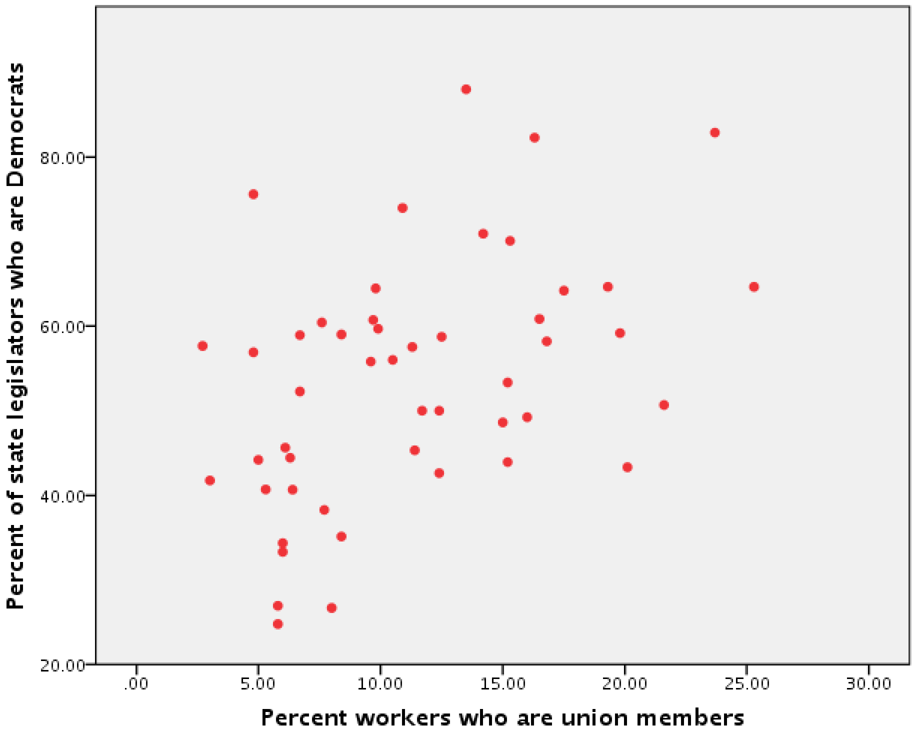


1.1 Scatterplots in SPSS

To do a scatterplot in SPSS, you simply need to use the Graphs → Chart Builder command. Pick the Simple Scatter and the just drag your independent variable to the x-axis and the dependent variable to the y-axis. Then hit OK. Once you see the scatterplot in your output, you can edit it to add lines or change colors by double-clicking on it.

1.2 Examples

1) You work get a job at a fancy newspaper and they ask you to write a story about the recent political unrest with regard to organized labor. To support your argument about the relationship between organized labor and Democrats, they ask you to include a scatterplot showing the relationship between the percent of workers who are union members in a state (`union`) and the percent of the state legislators who are Democrats (`demstate`). What direction is the relationship? Does it appear linear?



Positive, fairly linear.

2 Correlation

Pearson's correlation coefficient is a measure of the direction and strength of the relationship between two interval-level variables. If the two variables are X and Y with averages $\bar{X}$ and $\bar{Y}$, and estimated standard deviations s_x and s_y, then the correlation coefficient is:

$$r = \frac{\sum_i \left(\frac{X_i - \bar{X}}{s_x}\right)\left(\frac{Y_i - \bar{Y}}{s_y}\right)}{n-1} \tag{1}$$

which we could write as:

$$r = \frac{\sum_i (Z\text{-scores for } X)(Z\text{-scores for } Y)}{n-1} \tag{2}$$

That is, we want to know how often observations fall in same place in their distribution for both variables. For instance, if wealth and education have a high positive correlation this means that people who are on the upper end of the wealth distribution will be on the upper end of the education distribution.

If a country's GDP per capita has a high negative correlation with its levels of political violence (in, say, number of political violence deaths per 1,000 citizens), then those countries on the low end of the GDP distribution will be on the high end of the political violence distribution.

2.1 Example

Let's say you wanted to calculate the Pearson correlation coefficient for the following two variables:

x	y
1	17
2	14
3	16
4	10
5	8

Note that $\bar{X} = 3.0$, $\bar{Y} = 13.0$, $s_x = 1.6$, and $s_y = 3.9$.

First, calculate all the Z-scores for X:

$$\left(\frac{1-3}{1.6}\right), \left(\frac{2-3}{1.6}\right), \left(\frac{3-3}{1.6}\right), \left(\frac{4-3}{1.6}\right), \left(\frac{5-3}{1.6}\right)$$

which are just

$$-1.25, \ -0.625, \ 0, \ 0.625, \ 1.25$$

. Now, do the same for Y:

$$\left(\frac{17-13}{3.9}\right), \left(\frac{14-13}{3.9}\right), \left(\frac{16-13}{3.9}\right), \left(\frac{10-13}{3.9}\right), \left(\frac{8-13}{3.9}\right)$$

which is just

$$1.03, \ 0.26, \ 0.76, \ -0.76, \ -1.28$$

. Multiply each Z-score from the the X column by its partner in the Y column and add:

$$(-1.25 \times 1.03) + (-0.625 \times 0.26) + (0 \times 0.76) + (0.625 \times -0.76) + (1.25 \times -1.28) = -3.53.$$

Finally, just divide by $n-1$, which is $5-1=4$ in this case:

$$-\frac{3.52}{4} = \text{-0.88}$$

2.2 Correlation in SPSS

To get a correlation matrix in SPSS, simply use the Analyze → Correlate → Bivariate command.

2) After your article hits the front page, you get a letter from a large number of readers that want more detail than just the scatterplot and what they call your "impressions." Calculate the correlation between `union` and `demstate` to bolster your claims. Is the relationship significant?

The correlation is 0.439 and the p-value is 0.002, which is significant at the 0.05 and 0.01 levels.

3 Linear Regression

Linear regression is a way to to see how our dependent variables changes as our independent variable changes. We are going to assume that we can write the relationship down as a linear function:

$$Y = a + bX \tag{3}$$

which you might remember from algebra looks suspiciously like $y = mx + b$. All we are doing here is trying to estimate a line that summarizes the relationship between variables Y and X. In this class, the properties of the line will be parameters in the population that we want to estimate.

$$Y = \alpha + \beta * X \tag{4}$$

$$\alpha = \text{intercept} \tag{5}$$

$$\beta = \text{coefficient} \tag{6}$$

POPQUIZ!!!! Could we ever know α and β? Why or why not?

Not for certain! They are population parameters that we want to make inferences about, just like the population mean or the population correlation. We have to calculate sample statistics (or estimates) of these quantities: $\hat{\alpha}, \hat{\beta}$.

3.1 Linear Regression in SPSS

To run a linear regression in SPSS, simply use the Analyze → Regression → Linear. Input your independent and dependent variables and hit OK.

3) The last letter you get is an angry letter from your Gov50 TF asking you why you didn't run a linear regression to explain Democratic performance in the state legislature as a function of union membership. What would be the independent and dependent variables for this regression? What are the estimated coefficient and intercept? Interpret these values.

The dependent variable is the one we want to explain, so it is the Democrat performance (`demstate`). The independent variable is the one doing the explaining, so it is union membership (`union`). The coefficient in this case is 1.13, which means that a one-percent increase in union membership is associated with a 1.13 percentage point increase in the percent of state legislators that are Democrats. The intercept is 40.75, which is the percent of Democratic state legislators in states with no union membership.