
Network Causal Inference on Social Media Influence Operations

**Harvard Applied Statistics Workshop (Gov 3009)
Oct 31st, 2018**

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Joint work with Olga Simek, Danelle C. Shah, and Donald B. Rubin



HARVARD UNIVERSITY
DEPARTMENT OF STATISTICS

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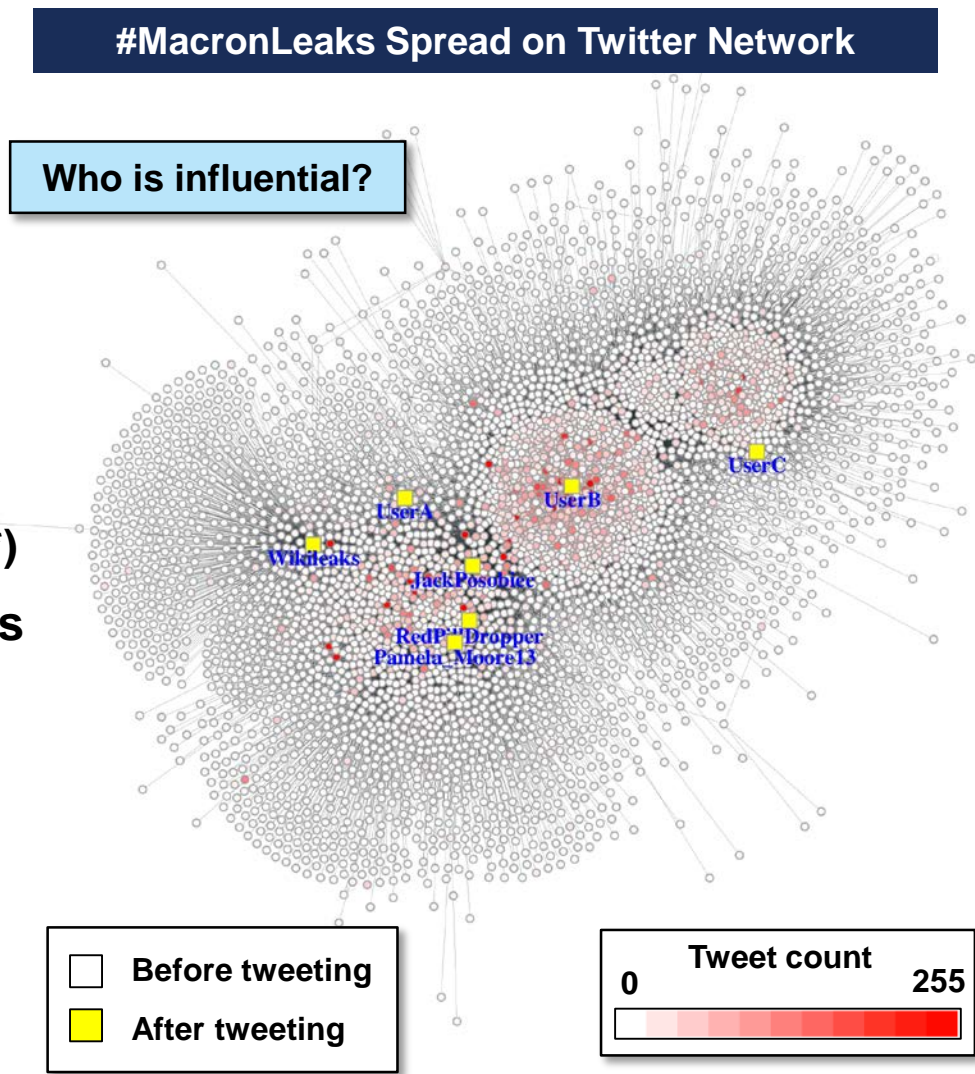
Outline

- ➔ • **Motivation and introduction**
- **Network potential outcome causal framework**
 - Basic building block: network potential outcomes
 - Theories for design and analysis to address network confounders
- **Application on social media influence operations**
 - Case study: 2017 French Presidential Election



Motivation for Network Causal Inference

- How do we quantify the social impact of certain individuals on a network?
- Network causal inference provides a framework to quantify impact
 - Attributes impact correctly
 - Correlation \neq Causation
 - Disentangle impact from network confounders (e.g. homophily*)
 - Predictive inference guides optimal “campaign” strategies
- Many applications
 - Marketing, public health, education, etc.
 - Security: influence operations on social media





Publications for This Talk

- Smith, Kao, Simek, Shah, and Rubin, *Influence estimation on social media networks using causal inference*, in *Proc. IEEE SSP* (2018) (patent pending)
- Kao, *Causal inference under network interference: A framework for experiments on social networks*. Ph.D. Thesis, Harvard University (2017)
- Kao, Airoidi, and Rubin, *Causal inference under network interference: A network potential outcome framework with Bayesian imputation*, in preparation



Causal Inference Under Network Interference: Open Area for Methodology Work

- **Early work (interference as nuisance):**

- Designs for interference (David & Kempton, 1996, Azais & Bailey, 1993)

- **Hypothesis testing on the presence of effects**

- Interference between units in randomized experiments (Rosenbaum, 2007, Bowers et. al. 2013)
- Exact P-values for network interference via artificial experiment (Athey, Eckles, & Imbens, 2015) and conditioning mechanism (Basse, Feller, & Toulis 2018)

- **Estimation of specific causal effects:**

- Two-staged randomization (Hudgens & Halloran, 2008)
- Inverse-probability weighting (Aronow & Samii, 2012)
- Graph cluster randomization (Ugander et al., 2013)
- Design and estimation under specific structures of network interference (Sussman & Airoidi 2017)

- **Entanglement with social confounders:**

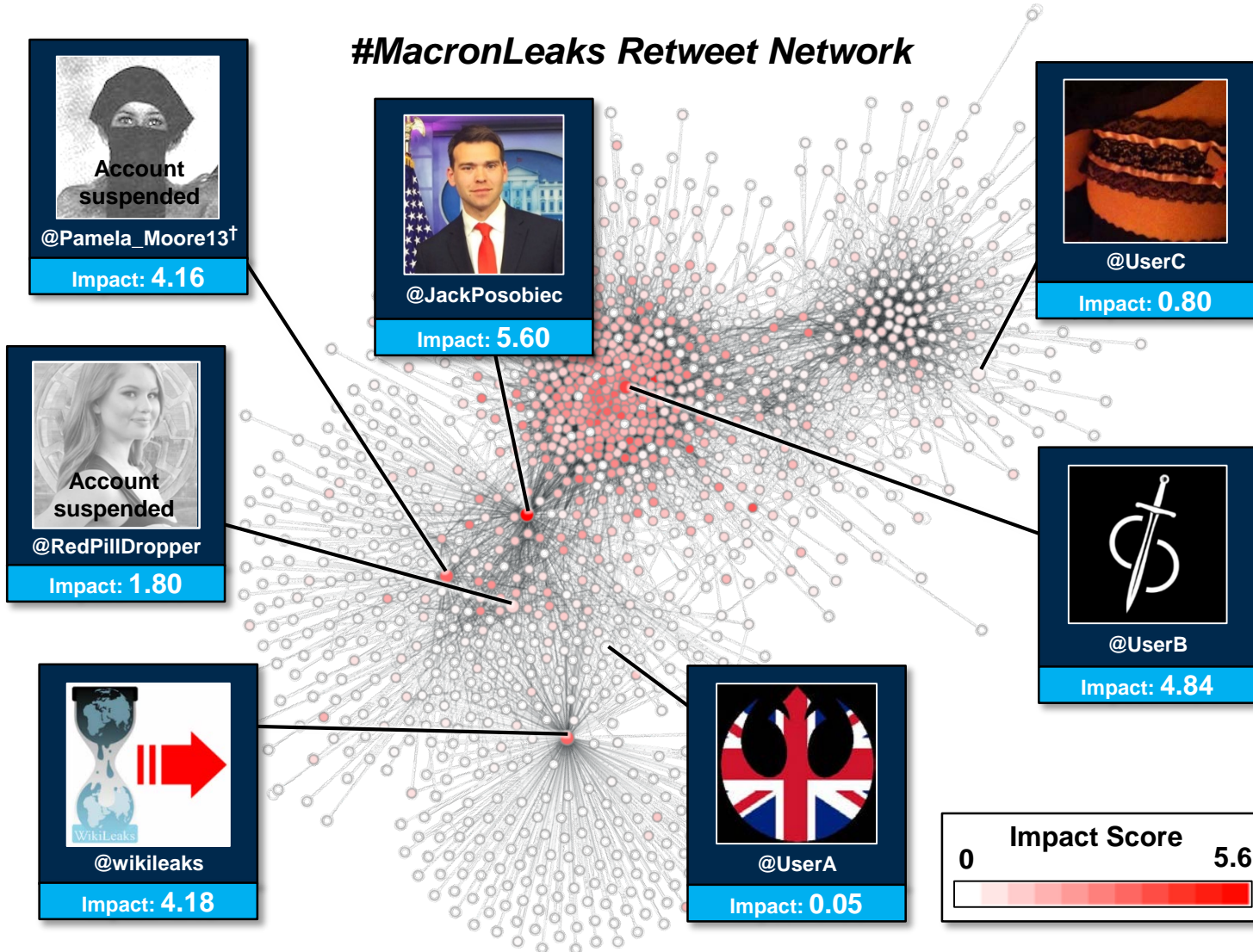
- Unidentifiability of peer effects among social confounders (Manski, 1993, Shalizi & Thomas 2011)
- Causal diagram for interference (Ogburn & Vanderweele, 2014)

We propose a framework for estimating general causal effects under network interference via principled design and estimation



Causal Impact Estimation on #MacronLeaks Narrative

#MacronLeaks Retweet Network



Screen name	T	RT	F	Earliest time	Pagerank Centrality	Impact*
@JackPosobiec	95	47k	261 k	18:49	2.84	5.60
@RedPillDropper	32	8k	8 k	19:33	2.86	1.80
@UserA	256	59k	1 k	19:34	27.08	0.05
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@wikileaks	25	63k	5515k	20:32	2.80	4.18
@Pamela_Moore13†	4	4k	54 k	21:14	2.79	4.16
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Tweets (T), Retweets (RT), Followers (F), Causal influence estimate*

- Causal impact score measures contribution to narrative flow on the network, beyond activity-based and topological statistics
- High impact accounts corroborated with evidence from the U.S. Congress† and journalistic reports



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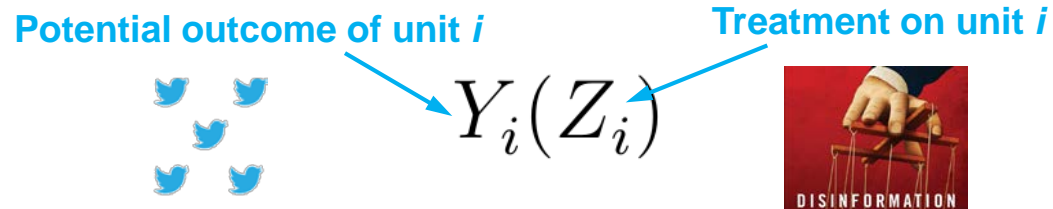
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Regular Causal Inference: Potential Outcome Framework* and Causal Estimand

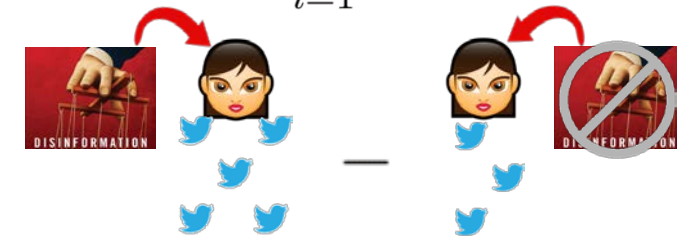
Regular potential outcomes:



Unit	Potential outcomes		Observed outcomes	
	$Z_i = 0$	$Z_i = 1$	$Z_i = 0$	$Z_i = 1$
1	$Y_1(0)$	$Y_1(1)$	$Y_1(0)$?
2	$Y_2(0)$	$Y_2(1)$?	$Y_2(1)$

Population average treatment effect:

$$\tau_{ATE} = \frac{1}{N} \sum_{i=1}^N Y_i(1) - Y_i(0)$$



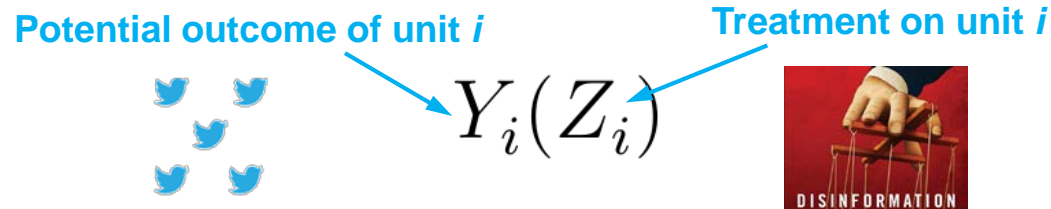
Averaged over the population of N accounts





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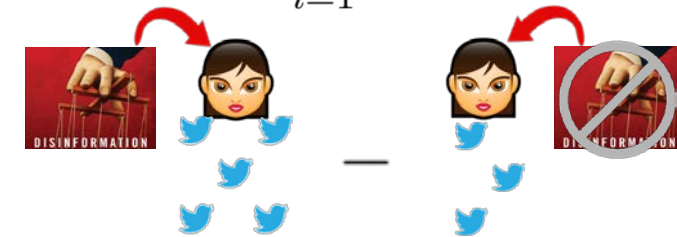
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Averaged over the population of N accounts



1. Outcome of a unit only depends on its own treatment
2. Estimating causal effect is essentially filling in the missing outcomes, by computing:

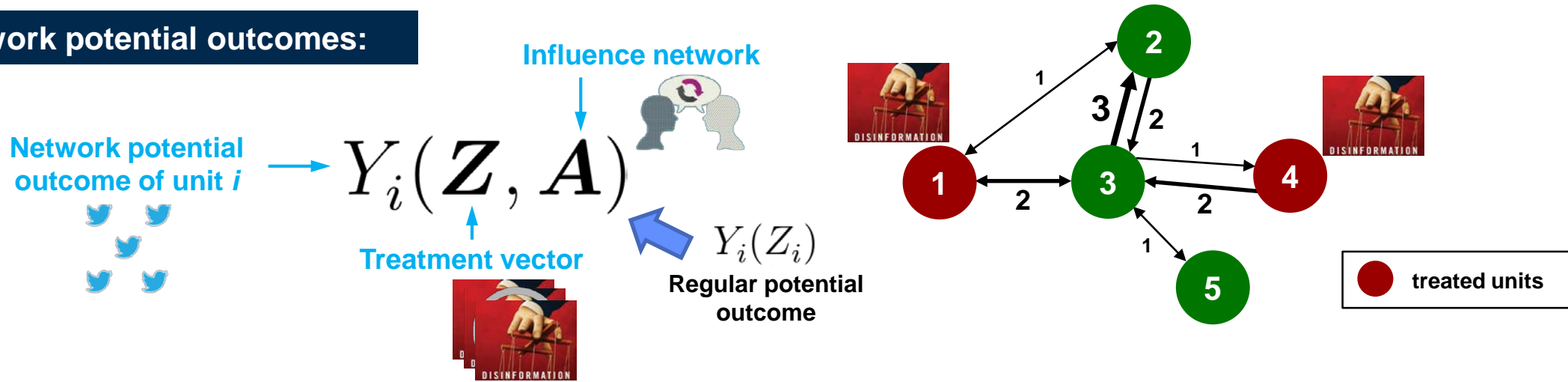
$$P(Y_{\text{mis}} | X, Z, Y_{\text{obs}})$$

Unit covariates (attributes) $\rightarrow X$
 Observed outcomes $\rightarrow Y_{\text{obs}}$
 Missing outcomes $\rightarrow Y_{\text{mis}}$
 Treatment vector $\rightarrow Z$



Network Causal Inference: Network Potential Outcome Framework*

Network potential outcomes:

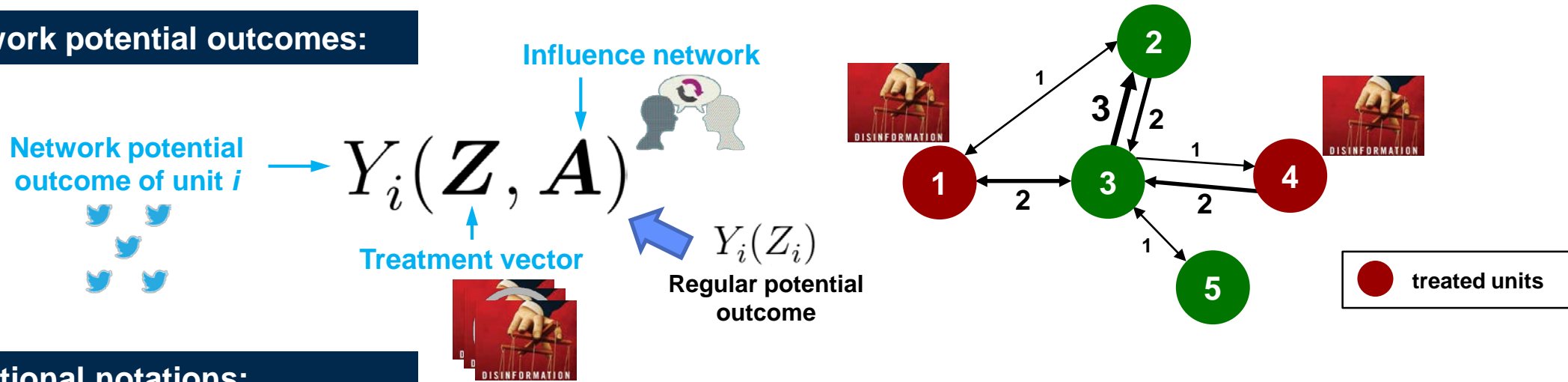


Network potential outcomes may be affected by treatments on other units due to social influence on the network



Network Causal Inference: Network Potential Outcome Framework*

Network potential outcomes:



Additional notations:

- The set of all network potential outcomes for unit i : $\mathbb{Y}_i = \{Y_i(\mathbf{Z} = \mathbf{z}, \mathbf{A} = \mathbf{a})\}$ for $\forall \mathbf{z}, \mathbf{a}$
- The set of all network potential outcomes: $\mathbb{Y} = \{\mathbb{Y}_i\}$ for $\forall i$
- The set of observed and unobserved network potential outcomes are \mathbb{Y}_{obs} and \mathbb{Y}_{mis}

Network potential outcomes may be affected by treatments on other units due to social influence on the network



Causal Estimands for Influence Operations

Average Effect of Treatment on One Individual (Individual Impact):

$$\zeta_k(\mathbf{z}) \stackrel{\text{def}}{=} \frac{1}{N} \sum_{i=1}^N (Y_i(\mathbf{z}_{k+}, \mathbf{A}) - Y_i(\mathbf{z}_{k-}, \mathbf{A}))$$

↑ Outcome of unit i , when k is treated Outcome of unit i , when k is NOT treated

where \mathbf{z}_{k+} is with unit k treated
and \mathbf{z}_{k-} without

Average Effect of Network Manipulation:

$$\zeta_{A \rightarrow A'}(\mathbf{z}) \stackrel{\text{def}}{=} \frac{1}{N} \sum_{i=1}^N (Y_i(\mathbf{z}, \mathbf{A}') - Y_i(\mathbf{z}, \mathbf{A}))$$

↑ Outcome of unit i , under network A' Outcome of unit i , under network A

Estimate the missing outcomes, by computing:

Unit covariates (attributes) Influence network

$$P(Y_{mis} | \mathbf{X}, \mathbf{Z}, \mathbf{A}, Y_{obs})$$

↑ Missing outcomes Treatment vector Observed outcomes

Define the causal estimands to quantify individual impact and the effect of a specific influence network manipulation



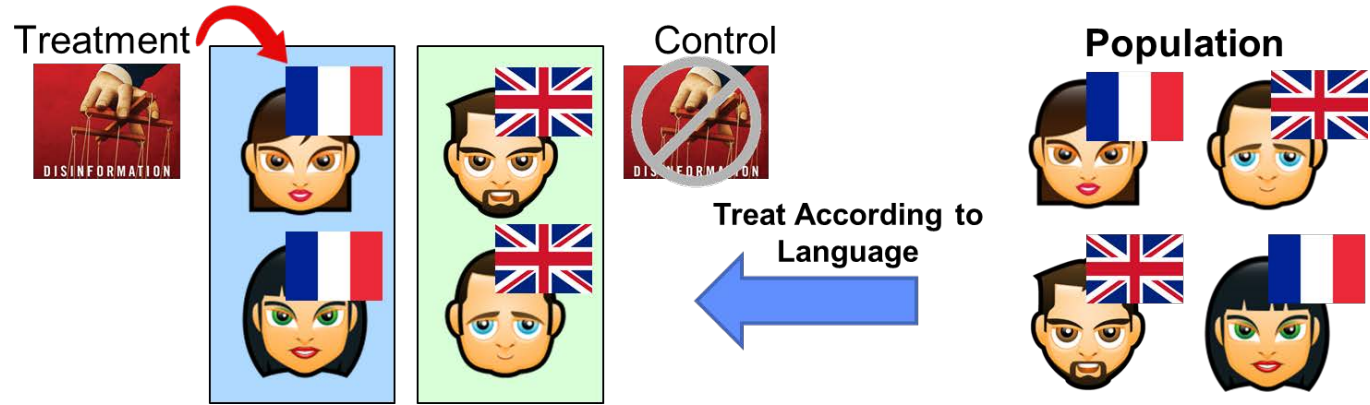
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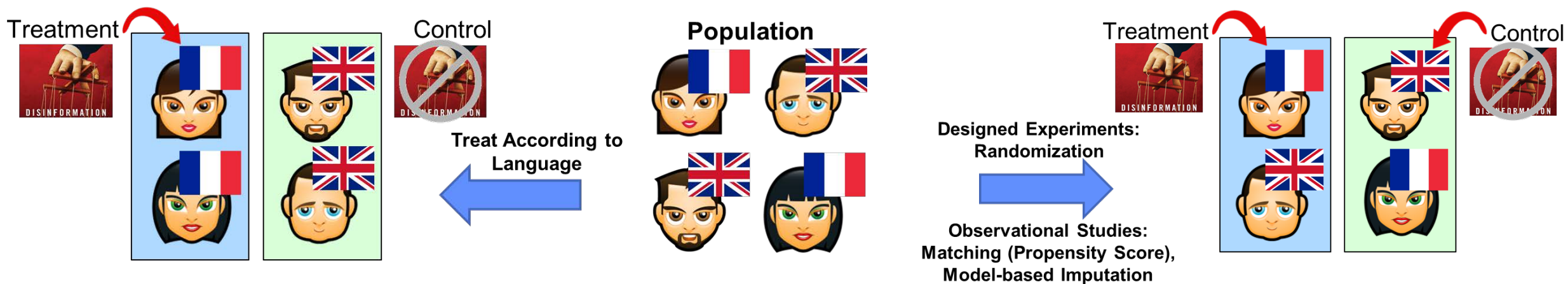
Regular Causal Inference: Overcoming Selection Bias



Problem: Language effect on outcomes leads to biased causal estimate, if language is a confounder.



Regular Causal Inference: Overcoming Selection Bias

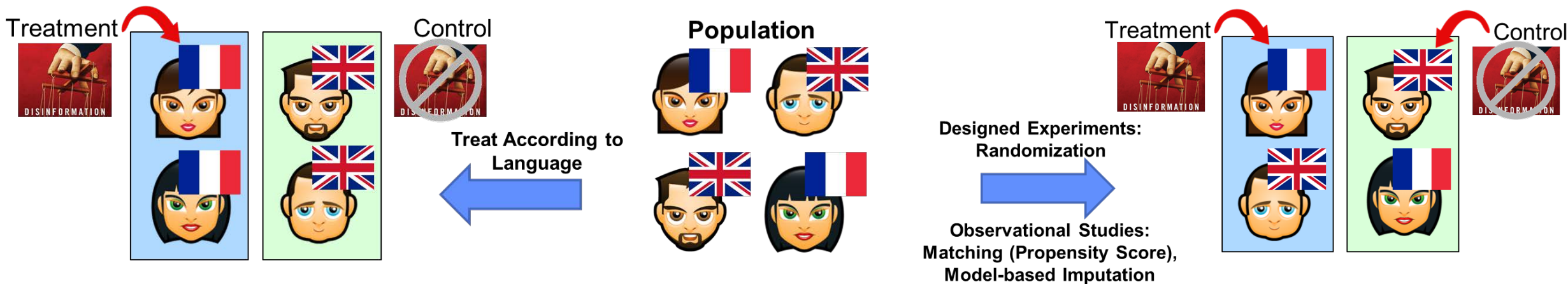


Problem: Language effect on outcomes leads to biased causal estimate, if language is a confounder.

Solution: Balancing language between treatment and control groups or adjusting for it in the estimation.



Regular Causal Inference: Overcoming Selection Bias



Problem: Language effect on outcomes leads to biased causal estimate, if language is a confounder.

Solution: Balancing language between treatment and control groups or adjusting for it in the estimation.

What are the confounding covariates that need to be accounted for?

They are the unit covariates \mathbf{X} , when conditioned on, leads to independence between treatment assignment \mathbf{Z} and the potential outcomes:

$$P(\mathbf{Z}|\mathbf{X}, \mathbf{Y}) = P(\mathbf{Z}|\mathbf{X}, \mathbf{Y}') \quad \text{for all } \mathbf{Z}, \mathbf{X}, \mathbf{Y}, \text{ and } \mathbf{Y}'$$

This also simplifies the computation of missing outcomes: $P(\mathbf{Y}_{\text{mis}}|\mathbf{X}, \mathbf{Z}, \mathbf{Y}_{\text{obs}}) = P(\mathbf{Y}_{\text{mis}}|\mathbf{X}, \mathbf{Y}_{\text{obs}})$



Overcoming Selection Bias from Treatment and Network Confounders

Desired property indicating all confounders are accounted for in X

$$P(\mathbb{Y}_{mis} | \mathbf{X}, \mathbf{Z}, \mathbf{A}, \mathbb{Y}_{obs}) = P(\mathbb{Y}_{mis} | \mathbf{X}, \mathbb{Y}_{obs})$$

Unit covariates (attributes) Influence network
Missing outcomes Treatment vector Observed outcomes

How to obtain this property?



Overcoming Selection Bias from Treatment and Network Confounders

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How to obtain this property?

Unconfounded Treatment Condition

$$P(\mathbf{Z} | \mathbf{X}, \mathbf{A}, \mathbb{Y}) = P(\mathbf{Z} | \mathbf{X}, \mathbf{A}, \mathbb{Y}')$$

for all $\mathbf{Z}, \mathbf{X}, \mathbf{A}, \mathbb{Y}$, and \mathbb{Y}'

Met if treatment is completely random or determined by the covariates and network



Overcoming Selection Bias from Treatment and Network Confounders

Desired property indicating all confounders are accounted for in X

$$P(Y_{mis} | X, Z, A, Y_{obs}) = P(Y_{mis} | X, Y_{obs})$$

Unit covariates (attributes) Influence network
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How to obtain this property?

Unconfounded Treatment Condition

$$P(Z | X, A, Y) = P(Z | X, A, Y')$$

for all Z, X, A, Y , and Y'

Unconfounded Network Condition

$$P(A | X, Y) = P(A | X, Y')$$

for all A, X, Y , and Y'

Disentangle network confounders such as homophily

Met if treatment is completely random or determined by the covariates and network

Meet this condition by including the confounding network model parameters in X :

$$A \sim H_G(X_G)$$

$$X \supseteq \tilde{X}_G$$

e.g. community membership, degree distribution



Overcoming Selection Bias from Treatment and Network Confounders

Desired property indicating all confounders are accounted for in X

$$P(Y_{mis} | X, Z, A, Y_{obs}) = P(Y_{mis} | X, Y_{obs})$$

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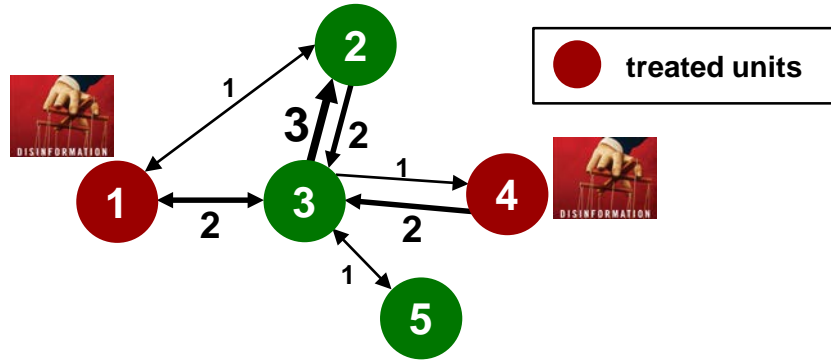
The confounders X should be accounted for via both balancing and estimation adjustment



Toy Example: Why is Unconfounded Influence Network Important?

Simulated experiment: estimate social impact

Treatments are completely randomized



$$Z = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Influence network

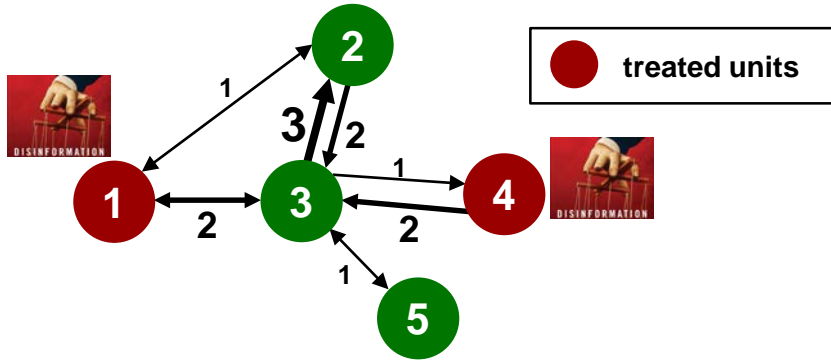
$$A = \begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 2 & 3 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



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Network confounder: activity level

$$X = \begin{bmatrix} 0.4 \\ 0.7 \\ 1.4 \\ 0.3 \\ 0.2 \end{bmatrix}$$

Outcome model:

$$Y(Z, A) = \underbrace{\tau Z}_{\text{Primary effect}} + \underbrace{\gamma A^T Z}_{\text{Social effect}} + \underbrace{\beta X + \mu + \epsilon}_{\text{Covariate effect, Constant effect, Random effect}}$$

$$Y_{obs} = \begin{bmatrix} 34.02 \\ 11.91 \\ 33.79 \\ 32.93 \\ 2.07 \end{bmatrix}$$

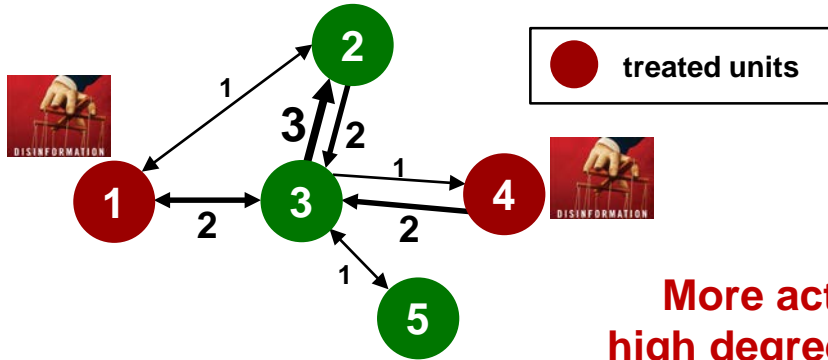
$$\begin{aligned} \tau &= 30 \\ \gamma &= 5 \\ \beta &= 10 \\ \mu &= 0 \end{aligned}$$



Toy Example: Why is Unconfounded Influence Network Important?

Simulated experiment: estimate social impact

Treatments are completely randomized



Network confounder: activity level

More active units have high degrees in the network

$$Z = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Influence network



$$A = \begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 2 & 3 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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Treatment effects Individual baseline

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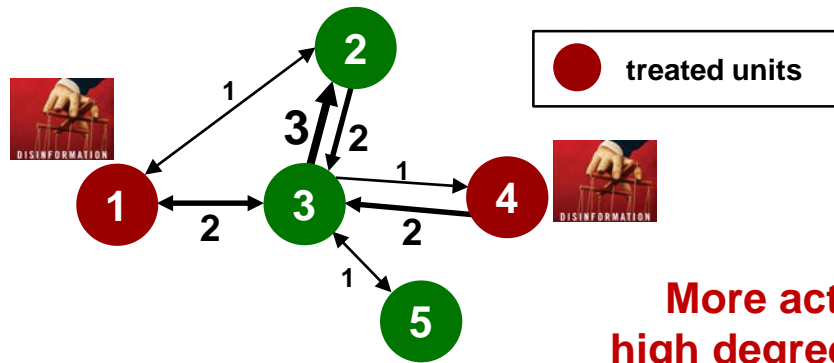
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● treated units

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$$\begin{aligned} \tau &= 30 \\ \gamma &= 5 \\ \beta &= 10 \\ \mu &= 0 \end{aligned}$$

Social effect coefficient	True Value	Estimation with X (90% PI*)	Estimation w/o X (90% PI*)
γ	5	(4.22, 5.97)	(7.59, 7.95)

* 90% posterior interval obtained through Bayesian regression with weakly informative prior

Not conditioning on X breaks the unconfounded influence network condition and leads to biased social effect causal estimate



Outline

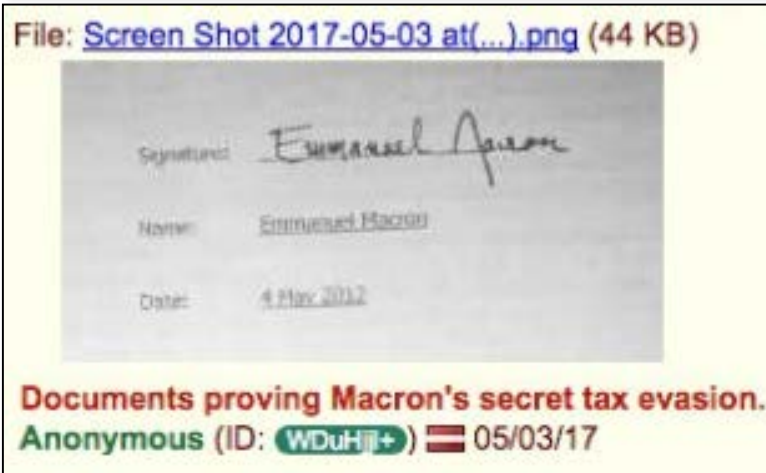
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IO Narratives in 2017 French Presidential Election

Cyber



Hacks and Leaks

*“Encrypted data flowing in public communication channels will be among the coveted targets for cyber-attacks”**

Social Media



False amplification via bots and inauthentic “sock puppet” accounts

Traditional Media



Legitimization of fringe narratives; Information manipulation through bias, slant, distortion, omission

During the 2017 French elections IO campaigns were waged on multiple fronts, exploiting global information technologies and networks.

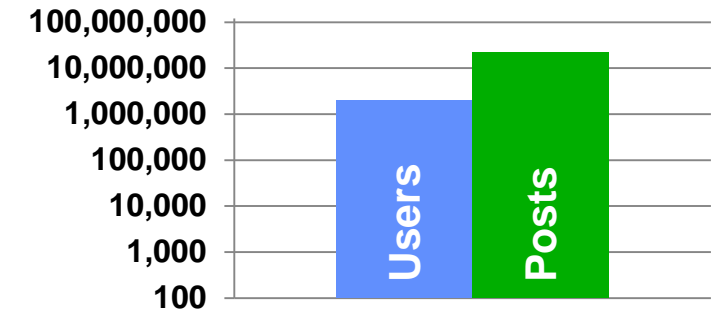


Detection of IO Narratives on Social Media

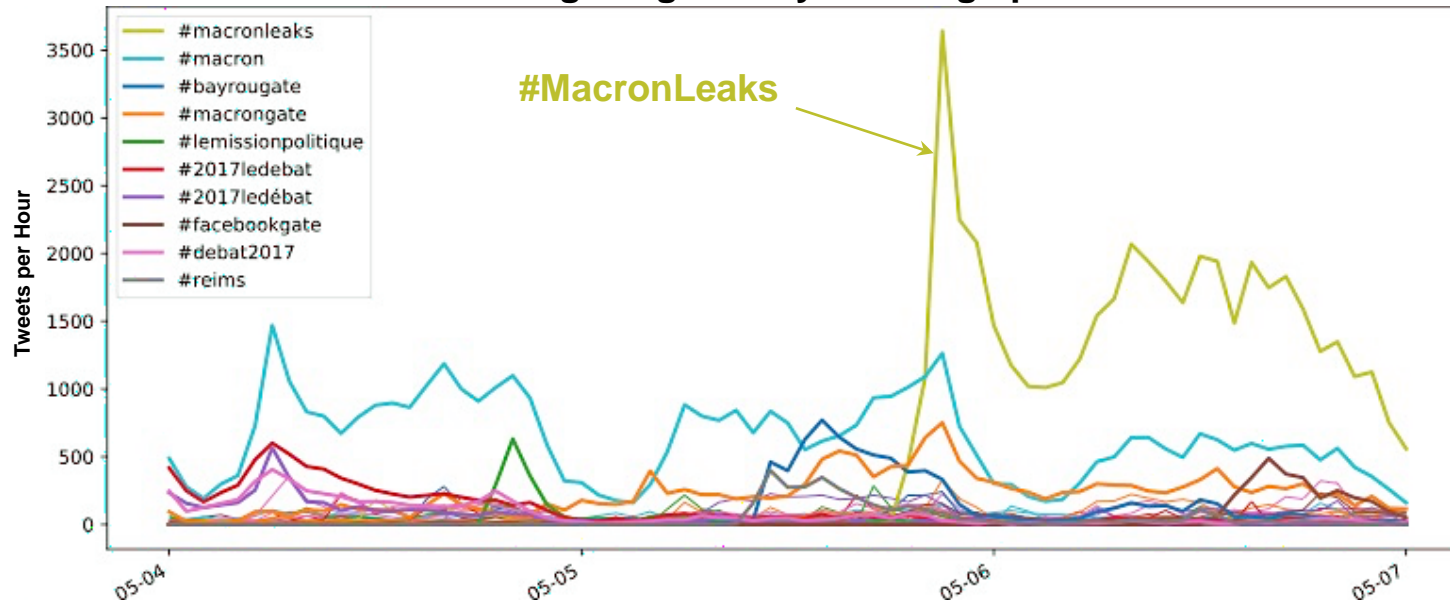
2017 French Elections

- On May 5, 2017 thousands of internal *En Marche!* documents were leaked online
- #MacronLeaks IO campaign involved sources, coordinated amplifiers, and bots

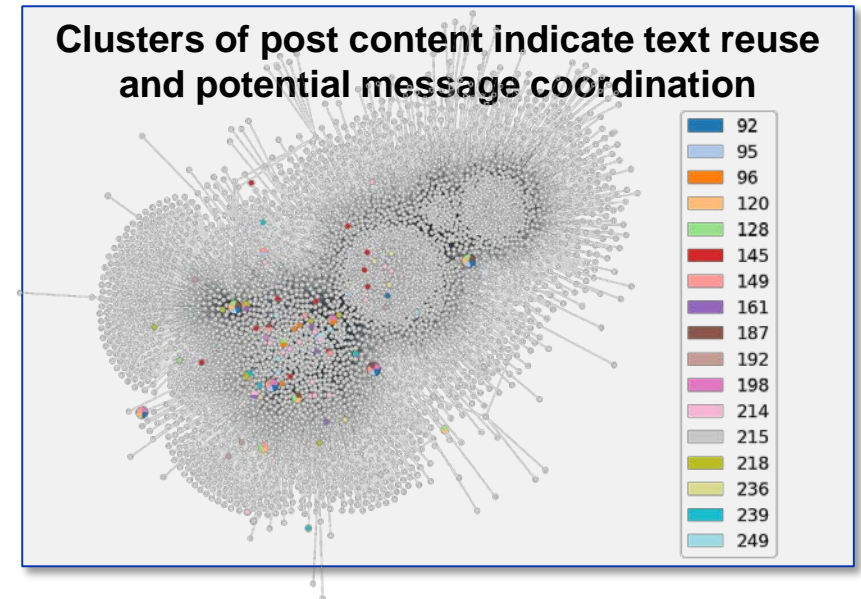
370 GB social media collected April–May 2017



Twitter hashtag usage in days leading up to election



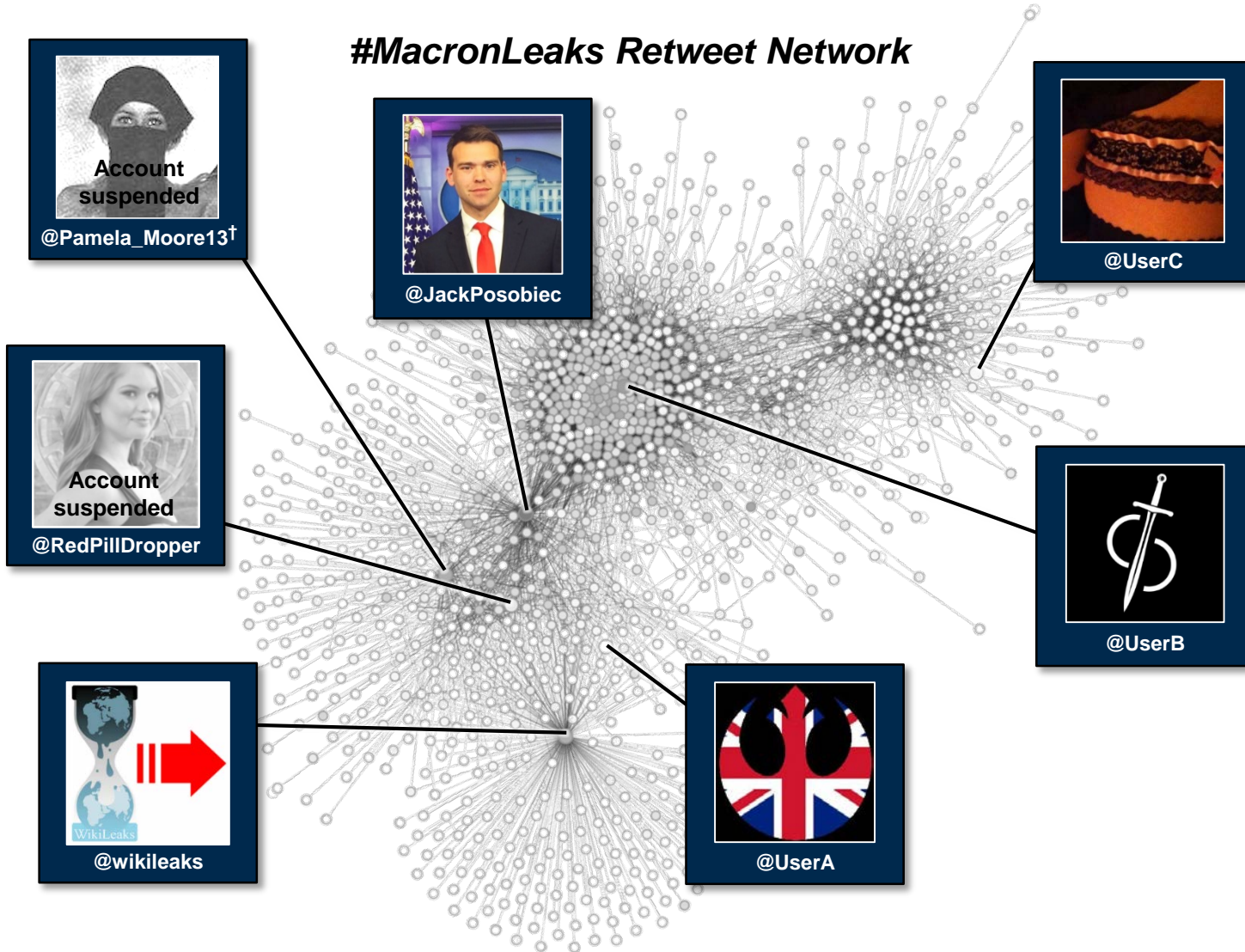
Clusters of post content indicate text reuse and potential message coordination





Causal Impact Estimation on #MacronLeaks Narrative

#MacronLeaks Retweet Network



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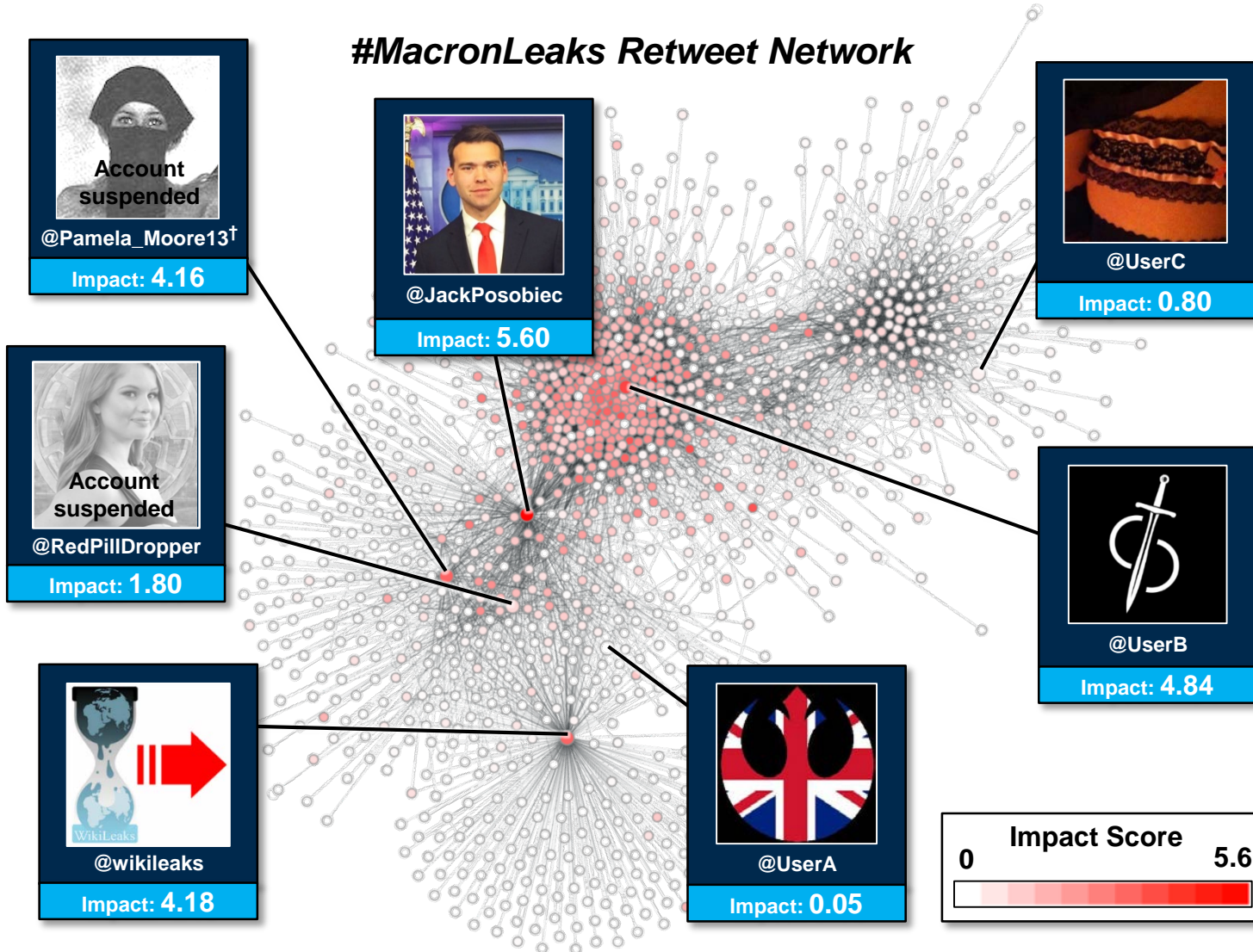
Tweets (T), Retweets (RT), Followers (F)

- “Impact” is often quantified by count-based statistics or network metrics (e.g., retweets or centrality)
- These measures do not fully capture the extent to which the network exposure of a narrative/rumor can be attributed to any particular individual(s)



Causal Impact Estimation on #MacronLeaks Narrative

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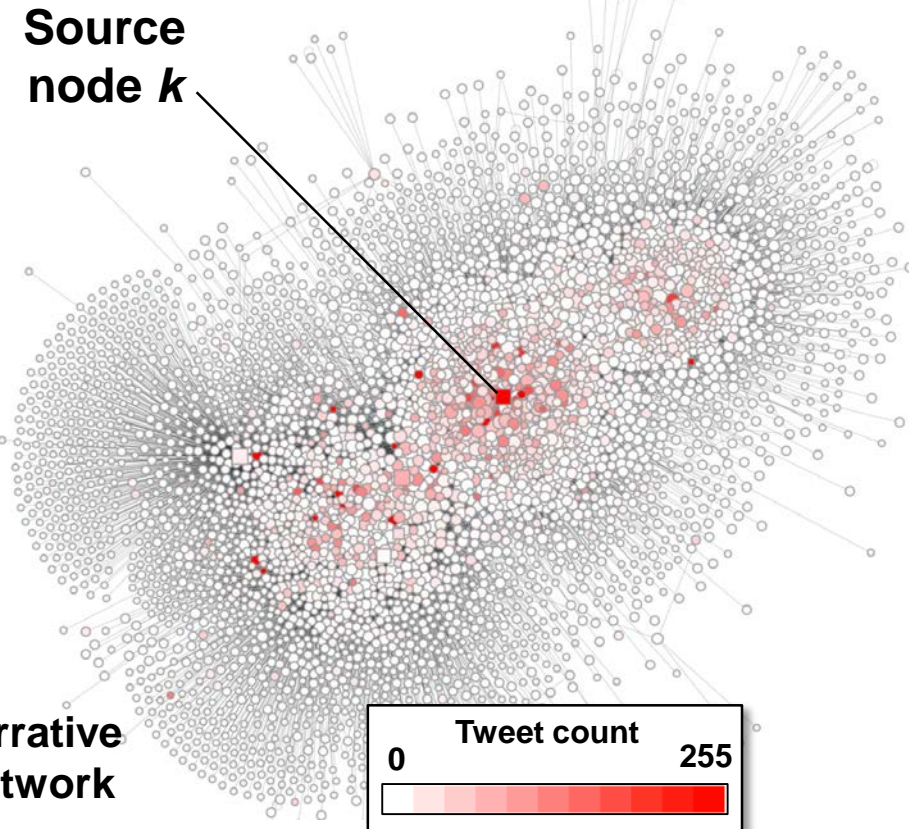


Network Causal Inference for Impact Estimation*

Impact Estimand:

$$\zeta_k = \text{Average}[Y_i(z_{k+}, A) - Y_i(z_{k-}, A)]$$

Observations Imputations



- Outcomes are the individual activities on the narrative (e.g. tweet counts)
- Explicitly measures each account's contribution to the outcomes



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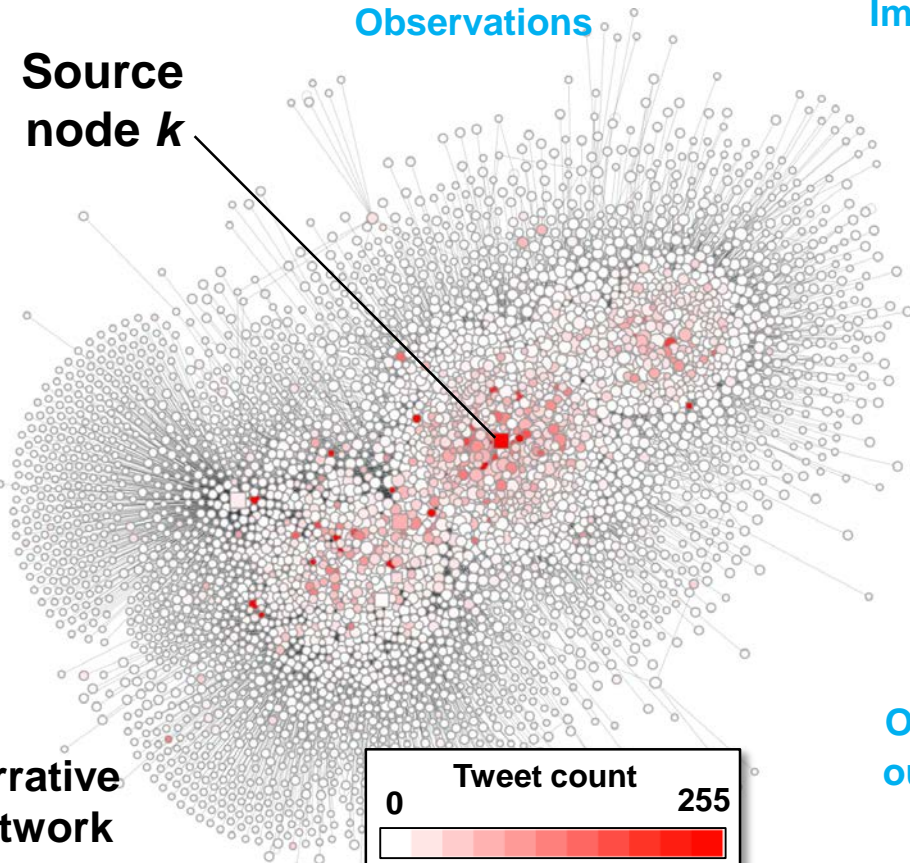
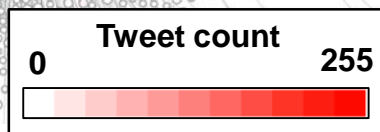
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Observations

Imputations

Source node k

Narrative Network



- Outcomes are the individual activities on the narrative (e.g. tweet counts)
- Explicitly measures each account's contribution to the outcomes
- Outcome model accounts for narrative propagation on the network

Network potential outcome model:

$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\log \lambda_i = \tau Z_i + \Sigma \Pi \tau \gamma_j s_i + \beta^T x_i + \mu + \epsilon_i$$

Exposure to source

Individual baseline

Observed outcomes $Y(z_{k+})$



MCMC



$P(\tau, \gamma, \beta, \mu, \epsilon | Y)$



$\lambda(\tau, \gamma, \beta, \mu, \epsilon)$



Imputed outcomes $Y(z_{k-})$

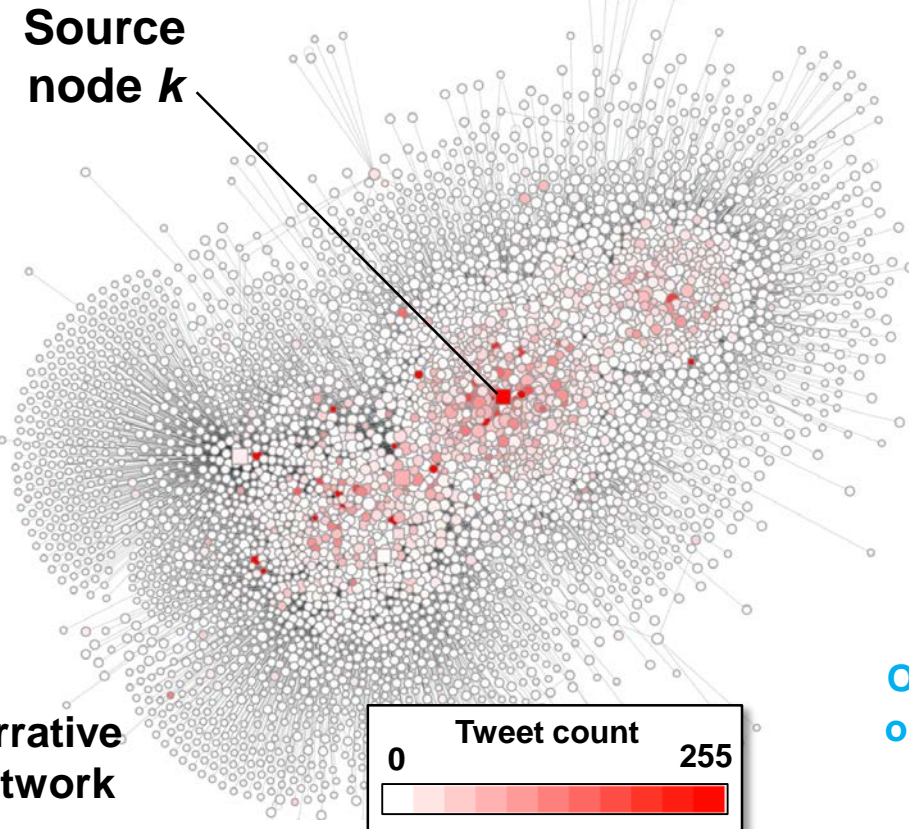


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Observations Imputations



- Outcomes are the individual activities on the narrative (e.g. tweet counts)
- Explicitly measures each account's contribution to the outcomes
- Outcome model accounts for narrative propagation on the network
- Causal framework disentangles confounders (e.g. homophily) from social influence

Network potential outcome model:

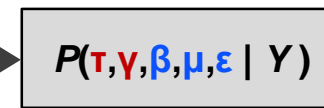
$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\log \lambda_i = \tau Z_i + \sum_j \Pi \tau \gamma_j s_j + \beta^T x_i + \mu + \epsilon_i$$

Exposure to source Individual baseline

Accounts for confounders (e.g. node degrees and community membership)

Observed outcomes $Y(z_{k+})$

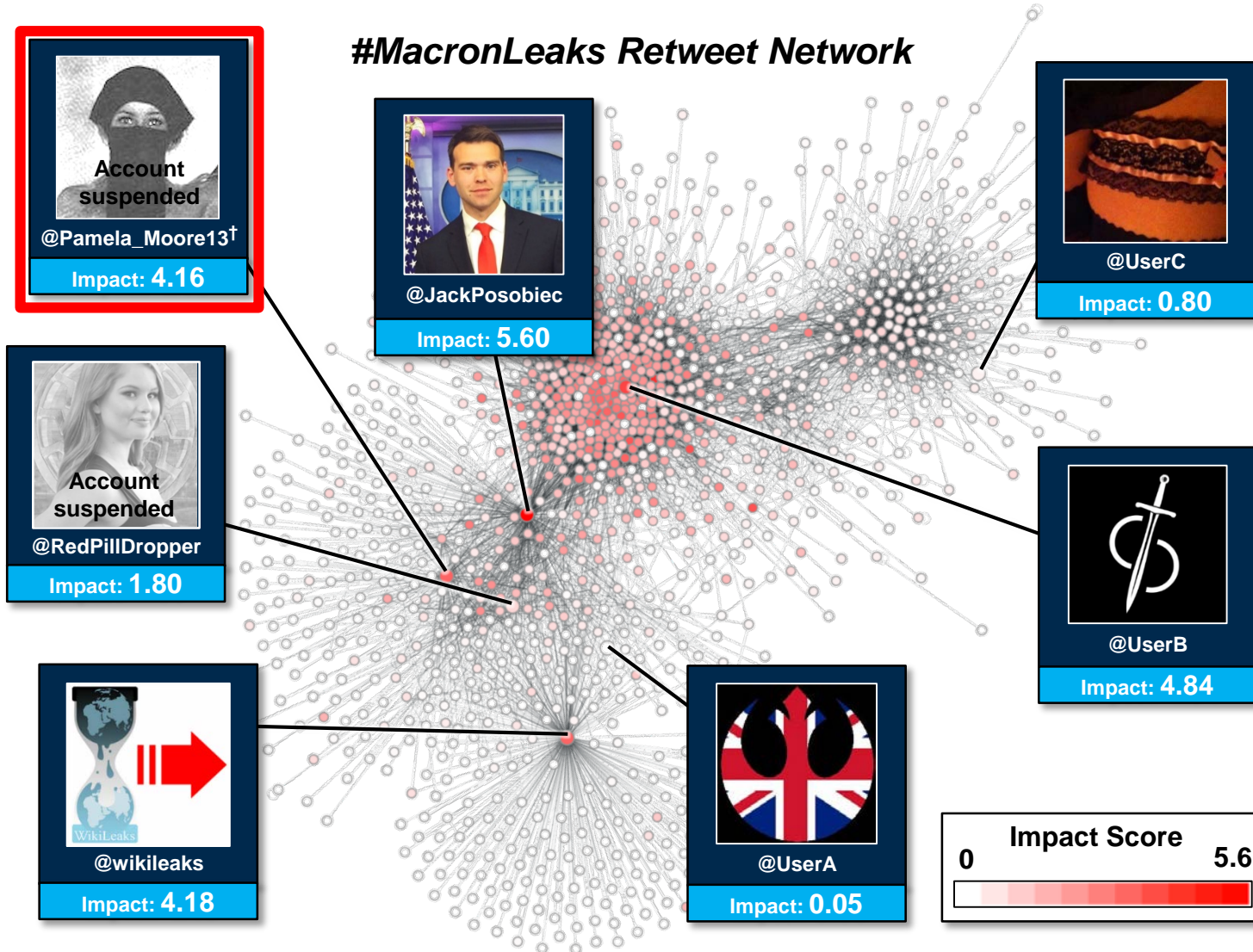


Imputed outcomes $Y(z_{k-})$



Causal Impact Estimation on #MacronLeaks Narrative

#MacronLeaks Retweet Network



Screen name	T	RT	F	Earliest time	Pagerank Centrality	Impact*
@JackPosobiec	95	47k	261 k	18:49	2.84	5.60
@RedPillDropper	32	8k	8 k	19:33	2.86	1.80
@UserA	256	59k	1 k	19:34	27.08	0.05
@UserB	260	54k	3 k	20:25	57.05	4.84
@wikileaks	25	63k	5515k	20:32	2.80	4.18
@Pamela_Moore13†	4	4k	54 k	21:14	2.79	4.16
@UserC	1305	51k	< 1 k	22:16	6.36	0.80

Tweets (T), Retweets (RT), Followers (F), Causal influence estimate*

- Causal impact score measures contribution to narrative flow on the network, beyond activity-based and topological statistics
- High impact accounts corroborated with evidence from the U.S. Congress† and journalistic reports



Summary and Future Work

- **Presented a network causal inference framework to quantify social impact**
- **Applied to finding key influencers in social media influence operations**
 - **Demonstrated on the 2017 French Presidential Election**
 - **On-going work:**
 - Detect and characterize more complex narratives**
 - Recommend intervention via predictive inference and network control**
- **Open questions:**
 - **How to effectively balance confounders across many treatment exposure groups?**
 - **How to best impute missing network potential outcomes and mitigate model mis-specifications?**
 - **Other applications for network causal inference?**



Backups



Modeling the Potential Outcomes With Network Propagation GLM (Net-Prop GLM)

Generalized linear model (GLM) with the appropriate link function $g()$ and distribution for the potential outcomes $Y_i(Z_{N_i}, A_{N_i})$

$$E[Y_i] = g^{-1} \left(\underbrace{\tau Z_i}_{\text{primary treatment effect}} + \underbrace{\tau \gamma_1 s_{1,i}}_{\text{1st-hop peer effect}} + \underbrace{\tau \gamma_1 \gamma_2 s_{2,i}}_{\text{2nd-hop peer effect}} + \dots + \underbrace{\tau \gamma_1 \dots \gamma_L s_{L,i}}_{\text{Lth-hop peer effect}} + \underbrace{\beta^T x_i}_{\text{covariate effect}} + \underbrace{\mu}_{\text{mean effect}} + \underbrace{\epsilon_i}_{\text{random effect}} \right)$$

$$A_{l,j_i} \stackrel{\text{iid}}{\sim} \text{Pois}(\lambda_{ji})$$

$$\epsilon_i \sim \mathbb{N}(0, \sigma^2 = c)$$

$$s_{1,i} = \sum_{j \in \mathcal{N}_i^1} Z_j A_{1,j_i} \sim \text{Pois}(\kappa_{1,i}) \quad \text{where} \quad \kappa_{1,i} = \sum_{j \in \mathcal{N}_i^1} Z_j \lambda_{ji}$$

$$s_{l,i} = \sum_{j \in \mathcal{N}_i^1} s_{l-1,j} A_{l,j_i} \sim \text{Pois}(\kappa_{l,i}) \quad \text{where} \quad \kappa_{l,i} = \sum_{j \in \mathcal{N}_i^1} s_{l-1,j} \lambda_{ji}$$

parameter estimation using MCMC with Bayesian regressions and M-H steps

Outcome Distribution	Link Function	Effects Property
Normal	Identity	Additive
Binomial	Logistic	Additive, slow start and diminishing return
Poisson	Log	Multiplicative



Causal Estimands for Primary Treatment Effects

Unit-Level Effect With Fixed Neighborhood Assignment:

$$\xi_i(\mathbf{z}_{\bar{i}}) \stackrel{\text{def}}{=} Y_i(Z_i = 1, \mathbf{Z}_{\bar{i}} = \mathbf{z}_{\bar{i}}, \mathbf{A}) - Y_i(Z_i = 0, \mathbf{Z}_{\bar{i}} = \mathbf{z}_{\bar{i}}, \mathbf{A})$$

Unit-Level Effect Averaged Over All Neighborhood Assignments:

$$\xi_i^{\text{ave}} \stackrel{\text{def}}{=} \frac{1}{2^{(N-1)}} \sum_{\mathbf{z} \in \mathcal{Z}_{\bar{i}}} \xi_i(\mathbf{z}_{\bar{i}})$$

Average Population Effect Over All Neighborhood Assignments:

$$\xi^{\text{ave}} \stackrel{\text{def}}{=} \frac{1}{N} \sum_{i=1:N} \xi_i^{\text{ave}}$$

Network potential outcomes are the basic building blocks for simple to more complicated causal quantities



Causal Estimands for Fixed Treatment Assignment

Average Peer Effect:

$$\delta^{\text{fix}}(\mathbf{z}) \stackrel{\text{def}}{=} \frac{1}{N} \sum_{i=1:N} Y_i(Z_i = z_i, \mathbf{Z}_{\bar{i}} = \mathbf{z}_{\bar{i}}, \mathbf{A}) - Y_i(Z_i = z_i, \mathbf{Z}_{\bar{i}} = \mathbf{0}, \mathbf{A})$$

Average Effect of Treatment on One Individual (Individual Impact):

$$\zeta_i(\mathbf{z}) \stackrel{\text{def}}{=} \frac{1}{N} \sum_{j=1}^N (Y_j(\mathbf{Z} = \mathbf{z}_{i+}, \mathbf{A}) - Y_j(\mathbf{Z} = \mathbf{z}_{i-}, \mathbf{A}))$$

where \mathbf{z}_{i+} is the fixed treatment with unit i treated and \mathbf{z}_{i-} without

Average Effect of Network Manipulation:

$$\zeta_{\mathbf{A} \rightarrow \mathbf{A}'}(\mathbf{z}) \stackrel{\text{def}}{=} \frac{1}{N} \sum_{j=1}^N Y_j(\mathbf{Z} = \mathbf{z}, \mathbf{A}') - Y_j(\mathbf{Z} = \mathbf{z}, \mathbf{A})$$

Define the causal estimands according to the question of interest



Simplification Through Unconfoundedness Assumptions

Theorem: Simplified Imputation Under Network Interference

If the unconfounded treatment assignment assumption and the unconfounded influence network assumption are both met, the network treatment mechanism (Z, A) does not enter the posterior distribution of the missing potential outcomes:

$$P(Y_{mis} | \mathbf{X}, \mathbf{Z}, \mathbf{A}, Y_{obs}) = P(Y_{mis} | \mathbf{X}, Y_{obs})$$

This simplification allows us to compute the posterior distribution of Y_{mis} by accounting for the critical unit covariates \mathbf{X}



Unconfounded Treatment Assignment Assumption

Unconfounded Assignment Assumption under Network Interference

Conditional on the relevant unit covariates \mathbf{X} and the influence network \mathbf{A} , the treatment assignment \mathbf{Z} does not depend on the potential outcomes:

$$P(\mathbf{Z}|\mathbf{X}, \mathbf{A}, \mathbb{Y}) = P(\mathbf{Z}|\mathbf{X}, \mathbf{A}, \mathbb{Y}') \quad \text{for all } \mathbf{Z}, \mathbf{X}, \mathbf{A}, \mathbb{Y}, \text{ and } \mathbb{Y}'$$

The assignment is unconfounded if the treatment is completely random or determined by the covariates and the influence network



Unconfounded Influence Network Assumption

Unconfounded Influence Network Assumption under Network Interference

Conditional on the relevant unit covariates \mathbf{X} , the influence network A does not depend on the potential outcomes:

$$P(\mathbf{A}|\mathbf{X}, \mathbb{Y}) = P(\mathbf{A}|\mathbf{X}, \mathbb{Y}') \quad \text{for all } \mathbf{A}, \mathbf{X}, \mathbb{Y}, \text{ and } \mathbb{Y}'$$

We will see how this assumption can be met with a parametric network model



Principled Methodology for Network Causal Inference

Theorem: Unconfounded Influence Network by Conditioning on Network Parameters

The unconfounded influence network assumption:

$$P(\mathbf{A}|\mathbf{X}, \mathbb{Y}) = P(\mathbf{A}|\mathbf{X}, \mathbb{Y}') \quad \text{for all } \mathbf{A}, \mathbf{X}, \mathbb{Y}, \text{ and } \mathbb{Y}'$$

is met if:

1. The distribution of the influence network \mathbf{A} can be characterized by a model H_G with nodal parameters \mathbf{X}_G and population parameters Θ_G :

$$\mathbf{A} \sim H_G(\mathbf{X}_G, \Theta_G)$$

2. The influence network \mathbf{A} correlates with the potential outcomes \mathbb{Y} only through a subset of the nodal parameters $\tilde{\mathbf{X}}_G \in \mathbf{X}_G$ and population parameters $\tilde{\Theta}_G \in \Theta_G$
3. The unit covariates \mathbf{X} contain these network parameters $\tilde{\mathbf{X}}_G, \tilde{\Theta}_G$

The confounding covariates \mathbf{X} should be accounted for in both the design and analysis phase of the experiment