

Problem Set #5  
Due: 9 October 2014

Instructor: David Laibson  
Economics 2010c

**Problem 1:** Solve the Merton Consumption problem assuming that

$$u(c) = \ln(c).$$

Do not set  $\gamma = 1$ , as we did in class. Instead, solve the problem from scratch assuming  $u(c) = \ln(c)$ . When you guess the value function, try this functional form:

$$V(x) = \psi \ln(x) + \text{constant}.$$

**Problem 2: True/false/uncertain. Quality of explanation determines grade.**

1. The variance of  $x(t) - x(0)$  increases linearly with  $t$  if  $x$  is any Ito Process.
2. An Ito Process can't be mean-reverting.
3. An Ito Process is not differentiable since it is not continuous.

**Problem 3 [you'll need the material from the lecture on October 7]:** Recall the stopping problem from Lecture 10. The lecture notes show that the optimal threshold rule is

$$x^* = -\frac{b^2}{a + \sqrt{a^2 + 2b^2\gamma}} - \frac{a}{\gamma}$$

Show that  $x^* < 0$ . Show that the following limit conditions apply. Provide intuitive (economic) explanations for all of these results.

$$\begin{aligned} x_{(a=0)}^* &= -\frac{b}{\sqrt{2\rho}} < 0 \\ \lim_{a \rightarrow \infty} x^* &= -\infty \\ \lim_{a \rightarrow -\infty} x^* &= 0 \\ \lim_{b \rightarrow 0} x^* &= \begin{cases} -\frac{a}{\rho} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases} \\ \lim_{b \rightarrow \infty} x^* &= -\infty \\ \lim_{\rho \rightarrow 0} x^* &= \begin{cases} -\infty & \text{if } a \geq 0 \\ \frac{b^2}{2a} & \text{if } a < 0 \end{cases} \\ \lim_{\rho \rightarrow \infty} x^* &= 0 \end{aligned}$$

Hint: you may need to apply L'Hopital's rule for some of these results.

**Problem 4 [you'll need the material from the lecture on October 7] (Stopping problem with geometric Brownian motion):** Consider a new variant of the stopping problem from lecture 10. Now assume that the price of the good evolves according to *geometric* Brownian motion.

$$dx = axdt + bxdz.$$

In addition, assume that the instantaneous profitability of production is

$$w(x, t) = x - c$$

where  $c$  is the (fixed) cost of production.

$$\Omega(x, t) = 0$$

Solve for the value function and the optimal threshold rule  $x^*$ .