Designing Markets for Prediction

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■ *We survey the literature on prediction mechanisms, including prediction markets and peer prediction systems. We pay particular attention to the design process, highlighting the objectives and properties that are important in the design of good prediction mechanisms.*

Mechanism design has been described as "inverse game theory." Whereas game theorists ask what outcome results from a game, mechanism designers ask what game produces a desired outcome. In this sense, game theorists act like scientists and mechanism designers like engineers.

In this article, we survey a number of mechanisms created to elicit predictions, many newly proposed within the last decade. We focus on the engineering questions: How do they work and why? What factors and goals are most important in their design?

The primary goal of a prediction mechanism is to obtain and aggregate dispersed information, which often exists in tacit forms as beliefs, opinions, or judgements of agents. Coalescing information is a necessary first step for decision making in almost all domains. For example, consider seasonal influenza, a significant cause of illness and death around the world. Although it recurs every year, the geographic location, timing, magnitude, and duration of outbreaks vary widely. Many people possess relevant pieces of the full information puzzle, including doctors who meet patients, clinical microbiologists who perform respiratory culture tests, pharmacists who fill prescriptions, people who have the flu, and people who know people who have the flu. Aggregating such information quickly and accurately is crucial for influenza surveillance since timing is very important for both prevention and treatment of influenza.

Prediction Markets

A prediction market is one type — probably the most common and well known type — of prediction mechanism. A prediction market offers contracts whose future payoff is tied to outcomes of an event of particular interest and attracts participants to trade the contract. For instance, a contract that pays $1 if avian flu is confirmed in the United States before March 31, 2011, and $0 otherwise can be used to predict the likelihood of an avian flu outbreak. Wagering on the event outcome through buying or selling the contract in the market, agents can express their

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opinions in a credible way. A risk-neutral agent who believes that the probability for avian flu to be confirmed by the deadline is α can make profits on expectation by buying the contract if the current market price is lower than α, and selling the contract if the current market price is higher than α. The market price hence incorporates the information of participants and approximately represents a real-time consensus forecast for the event. For example, the Iowa Electronic Markets (IEM) offer contracts on political elections and Intrade supports trading on a range of events including avian flu, global average temperature, and Osama bin Laden's capture.

The use of prediction markets for information aggregation was inspired by the informational efficiency of financial markets (Fama 1970, Hayek 1945). Most of the prediction markets nowadays use the same continuous double auction mechanism as stock markets. However, as prediction markets have shown great potential as highly effective information aggregation tools in their early adoptions — market forecasts often outperform other forecasting methods in a diverse array of settings (Forsythe et al. 1991; Berg et al. 2001; Chen and Plott 2002; Chen, Fine, and Huberman 2003; Cowgill, Wolfers, and Zitzewitz 2008; Wolfers and Zitzewitz 2004; Pennock et al. 2002; Chen et al. 2005) — recent research is not constrained by the framework of financial markets and has been focusing on understanding and achieving properties that are important for the purpose of information aggregation.

Peer Prediction Systems

Prediction markets elicit forecasts for events with a clear, objective outcome that can be reliably discerned after the fact: for example, the winner of an election as reported in the *New York Times* or the prevalence of flu according to the Centers for Disease Control and Prevention. Many information-aggregation tasks do not conform to this requirement, either because the outcome is subjective — the quality of a movie — or unmeasurable — the extinction of the human race. Peer prediction systems operate by evaluating each agent's prediction not against an objective reality but against the other agents' predictions. Remarkably, under certain conditions such systems can induce truth telling in equilibrium, meaning that if others are playing honestly the best response is to play honestly as well, yielding aggregate assessments of subjective or unmeasurable outcomes.

Design Objectives

The goal of a prediction mechanism is to acquire and aggregate information. Social efficiency, or making participants happier to the greatest extent possible, is not necessarily an objective at all. On this point, prediction mechanisms differ from almost every other mechanism we can think of, including auction, voting, and matching mechanisms. In a prediction mechanism, trade is a means to an end, not an end in itself. Thus prediction mechanisms must be compared against nonmarket approaches like polling, forecasting, modeling, machine learning, and belief aggregation designed to achieve the same ends (Goel et al. 2010).

Prediction mechanisms are unusual in other ways. A pure prediction mechanism may reasonably operate at a loss; maximizing revenue or even balancing the budget may not be a concern. If the operator wants information, she may be perfectly happy to pay for it. On the other hand, two somewhat nonstandard properties are important: expressiveness and liquidity. An expressive mechanism offers agents flexibility in how they communicate information; at the extreme, agents can provide any information they have in any form they like. Liquidity ensures that agents can be compensated for their information at any time, even when few others are around.

Prediction mechanisms share at least three more common objectives: incentive compatibility, computational tractability, and individual rationality. Incentive compatibility means that every agent's best strategy is to honestly report all of their information as soon as they have it, an important property that's difficult to achieve in general. Computational tractability means that the outputs of the mechanism, like allocations and prices, can be computed in a reasonable amount of time. Individual rationality simply means that agents are better off for playing the game than not.

Summarizing, the ultimate objective of a prediction mechanism is to aggregate information. Other objectives are in service of the primary. These include liquidity, incentive compatibility, expressiveness, computational tractability , and individual rationality. Common design objectives that are often not important for prediction mechanisms include social efficiency, revenue optimization, and budget balance.

In this article, we survey recent progresses on understanding and designing prediction mechanisms according to these objectives. In particular, many automated market maker mechanisms have been designed to provide (effectively infinite) liquidity for prediction markets; much effort has been put into understanding manipulation in prediction markets and designing prediction mechanisms to achieve incentive compatibility; and research on combinatorial prediction markets has advanced our understanding of the computational tractability of operating combinatorial prediction markets when we increase expressiveness.

Scoring Rules

The simplest prediction mechanism is a scoring rule, or payment to a single expert in return for her information. The payment amount depends on the expert's prediction and the actual outcome in a way that motivates the expert to be honest (Good 1952, Winkler 1969, Savage 1971, Gneiting and Raftery 2007). Formally, let v represent a discrete random variable with m mutually exclusive and exhaustive outcomes and $\mathbf{r} = \langle r_1, r_2, \ldots, r_m \rangle$ be a reported probability estimate for the random variable v. A scoring rule $S = \{s_1(\mathbf{r}), s_2(\mathbf{r}), \ldots, s_m(\mathbf{r})\}$ assigns a score $s_i(\mathbf{r})$ to the agent who reports $\mathbf{r}$ if outcome i is realized. A *regular* scoring rule implies that $s_i(\mathbf{r})$ is finite whenever $r_i > 0$. A regular scoring rule is (*strictly*) *proper* if truthful reporting (strictly) maximizes the expected score of a risk-neutral agent. In other words, proper scoring rules are incentive compatible for risk-neutral agents when eliciting probability assessments. For example, the logarithmic scoring rule,

$$s_i(\mathbf{r}) = a_i + b \log(r_i) \quad (1)$$

where $b > 0$, is a widely used proper scoring rule. In fact, every bounded convex and differentiable function of $\mathbf{r}$ defines a proper scoring rule (McCarthy 1956, Hendrickson and Buehler 1971, Savage 1971).

To obtain multiple forecasts, the operator could give separate scoring rule payments to everyone. Or, he could employ a *shared scoring rule* that rewards each expert according to only the difference between her prediction and the average of the others (Kilgour and Gerchak 2004). Now experts risk losing money in addition to gaining it — the system becomes a constant sum game — though the mechanism is still incentive compatible assuming experts don't revise their beliefs. Lambert et al. (2008) explore self-financing (budget-balanced) wagering mechanisms of this type in an axiomatic framework. Indeed, the line between scoring rules and markets becomes blurred: for example, the most common automated market maker used for prediction markets can be viewed as a *sequential shared scoring rule*, as we shall now see.

Liquidity and Market Makers

An auctioneer matches up willing traders with each other — the auctioneer never takes on any risk of his own. This is how most financial exchanges like the stock market operate, and how IEM, Intrade, and gambling exchanges like Betfair operate.

An automated market maker, on the other hand, will quote a price for any contract whatsoever. Even a lone agent can trade with the market maker as long as she accepts the price, greatly enhancing liquidity. The liquidity comes at a cost though: the market maker can and often does lose money, though as we'll see in the following the loss can be bounded.

Auctions work well for stock markets where there are a large number of buyers and sellers and finding a counterparty to trade is relatively easy. However, when there are fewer participants per outcome, auctions may suffer from illiquidity or the *thin market* problem, potentially preventing agents from revealing their information. An auction is a zero-sum game for traders. As such, according to the *no-trade theorem* (Milgrom and Stokey 1982) paradox, rational risk-neutral traders will never trade, each reasoning roughly that any willing trading partner must know something that he or she doesn't know. The market maker's loss is the traders gain, turning the mechanism into a positive-sum game that even rational risk-neutral agents should play.

Market scoring rules (MSRs) are a family of automated market maker mechanisms proposed by Hanson (2003, 2007). An MSR is a sequential shared version of a proper scoring rule. The market maker starts the market with some initial probability $\mathbf{r}^0$ over the outcomes. Given a proper scoring rule, every trader in the market may change the current probability estimate to a new estimate of its choice at any time as long as it agrees to pay the scoring rule payment associated with the current probability estimate and receive the scoring rule payment associated with the new estimate. If outcome i is realized, a trader that changes the probability estimate from $\mathbf{r}^{\text{old}}$ to $\mathbf{r}^{\text{new}}$ pays $s_i(\mathbf{r}^{\text{old}})$ and receives $s_i(\mathbf{r}^{\text{new}})$. The market maker only pays the last trader and receives payment from the first trader. Because there is a large class of proper scoring rules, MSRs are a large family of market makers. From the previous description, MSRs don't seem to closely resemble markets, as no contracts are traded and participants sequentially report probabilities. However, under mild conditions, any MSR is proven to be equivalent to a *cost-function-based market maker* that offers contracts to trade and is more natural for implementation purposes (Chen and Pennock 2007).

To predict the outcome of random variable v, a cost-function-based market maker offers m Arrow-Debreu contracts, one for each possible outcome. An Arrow-Debreu contract pays \$1 if the corresponding outcome is realized and \$0 otherwise. Let q_i be the total quantity of contract i held by all traders combined, and let $\mathbf{q}$ be the vector of all quantities held. The market maker utilizes a cost function $C(\mathbf{q})$ that records the total amount of money traders have spent as a function of the total number of shares held of each contract. A trader who wants to buy any bundle of contracts such that the total number of outstanding shares changes from $\mathbf{q}^{old}$ to $\mathbf{q}^{new}$ must pay $C(\mathbf{q}^{new}) - C(\mathbf{q}^{old})$

dollars to the market maker. Negative quantities encode sell orders, and negative "payments" encode sale proceeds earned by the trader. At any time, the instantaneous price of contract i is $p_i(\mathbf{q}) = \partial C(\mathbf{q})/\partial q_i$, representing the cost per share for purchasing or selling an infinitesimal quantity of contract i. Any $C(\mathbf{q})$ that is (1) differentiable, (2) monotonically increasing, that is, if $\mathbf{q} \geq \mathbf{q}'$, $C(\mathbf{q}) \geq C(\mathbf{q}')$, and (3) positive translation invariant, which is defined as $C(\mathbf{q} + k\mathbf{1}) = C(\mathbf{q}) + k$ for any $\mathbf{q}$ and k, defines a valid cost-function-based market maker such that $p_i(\mathbf{q})$ is nonnegative and

$$\sum_{i=1}^{m} p_i(\mathbf{q}) = 1$$

for all $\mathbf{q}$ (Chen and Vaughan 2010).

An MSR can be equivalently implemented as a cost-function-based market maker if an agent who changes the market prices from $\mathbf{p}$ to $\mathbf{p}'$ by trading in a cost-function-based market gets the same profit (under every outcome) as if it changes the market probability from $\mathbf{p}$ to $\mathbf{p}'$ in the MSR. For a logarithmic market scoring rule (LMSR) market maker that uses the logarithmic scoring rule (1) (Hanson 2003, Chen and Pennock 2007), the cost and price functions are

$$C(\mathbf{q}) = b \log \sum_{j=1}^{m} e^{q_j/b}$$

and

$$p_i(\mathbf{q}) = \frac{e^{q_i/b}}{\sum_{j=1}^{m} e^{q_j/b}}.$$

More recently, Chen and Vaughan (2010) have established a one-to-one mapping between the class of strictly proper MSR and the class of convex cost-function-based market makers. An MSR with strictly proper scoring rule $\{s_i(\mathbf{r})\}$ and a cost-function-based market maker with convex $C(\mathbf{q})$ map to each other if and only if

$$C(\mathbf{q}) = \sup_{\mathbf{p} \in \Delta_m} \left(\sum_{i=1}^{m} p_i q_i - \sum_{i=1}^{m} p_i s_i(\mathbf{p}) \right) \quad (2)$$

where Δ_m is the probability simplex. Given a strictly proper MSR, (2) gives the cost function for the corresponding cost-function-based market maker in terms of a convex optimization problem. The optimal $\mathbf{p}$ to the optimization problem gives the market prices $\mathbf{p}(\mathbf{q})$. Chen and Vaughan (2010) also provide the expression of $\{s_i(\mathbf{r})\}$ given any convex $C(\mathbf{q})$. The pair of markets are equivalent in terms of trader profits when prices for all outcomes are positive. This mapping allows the easy conversion between MSR and cost-function-based market makers.

The cost-function-based market makers and hence the MSRs have an interesting connection to no-regret learning. In the framework of learning from expert advice, an algorithm makes a sequence of predictions based on the advice of a set of experts and receives a corresponding sequence of losses. At every time step t, every expert i receives a loss $l_{i,t}$. The algorithm maintains a weight $w_{i,t}$ for each expert i at time t, where

$$\sum_i w_{i,t} = 1 .$$

The loss received by the algorithm at time step t is the weighted sum of the expert losses. The goal of the algorithm is to adjust weights of experts to achieve a cumulative loss that is "almost as low" as the cumulative loss of the best performing expert, even if expert losses are chosen by an adversary. Chen and Vaughan (2010) show that any cost-function-based market maker can be interpreted as an algorithm for the learning from expert advice problem by treating outcomes as experts and equating trades made in the market with expert losses observed by the learning algorithm. Moreover, there is a one-to-one mapping between the class of convex cost-function-based market makers and the class of Follow the Regularized Leader algorithms for the learning from expert advice problem.

Two other families of market maker mechanisms have been proposed based on different rationales but have some equivalence relationships with MSRs and cost-function-based market makers. Chen and Pennock (2007) introduced *utility-based market makers*. A utility-based market maker has a utility function of money and a subjective probability distribution of the event. It sets the instantaneous market prices of Arrow-Debreu contracts as its risk-neutral probabilities and hence keeps its expected utility constant at any time of the market. For the class of hyperbolic absolute risk aversion utility functions, which contains many frequently used utility functions, and the class of weighted pseudospherical scoring rules, there is a one-to-one mapping between the utility-based market makers and MSRs. The *sequential convex parimutuel mechanism* (SCPM) (Agrawal et al. 2009) is a market maker designed for limit orders. In a SCPM, traders specify a maximum quantity of shares that they would like to buy and a maximum price per share that they are willing to pay. The market maker decides how many shares of trade to accept by solving a convex optimization problem. The payment of the accepted trade is determined by a generalized VCG mechanism. Although SCPM is defined differently and can more naturally incorporate limit orders and batch orders, the underlying mathematics of SCPM are analogous to those of cost-function-based market makers.

Another automated market maker mechanism is the *dynamic parimutuel market* (DPM) (Pennock 2004; Chen, Pennock, and Kasturi 2008). DPM is a dynamic-cost variant of a parimutuel market, which is often used in horse racing. There are m

contracts offered in a DPM market, each corresponding to one outcome. As in a parimutuel market, traders who wager on the true outcome in DPM split the total pool of money at the end of the market, in proportion to the amount they wagered. However, unlike a parimutuel market, the price of a single share in DPM varies dynamically according to a price function, thus allowing traders to sell their shares prior to the determination of the outcome for profits or losses. From a trader's perspective, DPM acts as a market maker in a similar way as cost-function-based market makers, the major exception being that the payoff of a contract is not fixed. The commonly used cost and price functions of DPM are

$$C(\mathbf{q}) = \sqrt{\sum_{j=1}^{m} q_{i'}^2}$$

and

$$p_i(\mathbf{q}) = \frac{q_i}{\sqrt{\sum_{j=1}^{m} q_j^2}}$$

The payoff of contract i when outcome i happens is

$$o_i\left(\mathbf{q}^f\right) = \frac{C\left(\mathbf{q}^f\right)}{q_i^f}$$

where $\mathbf{q}^f$ is the quantity vector at the end of the market. DPM needs the market maker to seed the market with some initial shares (money), which can be arbitrarily small, because the price function is not defined at $\mathbf{q} = 0$.

Because automated market makers accept orders without knowing the realized outcome of the event, they can potentially lose money. A key property that research on designing automated market makers has focused on is bounded loss, which ensures that no matter what happens in the market and no matter which outcome is realized, the loss of the market maker is bounded. For an MSR, as the market maker pays the last trader and gets paid by the first trader, the worst-case loss of the market maker happens when traders change the market probability of the realized outcome to 1. The worst-case loss of an MSR with scoring rule $\{s_i(\mathbf{r})\}$ and initial market probability $\mathbf{r}^0$ is bounded by

$$\max_i \max_{\mathbf{r}\in\Delta_m} s_i(\mathbf{r}) - s_i\left(\mathbf{r}^0\right) = \max_i s_i\left(\mathbf{e}_i\right) - s_i\left(\mathbf{r}^0\right)$$

where $\mathbf{e_i}$ is the vector with 1 assigned to its i-th element and 0 everywhere else. Thus, any MSR market maker with a regular proper scoring rule has bounded loss. Given uniform initial market probability, the loss of an LMSR market maker is bounded by $b \log m$, where m is the number of outcomes (Hanson 2003). For cost-function-based market makers, the worst-case loss bound can be characterized as $\sup_{\mathbf{q}\in\mathbb{R}^m} (\sup_i q_i - (C(\mathbf{q}) - C(\mathbf{0})))$. For DPM, the loss of the market maker is bounded by its initial subsidy as the market is parimutuel.

LMSR has become the de facto market maker mechanism for prediction markets. It is used by many companies including Inkling Markets, Consensus Point, Yahoo!, and Microsoft. However, setting the value of b, often called the liquidity parameter, in LMSR is more art than science in practice. The b parameter determines how quickly prices move with trades as well as the market maker's worst-case loss. If b is too small, the price of a contract changes dramatically after a small number of shares is traded. If b is too large, the price of a contract barely moves even with a large volume of trades. Othman et. al. (2010) propose a modified LMSR where the liquidity parameter no longer needs to be set fixed *a priori*. In this modified LMSR, the value of b increases continuously with the total number of shares of all contracts purchased by all traders. Unlike the original LMSR where the prices of all contracts always sum to 1, the modified LMSR allows the sum of the prices to be greater than 1. Intuitively, the modified LMSR behaves as if it charges some transaction fees for every infinitesimal trade and uses the collected transaction fee to increase the value of the liquidity parameter.

Incentive Compatibility

For prediction markets, incentive compatibility means that a risk-neutral agent maximizes its expected profit by changing the market probability to its probability assessment immediately. In other words, the agent will reveal its information *truthfully* and *immediately*. Incentive compatibility provides simplification for mechanisms, because agents do not need to strategize and simply revealing their information is the best response. Moreover, if agents lie about their information, information aggregation may be put into question.

Prediction markets are not incentive compatible in general. The no-trade theorem means that rational traders should not trade at all in CDA. The Kyle model of financial markets posits two types of traders: rational traders and noisy traders (Kyle 1985). The existence of noisy traders makes CDA a positive-sum game for the rational traders and hence circumvents the no-trade theorem. However, the mechanism is still not incentive compatible. For example, monopolist information holders will not fully reveal their information right away: instead, they will leak their information into the market gradually over time to obtain a greater profit (Chakraborty and Yilmaz 2004). Market scoring rules and most cost-function-based market makers

are myopically incentive compatible — a risk-neutral agent will report its probability truthfully if it only participates once.[1] But because an agent can potentially influence other agents by its trading action and can trade more than once, a forward-looking agent may lie about its information to mislead other agents ("bluff") with the hope to obtain greater profit by correcting their mistakes later.

While incentive compatibility in mechanism design often means dominant-strategy incentive compatibility, incentive compatibility for prediction mechanisms typically refers to Bayesian-Nash incentive compatibility. In fact, all existing prediction mechanisms that involve direct or indirect interactions of agents are not dominant-strategy incentive compatible.[2] Researchers hence focus on understanding and designing prediction mechanisms to implement Bayesian-Nash incentive compatibility. Some consider an even weaker notion of truthfulness — whether the mechanism converges to full information aggregation or obtains a representative sample of agent opinions at an equilibrium even if individual agents may not play honestly.

Chen et. al. (2009) attempt to understand whether there exists game-theoretic equilibrium at which agents truthfully reveal their information as soon as they can in prediction markets. They consider a two-outcome LMSR market and model it as an n-player, incomplete-information, dynamic game. At the beginning of the market, each risk-neutral player i gets a private signal s_i that is stochastically related to the outcome of the event ω. The joint distribution of s_i's and ω is common knowledge. Players trade in the LMSR market according to a prespecified sequence. The equilibrium behavior of the game depends on the information structure of the players. When players have conditionally independent signals (that is, conditional on ω, s_i's are independent), the unique Perfect Bayesian Equilibrium (PBE) of the game is the truthful betting equilibrium where every player truthfully reveals its information in the first round it can trade. Information is fully aggregated after everyone has traded. However, when the signals are unconditionally independent, a complete characterization of the equilibrium is unknown, but it is known that truthfully revealing one's information is not an equilibrium strategy. In fact, it is shown that there does not exist an equilibrium where all information is aggregated within a finite number of trades. A discounted LMSR where the b parameter in the logarithmic scoring rule decreases over time is then proposed to ensure that information is fully aggregated in the limit with unconditionally independent signals. Jain and Sami (2010) conduct lab experiments to test the previously mentioned theoretical results. They find that the assumption of prespecified trading sequence is crucial for the different behavior under the two information structures. Information is better aggregated in the experiments when the trading sequence is prespecified, compared with when traders endogenously decide when to trade. Moreover, when trading sequence is prespecified, Jain and Sami find that there are more manipulative behaviors with unconditionally independent signals than with conditionally independent signals, while the difference is not observed without the prespecified trading sequence.

Ostrovsky (2009) characterizes the condition of a contract and an information structure under which prediction markets converge to full information aggregation at PBEs, even if traders may not truthfully reveal their information at their first round of trade. He considers both Kyle's noisy-trader model of CDA and MSR market makers. He analyzes a dynamic game of n risk-neutral agents each receiving a piece of private information. He shows that if the contract together with the information structure satisfies a *separability* condition, information gets aggregated in the limit at any PBE; if the separability condition is not satisfied, there exists some prior distribution of event outcomes such that at some PBE information is not aggregated. Iyer, Johari, and Moallemi (2010) extend Ostrovsky's work by considering risk-averse agents. They study a setting where n risk-averse agents with conditionally independent private signals participate in an automated market maker mechanism. With risk-averse agents, Iyer, Johari, and Moallemi identify a *smoothness* condition of the prices that together with some other reasonable conditions of the market can ensure full information aggregation in the limit at any PBE. Loosely speaking, smoothness requires that there is no bid-ask spread for purchasing or selling an infinitesimal quantity of any contract. For cost-function-based market makers, this is always satisfied due to differentiability of the cost function. In fact, Iyer, Johari, and Moallemi prove that for any cost-function-based market maker that has bounded loss, if the signal space of agents is finite, information is always aggregated in the limit with risk-averse agents at any pure-strategy PBE. In addition, if there is at least one risk-neutral agent in the market, the market price eventually reflects the posterior probability of the event conditional on the pooled information. It's worth mentioning that although both Ostrovsky (2009) and Iyer, Johari, and Moallemi (2010) characterize conditions for full information aggregation at PBEs, the existence of such PBEs is still an open question.

Manipulation

The work surveyed in this section so far all implicitly assumes that agents could not take actions to influence the outcome of the event. This is often

not true in the real world. For example, with an internal prediction market to predict a software delivery date, a developer of the software who purchases contracts in the market may deliberately take actions outside of the market to affect the software delivery date so that the contracts he purchased will pay off. Shi, Conitzer, and Guo (2009) attempt to avoid such incentive misalignment. They consider a setting where there is a principal (for example, the company) who sets up a prediction mechanism to collect information about an event of interest. A group of agents who have information about the event of interest can also take actions to affect its outcome. The principal has a preference over the event outcomes (for example, on-time delivery of the software is preferred) and hence requires the prediction mechanism to not only elicit information from agents but also not incentivize the agents to take actions that may harm the principal. Shi, Conitzer, and Guo consider one-round MSR markets where every agent can only participate once. Focusing on the one-round mechanisms removes the complication of strategic play within the market because agents should play truthfully when they only participate once. Given the utility vector of the principal for event outcomes, they characterize "principal-aligned" proper scoring rules that do not incentivize the agent to take any action that may harm the principal in expectation. When using a "principal-aligned" proper scoring rule for a one-round MSR with n agents, the principal in the worst case needs to pay $\Theta(n)$ times as much as he would pay if incentive alignment is not required. The intuition is that the principal needs to subsidize each agent in a way such that the benefit that the agent can get when the desired outcome happens is higher than the possible benefit it can get within the market by manipulating the event outcome.

Ultimately, information elicitation and aggregation are conducted to assist decision making. If decisions will be made based on the information conveyed by a prediction market, agents may have incentives to manipulate the market in order to achieve the desired decision outcome. Dimitrov and Sami (2010) consider manipulations of prediction markets due to a certain type of conflicting incentives. Unlike previous work where strategic agents only care about profits within a single market, Dimitrov and Sami model a situation where an agent's trade in one market can influence the trading decision of another agent in a second market. Specifically, they consider a simple two-player three-stage model, where Alice trades in the first market, Bob observing Alice's action in the first market makes his trading decision in the second market, and Alice can then trade in the second market. Alice can potentially manipulate the price in the first market so that Bob's trading decision in the second market leaves the market in a more profitable state for Alice. Dimitrov and Sami characterize the weak perfect Bayesian equilibria of the game and find that the payoffs of Alice and Bob are unique across all equilibria.

Peer Prediction

Prediction markets rely on a verifiable "ground truth" of the event of interest to evaluate reports of agents. However, many events in the real world are either subjective or nonverifiable. In the past six years, a stream of work develops scoring rule based methods that evaluate the report of an agent against the reports of its peers to truthfully elicit information for events where ground truth does not exist or can not be obtained.

Miller, Resnick, and Zeckhauser (2005) propose the first peer prediction method. Suppose ω is the event of interest, which is represented as a discrete random variable. Each agent receives a signal s_i that is independent, and randomly drawn from a probability distribution conditional on the true state of ω. Both the prior distribution of ω and the conditional probability distribution of signals $s_i|\omega$ are common knowledge. The true state of ω is not verifiable. The peer prediction method makes use of the stochastic correlation between signals of agents to achieve a Bayesian Nash equilibrium (BNE) where every agent truthfully reports its signals to the mechanism. If an agent i truthfully reports its signal s_i, knowing the prior distribution the mechanism calculates agent i's posterior probability of the signal of a reference agent j, $\mathbf{P}(s_j|s_i)$, which can then be evaluated and rewarded using a proper scoring rule according to agent j's reported signal. Thus, if all other agents report truthfully, agent i maximizes its expected reward by reporting truthfully. However, truthful reporting is not a unique equilibrium. There exist lying equilibria. Jurca and Faltings (2006) improve the peer prediction method by finding the incentive payment computationally to reduce the total payment of the mechanism. They further show that the peer prediction method can be extended to make incentive payment based on more than one reference report, to deal with collusion and sybil attack, and to ensure that truthful reporting is the unique equilibrium (Jurca and Faltings 2007). Goel, Reeves, and Pennock (2009) propose a collective revelation mechanism that not only admits a truthful reporting BNE but also weights the estimates of agents by their relative information content.

The peer prediction method, its extensions, and the collective revelation mechanism are based on the assumption of common knowledge of common prior. Agents are assumed to have a common prior of the event and signals, and the mechanism makes use of the common prior in determining

incentive payments. The Bayesian Truth Serum (BTS), introduced by Prelec (2004), has a slightly weaker assumption. It still assumes that agents have common prior, but the prior can be unknown and the mechanism does not explicitly use the prior distribution. Consider an opinion poll. BTS works by asking each agent to report its subjective answer to the poll and an estimate of the final distribution over possible answers. The reward of the agent consists of two parts: an information score for the answer, which is higher for answers that are surprisingly more common than collectively predicted, and a prediction score that is inversely proportional to the Kullback-Leibler divergence of the estimated answer distribution from the actual answer distribution of the poll. Truthful reporting of both the subjective answer and the estimate of the answer distribution is a BNE for BTS. One limitation of BTS is that partial poll results can not be revealed before the end of the poll. To overcome this limitation, Jurca and Faltings (2008) propose a mechanism that encourages a more general notion of truthful reporting for online polls where each agent knows the partial poll result before its participation. At a BNE of this mechanism, the reports of agents are not necessarily truthful, but they always reduce the gap between the updated partial poll result and the subjective belief of the reporter regarding the poll outcome. The poll result hence converges to the correct outcome, that is, the true fractions of agents who endorse different poll answers.

Lambert and Shoham (2008) take a different approach. Instead of seeking truthful reports of agents, they propose a mechanism that can extract a representative sample of opinions. The mechanism selects a group of agents and the payment to each agent depends only on the reports of the selected agents. Unlike the previous methods, the mechanism does not need the existence of a common prior. When at least one participant may be trusted, the mechanism ensures that at all Nash equilibria true samples of opinions are obtained. Lambert and Shoham (2009) further give necessary and sufficient conditions for the existence of incentive payments that induce truthful answers in online questionnaires at an equilibrium, and provide characterizations of such payments.

Expressiveness and Computational Tractability

Prediction mechanisms with more than a few outcomes become unwieldy if agents must provide information about each outcome individually, one at a time. An expressive mechanism allows agents to place *combinatorial bids* that say things about sets of outcomes together, greatly simplifying and reducing the communication needed.

For example, imagine a 539-outcome prediction market for the U.S. presidential election with one outcome for every possible number of electoral votes between 0 and 538 that the Democratic candidate will receive. A prediction like "the Democrat will receive between 269 and 312 electoral votes" becomes tedious and inefficient if each of the intervening 44 outcomes is traded separately. A natural form of expressiveness here is to allow the entire interval to be bought in a single transaction.

Combinatorial bids are useful in any market but they are almost necessary when the outcome space is itself combinatorial, for example, all possible permutations of a horse race. A race among ten horses has 10! outcomes and a prediction like "horse A will finish ahead of horse B" involves half of them, or over 1.8 million outcomes, too many to deal with individually.

Combinatorial bids also allow for smarter accounting so traders' funds aren't unnecessarily locked up to cover two bets that provably can never lose together, for example "Horse A will not win" and "Horse B will not win."

Another form of expressiveness allows traders to place *indivisible* bids that the mechanism must fill either completely or not at all. In the context of prediction markets, this option may not be so important. Traders may be happy to receive partially filled bids with both less risk and proportionally less reward than they requested. If they are willing to risk $100 to win $200, many would also be willing to risk $50 to win $100 instead. Still, some traders may want indivisible bids to guarantee a minimum level of risk or insurance, otherwise opting out and going elsewhere.

Allowing greater expressiveness comes at a potential cost in the computational burden on the mechanism. A prediction market auctioneer can process combinatorial bids in time polynomial in the number of outcomes using linear programming if bids can be partially filled. If traders can place indivisible bids, the problem becomes NP-hard (Bossaerts, Fine, and Ledyard 2002, Fortnow et al. 2004). The automated market maker algorithms previously mentioned also run in polynomial time in the number of outcomes. Optimal accounting, or computing the maximum a trader can lose in the worst case and thus the minimum amount of cash or credit the center needs to reserve for that trader, is typically polynomial in the number of outcomes.

So, adding (divisible) combinatorial bids to markets with hundreds or even thousands of outcomes is feasible, with almost no downside. Still, combinatorial bidding is not supported by the vast majority of fielded prediction markets, including IEM, Intrade, Inkling, Newsfutures, and HSX, and the majority of financial and betting markets broadly speaking. Exceptions include Othman and

Sandholm's Gates Hillman Prediction Market (Othman and Sandholm 2010); the Policy Analysis Market (PAM)[3]; Bossaerts, Fine, and Ledyard's (2002) combined value-trading mechanism; the parimutuel call market mechanism (Agrawal et al. 2009; Baron and Lange 2005; Lange and Economides 2007; Peters, So, and Ye 2007); and Yahoo!'s Yoopick[4] and Predictalot[5] systems.

When the outcome space is combinatorial, expressivity poses a more difficult computational challenge. A running time that's polynomial in the number of outcomes is not good enough, since the number of outcomes is exponential in the number of base objects. Across a range of combinatorial outcome settings, the auctioneer problem is NP-hard and the LMSR market maker pricing problem is #P hard. There are a few special cases where limiting expressivity enough can render the problem tractable. Optimal accounting in the combinatorial outcome setting is NP-hard.

Some of the known results include Boolean betting, tournament betting, permutation betting, and taxonomy betting.

Boolean Betting. Base objects are binary events, for example whether the Democratic candidate wins Alabama, Alaska, and so on, for all 50 states. Outcomes are all possible combinations, in this case all 2^{50} ways the election might swing. Predictions are phrased in Boolean logic, for example "Ohio and Florida but not Virginia." (Conditionals like "Nevada if California" can also be handled without affecting the complexity.) The auctioneer problem is NP-hard and remains hard even if the most complicated bet allowed is conjoining two events. Allowing indivisible bids makes the problem NP-hard even for a small number of outcomes polynomial in the number of base objects (Fortnow et al. 2004). LMSR pricing is #P-hard and inapproximable in general (Chen et al. 2008).

Tournament Betting. This is a special case of Boolean betting where base objects are matches in a single-elimination tournament, a common structure in sports playoffs. LMSR pricing remains hard, though if bets are restricted to "team *A* advances to round *k*" prices can be computed in polynomial time using a Bayesian network (Chen, Goel, and Pennock 2008).

Permutation Betting. Outcomes are permutations of base objects, for example all 10! possible finish orderings in a 10 horse race. Predictions are properties of the final ordering, for example Horse B will finish ahead of horse D, or Horse B will finish between 3rd and 7th place. The auctioneer and LMSR pricing problems are both intractable and remain so even if all bets are pairwise: "X will finish ahead of Y." Interestingly, the auctioneer problem becomes tractable if all bets encode one-to-many subsets of the form "Horse A will finish in positions 1, 3, or 7" or "Horses C or E will finish in position 2," although the pricing problem remains hard (Chen et al. 2007). Ghodsi et al. (2008) show that even subset betting is hard for indivisible bids, unless bets are further restricted to candidate-rank specifications like "Horse B will finish in position 3." Agrawal, Wang, and Ye (2008) give a polynomial-time convex optimization algorithm for the auctioneer problem when bets are linear combinations of candidate-rank specifications. They also show how to use maximum entropy to approximate the full joint distribution over all $n!$ permutations from the n^2 marginal prices for each candidate-rank pair maintained by their mechanism.

Taxonomy Betting. Base objects are (discretized) numbers at the base or leaves of a tree. An internal node in the tree represents the sum of its children. For example, the numbers might represent page views of a sports website organized in a hierarchy by topic (football, basketball, baseball), subtopic (college, professional), subsubtopic, and so on. Outcomes are the cross product of the numbers at the leaves. Bets can be placed on the range of any node in the tree, for example "page views of the NBA subsection will be between 100K and 150K." LMSR pricing in this context is tractable using dynamic programming, though slight generalizations of the betting language render it hard (Guo and Pennock 2009).

Why do we need or want combinatorial-outcome markets? Simply put, they allow for the collection of more information. Combinatorial outcomes allow traders to assess the correlations among base objects, not just their independent likelihoods, for example the correlation between Democrats winning in Ohio and Pennsylvania. Understanding correlations is key in many applications, including risk assessment.

Although financial and betting exchanges, bookmakers, and racetracks are modernizing, turning their operations over to the computers and moving online, their core logic for processing bids hasn't changed much since auctioneers were people. For simplicity, they treat all bets like apples and oranges, processing them independently, even when they are related. For example, bets on a horse to win and to finish in the top two are managed separately at the racetrack, as are options to buy a stock at strike price 30 and strike price 20 on the Chicago Board Options Exchange. In both cases it's a logical truism that the first is worth less than the second, yet the market pleads ignorance, leaving it to traders to enforce consistent pricing.

In a combinatorial market, a bet on Democrats to win Ohio and Florida automatically increases the odds on Ohio alone, as it logically should. Mindless mechanical tasks like this are handled automatically, by algorithms that are far better at it anyway, freeing up traders for the primary task a prediction market asks them to do: provide information. Traders are free to express their information in whatever form they find most natural, and it all flows into the same pool of liquidity. Especially in the context of a prediction market, it makes sense to focus traders on giving information rather than content-free strategies like arbitrage.

It's hard to imagine a combinatorial-outcome market working in practice without an automated market maker: otherwise, traders are unlikely to find each other in the sea of choices. We don't believe that markets need to restrict themselves to polynomial-time bidding languages, often a severe constraint. Instead, we believe that computing approximate market maker prices via sampling, the approach taken by Yahoo!'s Predictalot system, offers a route to practical general-pur-

pose systems. The sampling problem in this setting is difficult and unsolved, and requires care in order to ensure that traders cannot game the market maker for unbounded profit.

Beyond computational concerns, the market operator should weigh any potential gains in information against the fact that traders' attention and liquidity will be severely fractured across the nearly limitless things available to bet on.

Conclusion

We surveyed the literature on prediction markets and peer prediction systems with a focus on the properties that are important for information elicitation and aggregation, the ultimate objective of prediction mechanisms. Prediction markets are used to elicit and aggregate information about uncertain events whose outcome can be verified at a specific time in the future. We reviewed recent research on designing automated market maker mechanisms that provide liquidity for the market, understanding whether information is truthfully aggregated in prediction markets with strategic agents, and developing combinatorial prediction markets where market participants have more expressiveness to reveal their information. Peer prediction systems are designed for eliciting information on events where ground truth does not exist or is unobtainable. The most important challenge here is how to elicit truthful reports from strategic agents. We reviewed peer prediction systems where truth telling is induced in equilibrium.

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Notes

1. The requirement for a cost-function-based market maker to be myopically incentive compatible is that the prices span the probability simplex. In other words, it is always possible for a trader to change the market probability to its probability assessment.

2. Proper scoring rules for eliciting information from individual agents are dominant-strategy incentive compatible. There are no interactions among agents.

3. See hanson.gmu.edu/policyanalysismarket.html.

4. See sandbox.yahoo.com/yoopick.

5. See sandbox.yahoo.com/predictalot.

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