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An in-depth analysis of information markets with aggregate uncertainty

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Abstract The novel idea of setting up Internet-based virtual markets, information markets, to aggregate dispersed information and predict outcomes of uncertain future events has empirically found its way into many domains. But the theoretical examination of information markets has lagged relative to their implementation and use. This paper proposes a simple theoretical model of information markets to understand their information dynamics. We investigate and provide initial answers to a series of research questions that are important to understanding how information markets work, which are: (1) Does an information market converge to a consensus equilibrium? (2) If yes, how fast is the convergence process? (3) What is the best possible equilibrium of an information market? and (4) Is an information market guaranteed to converge to the best possible equilibrium?

Keywords Information market · Information aggregation · Prediction · Aggregate uncertainty

1. Introduction

Predicting outcomes of uncertain future events is a crucial task in all walks of life. Traditional approaches to this problem include statistical methods based on historical data and methods to elicit expert opinions. However, both of these methods have limitations. Using historical data to make forecast does not take into account of information that is not contained in the past data. Eliciting expert opinions means identifying experts, soliciting their participation, and determining how to combine different opinions when experts are not in agreement, which are often not easy [Ashton and Ashton, 3; Batchelor and Dua, 5; French, 19; Genest and Zidek, 23]. For example, should we weigh predictions of experts based on their past performance or some other metrics? The fast development and wide use of Internet

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and electronic commerce technology give rise to a promising alternative tool for predicting future events—information markets. Depending on the context they are used, information markets are also called prediction markets, forecasting markets, decision markets, virtual stock markets, or game markets. In this paper, we adopt the name information markets for consistency.

An information market can be roughly defined as a virtual financial market that ties to a future event and is specifically designed for forecasting its outcome. The presidential election markets at the Iowa Electronic Markets (IEM) [31] are examples of information markets. The markets tie payoff of securities to the result of the election. To predict which political party will win the election, the winner-takes-all market at IEM trades two securities. One security is for the Republican Party. It pays off only when the Republican Party nominee, George W. Bush for 2004, wins the presidential election. The other security is for the Democratic Party. It pays off only when the Democratic Party nominee, John Kerry for 2004, wins the election. Market participants trade securities based on their expectations about Bush and Kerry's winning chances. The price of the security for the Republican (Democratic) Party is a money-weighted average of market participants' expectations on how likely Bush (Kerry) will win. Thus, we can view security prices as a group prediction of which political party will win the presidential election. Similarly, to further predict how many votes each candidate will win, the vote share market at IEM trades two securities whose payoffs are proportional to the percentage of the total popular votes received by Bush and Kerry respectively in the election.

Theoretical constructs that support the idea of using information markets to predict outcomes of future events are Hayek hypothesis [Hayek, 26] and efficient market hypothesis [Fama, 13]. Hayek's classic critique of central planning is based on the assumed information efficiency of markets, that is, the price system in a competitive market is a very efficient mechanism to aggregate dispersed information among market participants. The efficient market hypothesis further states that market price of a security almost instantly incorporates all available information of all market traders. Both hypotheses have been supported to a large extent by empirical and theoretical work [Grossman, 24; Jackwerth and Rubinstein, 27; Plott and Sunder, 38]. If the price of a security can incorporate all available information of market traders, it can be viewed as a consensus prediction about the value of the security. This implies that financial market mechanisms can be used to aggregate information and make predictions.

Information markets have been empirically proven effective in many domains, including politics [Forsythe et al., 16–17; Forsythe, Rietz, and Ross, 18], entertainment [Thaler and Ziemba, 43], and sports [Debnath et al., 9; Gandar et al. 21]. Hollywood Stock Exchange (HSX) [12] trades securities to predict future box office proceeds of new movies. TradeSports [44], a betting exchange registered in Ireland, hosts markets for sports, politics, entertainment, and financial events. Although current evidence from these markets strengthens the future possibility of using information markets to assist businesses, universities, and governments in making critical decisions, we must be cautious to proceed since many general issues on information markets are awaiting investigation. The following questions, not intended to be a complete list, reflect some of these issues.

- How do information markets fundamentally differ from traditional financial markets? Financial markets are mainly designed for the purpose of risk management. As the primary functionality of information markets shifts from risk management to prediction and information aggregation, how do mechanisms of information markets deviate from those of traditional financial markets? Are theoretical results of financial markets still applicable to information markets?

- What are relevant properties of information markets? For example, Hayek hypothesis and efficient market hypothesis only state that available information is reflected in market prices, but how dispersed information is incorporated into market prices has not been satisfactorily studied. Understanding the dynamic information incorporation process is crucial for designing and implementing effective information markets, especially when mechanisms of information markets can be different from those of financial markets. This understanding requires developing various models of information markets, analyzing their information dynamics, and evaluating the appropriateness of models.
- How should we design information markets in practice? What factors affect performance of information markets? Such design issues arise from both theoretical results and empirical experience of information markets. Spann and Skiera [41] and Wolfer and Zitzewitz [45] have begun to consider design issues of information markets, including specification of securities, incentives for participation and information revelation, and mechanisms to match buyers with sellers.

This paper is an effort to investigate the second general question. Results from this question can also inform the third question above. For example, understanding the dynamic information incorporation process would have implication on designing "better" marketplaces. Aiming at obtaining deeper understanding of how information markets work, we theoretically characterized some fundamental properties of information markets to answer the following specific research questions:

1. Does an information market converge to a consensus equilibrium?
2. If yes, how fast is the convergence process?
3. What is the best possible equilibrium for an information market?
4. Is an information market guaranteed to converge to the best possible equilibrium?

In an early study, Feigenbaum et al. [14] examined properties of information markets. However, their model does not allow for *aggregate uncertainty*. Aggregate uncertainty occurs when even if information of all traders is pooled together, the state of the world is still not fully determined. It is very common to most real world situations. The contributions of this study are that we develop a simple model of information markets that captures aggregate uncertainty and is able to demonstrate how dispersed information flows into market prices. Based on the model we provide initial answers to the four specific research questions outlined above. Introducing aggregate uncertainty into our model of information markets makes it more realistic.

The remainder of the paper is organized as follow. Section 2 positions our work within the related literature, especially theoretical work on information efficiency of markets. In Section 3, we provide a mathematical representation of information markets with aggregate uncertainty, on which the later analysis is based. Convergence properties of information markets are derived in Section 4 with the help of results from common knowledge literature [Aumann, 4; Geanakoplos and Polemarchakis, 22; McKelvey and Page, 32; Nielsen et al., 34]. In Section 5, we discuss the validity and limitations of our model, and compare and contrast our results with those without aggregate uncertainty. Finally, Section 6 concludes our findings and pinpoints future research directions.

2. Related work

Research on information markets can be roughly divided into two categories: theoretical work and empirical studies, with the empirical studies being the currently dominant strength.

In Section 2.1, we discuss related theoretical work, emphasizing its relationship to our work. In Section 2.2, we review some experimental work.

2.1. Relationship to other theoretical work

Theoretical work directly targeting information markets is still rare. But research on rational expectations equilibrium (REE) and common knowledge is closely related to and provides a strong background for investigating information markets. There is a rich resource of prior work on these topics. The works reviewed here are only intended to position our paper related to previous works, which by no means represent a complete list.

REE models have been the main approach to understanding and formalizing the efficient market hypothesis and Hayek hypothesis. They provide important explanations of certain macroeconomic and financial phenomena. REE models are an extension to general equilibrium models, but take account of potential informational feedbacks from market prices. At the *fully revealing REE*, equilibrium market price reveals information of all market traders. Traders' actions are based on all revealed information. Much work has been done to examine the existence and stability of REE [Allen, 1–2; Jordan and Radner, 29; Radner, 39]. Jordan [28] provides a more detailed and complete review of REE models used in microeconomics. However, REE models are criticized for two paradoxes that they imply [Dubey et al., 10]. First, how can market traders take into account of the equilibrium price in making decisions when it is those decisions that determine the price? REE generally requires the simultaneous determination of equilibrium price and available information. It does not consider how information flows into the market. Second, there is no incentive for individuals to gather costly private information since it is going to be reflected in the price. Dubey et al. [10] proposed market games to overcome the difficulties of REE models. Our work, due to its purpose of investigating convergence properties of information markets, models information markets as restricted market games. We try to examine whether information markets converge to an equilibrium at which all information is aggregated as at fully revealing REE.

Dating back to 1976, Aumann [4] presents the formal definition of common knowledge and studies how two people with asymmetric information agree with each other. Aumann proves that if two people have the same priors, and their posteriors for some event are common knowledge, then these posteriors must be equal. However, it is very rare that two people can have common knowledge about their posteriors at the very beginning. Geanakoplos and Polemarchakis [22] extend Aumann's work by demonstrating that if two people with common priors successively announce their posteriors to each other, eventually this leads to a situation of common knowledge where their posteriors are equal. McKelvey and Page [32] generalize the previous results to n persons and only require successively announcing an aggregate statistic of individuals' posteriors. When this statistic eventually becomes common knowledge, all posteriors of n persons are equal. Nielsen et al. [34] contribute by extending the conditional probability (posterior) to the case of conditional expectation. The above mentioned papers study how people disagree with each other can eventually reach an agreement. This process is analogous to the process of information aggregation. Market traders with different information disagree with the expected value of the security at the beginning. By trading in the market, they gradually reach an agreement, which is represented by the market price. Thus, results from research on common knowledge provide useful tools to analyze information dynamics of information markets. The results from McKelvey and Page [32] and Nielsen et al. [34] are used in our arguments on properties of information markets.

The work by Feigenbaum et al. [14] appears to be the only one that explicitly deals with information markets rather than traditional markets. Using an innovative computational approach, the authors view the information aggregation process as a distributed computation. Private information held by market traders are treated as inputs to a function. Equilibrium market price is the value or output of the function in ideal situations. Thus, an information market is modeled as attempting to correctly compute the value of the function. They prove that when the function takes a certain form, weighted threshold, the equilibrium market price is guaranteed to equal to the value of the function. The number of rounds for the market to converge to this equilibrium equals the number of traders in the market. Their model, however, does not consider aggregate uncertainty. Our work is closely related to their work, but generalizes it by introducing aggregate uncertainty.

2.2. Empirical studies of information markets

Laboratory experiments, by systematically controlling some of the market parameters, provide simplified environments for understanding information market performances. Plott and Sunder [38] set up experiments to examine issues of information aggregation when different traders have diverse information about an underlying state of the world. The information structure does not have aggregate uncertainty, which means that although no individual trader knows the state of nature, if traders pool their information together the state can be identified with certainty. Their results demonstrate that market structure is important for information aggregation. Only with appropriate market structure can markets aggregate diverse information. Lundholm [30] examines the effect of aggregate uncertainty and finds that markets aggregate information less efficiently when there is greater aggregate uncertainty. Forsythe and Lundholm [15] study the effect of traders' preferences on information aggregation. They find that if participants have heterogeneous preferences, experience of participants is a necessary condition for information aggregation. O'Brien and Srivastava [35] focus on the relationship between asset structure and information aggregation ability of the market. Their results show that information aggregation ability decreases when asset structure of the market is sufficiently complex. Sunder [42] extensively summarizes experimental work on information aggregation. He concludes that the difficulties of the state of research are to understanding what factors facilitate or prevent information aggregation.

3. Information market modeling

A generic model of information markets should include at least three indispensable components, information structure, market mechanism, and trader behavior, as shown in Figure 1. Our model generalizes Feigenbaum et al.'s model [14] to capture aggregate uncertainty. The market mechanism and assumptions on trader behavior in our model are basically the same as those of their model. But the information structure of our model is different. In this section, we only lay out the three components of our model, and leave the justification of the model to Section 5.

3.1. Information structure

Information structure of the market specifies what the state space of the world is, how much information traders know about the true state of the world, and how information of traders

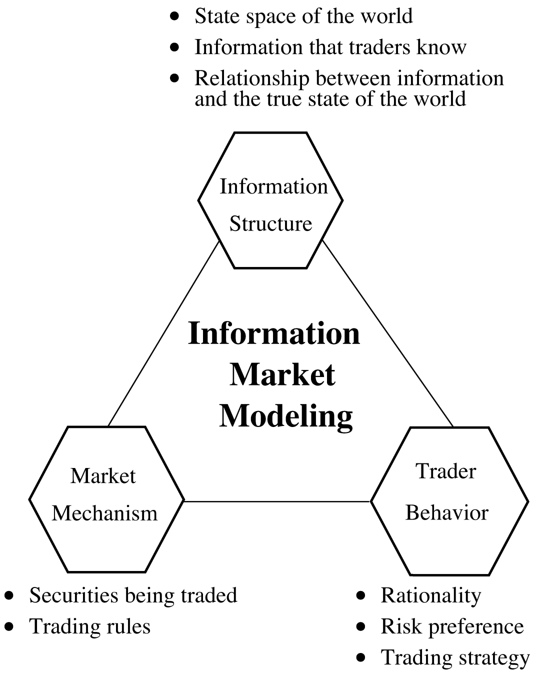

Fig. 1 Main Components of Information Market Modeling

relates to the true state of the world. For example, in the presidential election markets at IEM, state space of the world might be who is going to be nominated by each party, nominees' stances on important policy issues, population demographics, current strength of economy, previous voting records, and etc. Traders might have some information about the state of the world, such as their own regional demographics and local economy.

Usually information structure of markets is modeled using prior probability distributions of the state of the world and of the information that traders possess. Let S represents the state space of the world, where $s = (s_1, s_2, \ldots, s_m) \in S$ is a state vector of m dimensions. Assume there are n traders in the market, where all traders have a common prior probability distribution regarding to state of the world, $\mathcal{P}(s)$: $S \to [0, 1]$.

The trader's information space is X. Each trader $i = 1, \ldots, n$ gets a piece of information x_i about the state of the world, where $x = (x_1, x_2, \ldots, x_n) \in X$ is the information vector for all traders. Traders have common knowledge of the probability distribution of x, conditional on state of the world s, $\mathcal{Q}(x|s)$: $X \times S \to [0, 1]$. For example, suppose we have a one dimensional state space (i.e., $s = s_1$). The state of the world can be either 1 or 0. Conditional on $s = 1$ $(s = 0)$, the probability to get $x_i = 1$ $(x_i = 0)$ is 0.9, and the probability to get $x_i = 0$ $(x_i = 1)$ is 0.1. If the trader i gets $x_i = 1$, although he does not know the value of s for certain, he knows that, with probability 0.9, s equals 1. This uncertainty in individual information introduces aggregate uncertainty to our model. The true state of the world is uncertain even with pooled information.

We make a further simplifying restriction: The state variables, s_i's, and the information variables, x_i's, can only take Boolean values 0 or 1. Thus, the state space of the world in our model is $S = \{0, 1\}^m$. $X = \{0, 1\}^n$ is the information space. The prior probability distribution of the state of the world is $\mathcal{P}(s)$: $\{0, 1\}^m \to [0, 1]$, and $\mathcal{Q}(x|s)$: $\{0, 1\}^n \times \{0, 1\}^m \to [0, 1]$ is the conditional distribution of information.

In models of information markets without aggregate uncertainty, market traders hold accurate information about the state of the world s. For example, Feigenbaum et al.'s model [14] specifies that trader i knows s_i in an information market with n traders. Pooling information of all traders together makes the state of the world, s, uniquely determined. Their model can be viewed as a special case of our model by setting $m = n$ and $x_i = s_i$ with probability 1.

3.2. Market mechanism

Market mechanisms specify what securities are being traded and trading rules of the market. We model our market as predicting the value of a function $f(s)$. The value of the function is determined by the true state of the world, which will only be revealed some time in the future. One security is traded in the market, whose payoff is contingent on the value of $f(s)$. Specifically, the security pays off $\$f(s)$ in the future. The form of f is common knowledge to all traders. In our model, we restrict the value of the function $f(s)$ to be Boolean. Thus, $f(s) : \{0, 1\}^m \rightarrow \{0, 1\}$.

To explain why we model the payoff of securities as related to a function, we go through the abstract process of setting up an information market. Suppose we have an event of interest to predict, we can turn it into a random variable, create a security whose payoff equals the realized value of the random variable, and bring a group of participants together via an Internet marketplace to let them trade shares of the security[1]. Typically, the random variable is a function of the underlying state of the world. Using the presidential election winner-takes-all market as an example, we are interested in predicting the event whether the Democratic Party will win the presidential election. Turning the event into a random variable a, we have

$$a = \begin{cases} 1 \text{ if the Democratic Party nominee wins the presidential election;} \\ 0 \text{ if the Republican Party nominee wins the presidential election.} \end{cases}$$

The random variable a is determined by more fundamental variables such as who is going to be nominated by each party, what their stances on important policy issues are, and population demographics. These fundamental variables are characterized as state vector s in our model. Thus, a can be viewed as a function of s, (e.g. $a = f(s)$). The payoff of a share of the security for the Democratic Party will be $\$a$, which is the value of the function $f(s)$.

Following Dubey et al. [10] and Feigenbaum et al. [14], we model the market mechanism as a *Shapley-Shubik market game* [Shapley and Shubik, 40] with restrictions. The market game proceeds in rounds. In each round, each trader puts up quantities of the security to be sold and simultaneously puts up a positive amount of money to buy the security. For simplicity, we require each trader to offer selling 1 share of the security in each round, and assume that there are no restrictions on credit. Then, traders' bids can be represented as a vector $b = (b_1, b_2, \ldots, b_n)$, where b_i is the amount of money trader i offers to buy securities. The market determines the price of the security by taking the average of all bids in a round, thereby clearing demand and supply. Thus, the price for a round is $p = (\sum_{i=1}^{n} b_i)/n$. Only this price p, not individual traders' bids, is publicly announced in each round. All trading occurs at the market price. At the end of the round, trader i holds b_i/p shares of the security. He or she profits p dollars through selling the security and loses b_i dollars from buying the security. Thus, net money gain (loss) of trader i is $(p - b_i)$ dollars. The market then enters

[1] In practice, transferring an event into a random variable and choosing appropriate market trading rules can be complicated, which deserves separate discussion.

a new round. The process continues until an equilibrium is reached, after which prices and bids do not change from round to round.

3.3. Trader behavior

Modeling trader behavior can be achieved by specifying trader's risk preference, rationality, or trading strategy. In our model, we make the assumption that traders will always "tell the truth" rather than behave strategically. In other words, a trader will truthfully bid what he/she thinks the value of the security is in each stage of the market. This value is his/her expected payoff of a share of the security based on information available to him/her. The time value of money is ignored since information markets are usually alive only for a short period of time. Expectations are calculated based on probability distribution of the state of the world $\mathcal{P}(s)$, conditional probability distribution of information $\mathcal{Q}(x|s)$, and information inferred from market prices. As market prices may contain extra information, traders revise their expectations as the market proceeds.

4. Convergence properties of information markets

Based on the model of information markets in Section 3, we examine several convergence properties of information markets to answer the four research questions raised in Section 1.

4.1. Price convergence

As prices in information markets are predictors of future events, it is desirable that market prices are stable as long as no new information enters the markets. Thus, the first important question to ask is: Can an information market converge to an equilibrium, at which the price is stable if no new information arrives? With the aid of the results from McKelvey and Page [32] and Nielsen et al. [34], we present the answer to this question as Property 1.

Property 1. *Without the arrival of new information, an information market converges to an equilibrium in finite steps. At equilibrium, all traders have the same expectation about the value of $f(s)$, which equals the equilibrium market price.*

McKelvey and Page [32] and Nielsen et al. [34] studied how people who disagree with each other eventually reach an agreement. This process is analogous to the market trading process in our information market model. Roughly speaking, their results state that if the initial information partition of each trader is finite, and traders refine their information partition through an iterative process, in which a market statistic of traders' expectations of an event is made public in each period, then the market converges to an equilibrium in finite rounds. Further, if the market statistic satisfies some conditions, each trader's conditional expectation of the event must be identical at the equilibrium. We restate their results in our information market settings as Theorems 1 and Theorem 2, and apply them to obtain Property 1.

Let the initial information structure of an information market be as follows:

$$(\Omega, F, \rho) \quad \text{(a probability space)}, \tag{1}$$

$$P^0 = (P_1^0, \ldots, P_n^0) \quad \text{(initial information partitions)}, \tag{2}$$

$$h : \mathcal{R}^n \to \mathcal{R} \quad \text{(an aggregate function)} \tag{3}$$

For any individual i, P_i^0 is a finite partition of the probability space Ω. For any $\omega \in \Omega$, $P_i^0(\omega)$ denotes the element of P_i^0 that contains ω. The random variable that the market tries to predict is A. The market proceeds in rounds. Inductively, on round t, for each individual i and any state $\omega \in \Omega$, define

$$b_i^t(\omega) = E\big(A|P_i^t(\omega)\big) \tag{4}$$

to be individual i's expectation of the random variable A based on his current information partition.

$$b^t(\omega) = \big(b_1^t(\omega), \ldots, b_n^t(\omega)\big) \tag{5}$$

is the expectation vector for all agents.

Theorem 1. *(McKelvey and Page [32] and Nielsen et al. [34]) Assume an initial information structure as in (1), (2), and (3). Assume the market proceeds in an iterative process such that:*

(a) In every round t, a market statistic $\Phi^t = h(b^t(\omega))$ is made public;
(b) Traders refine their information partitions according to the information brought by the market statistic;
(c) Traders revise their next round expectation b_i^{t+1}'s according to their new information partitions.

Then, for all $\omega \in \Omega$, there is a round T such that Φ^T is common knowledge at ω.

Theorem 2. *(McKelvey and Page [32] and Nielsen et al. [34]) If the function h in (3) is stochastically regular, for any T, at which $\Phi^T = h(b^T(\omega))$ becomes common knowledge, and for all $\omega \in \Omega$, it must be the case that*

$$b_1^T(\omega) = b_2^T(\omega) = \cdots = b_n^T(\omega) = \Phi^T.$$

Mckelvey and Page [32] define that a function $g : \mathcal{R}^n \to \mathcal{R}$ is stochastically regular, if it can be written in the form $g = l \circ g'$, where g' is stochastically monotone and l is invertible on the range of g'. According to Bergin and Brandenburger [8], a function $g : \mathcal{R}^n \to \mathcal{R}$ is stochastically monotone if it can be written in the form $g(\mathbf{x}) = \sum_{i=1}^n g_i(x_i)$, where each $g_i : \mathcal{R} \to \mathcal{R}$ is strictly increasing.

By mapping the settings of the theorems to our information market model, we find that all requirements of the theorems are met by our information market. First, the elements of the probability space (Ω, F, ρ) can be interpreted as: Ω includes both the state space S and the information space X, i.e. $\Omega = \{S, X\} = \{0, 1\}^m \times \{0, 1\}^n$; F is the measurable space of Ω; and ρ is the joint probability distribution of s and x, which can be derived from the prior distribution of s, $\mathcal{P}(s)$, and conditional distribution of x, $\mathcal{Q}(x|s)$. (1) is thus well-defined.

Second, the finite initial information partition requirement in (2) is met in our model, because the initial information partition for each trader i is simply a bi-partition of the sample space according to the trader's bit of information x_i, that is $P_i^0 = \{\{S, X|x_i = 0\}, \{S, X|x_i = 1\}\}$.

Third, in our model, the event to be predicted is the value of $f(s)$. In other words, it is the event that $f(s) = 1$. Since we assume that traders will truthfully bid their expectation of the function $f(s)$, for each trader i and for any state $\omega \in \Omega$, $E(f(s)|P_i^t(\omega))$ would be individual i's bid at period t. It is exactly $b_i^t(\omega)$ as defined in (4).

Forth, in our information market, market clearing price p^t is announced as the market statistic Φ^T in each round of trading. Thus, the aggregation function h in (3) is the function to calculate the market clearing price. It is the mean function of all traders' bids at round t, $p^t = \sum_{i=1}^{n} b_i^t(\omega)/n$. This mean function satisfies the stochastically regularity condition required by Theorem 2.

Hence, Theorem 1 and Theorem 2 are applicable to our information market. Applying Theorem 1, we conclude that at some round T, the market price p^T becomes common knowledge. Loosely speaking, common knowledge is the knowledge that can be inferred by every trader before it is observed from the market. It does not bring any new information to traders. Traders' information partitions can not be further refined. Thus, their bids won't change, and the market price will remain at the same level in later rounds. The market reaches its equilibrium at the round T. Theorem 2 tells us that, at equilibrium, all traders have the same expectation about the value of $f(s)$, which equals the equilibrium market price.

4.2. Convergence speed

Since an information market is guaranteed to converge to an equilibrium in finite steps, how fast does it converge? Property 2 answers this question.

Property 2. *An information market converges to an equilibrium after at most n rounds of trading.*

Derivation of Property 2 is based on the nature of common knowledge possibility sets. It uses similar technique as that of Feigenbaum et al. [14] in proving the convergence time bound for information markets without aggregate uncertainty. We describe our inference process for Property 2 below.

The knowledge of trader i at time t can be viewed as the set of states in the space Ω that trader i considers possible to be the true state at time t. We call this set *trader i's knowledge possibility set at time t*, and denote as S_i^t. Common knowledge of traders at time t can be described as the set of states in the space Ω that are considered possible to be the true state by an outside observer at time t who only observes market prices without possessing any private information. We use S^t to denote the *common knowledge possibility set at time t*.

Before the market starts, the common knowledge possibility set is simply the whole space Ω, i.e. $S^0 = \Omega$. Each trader i submits his/her bid based on his/her knowledge possibility set S_i^0. After observing the market clearing price of round 1, p^1, an outside observer can logically eliminate those states in Ω that are not possible to have resulted in p^1. Common knowledge possibility set after round 1 contains less or equal elements than the initial common knowledge possibility set, i.e. $|S^0| \geq |S^1|$. According to the common knowledge possibility set after round 1, each trader can eliminate impossible states from his own knowledge possibility set. Trader i's knowledge possibility set after round 1 is $S_i^1 = S_i^0 \bigcap S^1$. Traders can revise their expectation in the next round of trading based on their updated knowledge possibility sets. This process continues. We thus have a sequence of common knowledge possibility sets: S^0, S^1, $S^2 \ldots$ Since knowledge needs to be consistent,

$$|S^0| \geq |S^1| \geq |S^2| \geq \cdots \tag{6}$$

must be satisfied.

We can show that the inequality in (6) is strict before the market reaches its equilibrium. After the equilibrium, the common knowledge possibility sets remain the same. Suppose that for some round T, $|S^T| = |S^{T+1}|$, it means that the market price of the $T + 1$ trading round does not provide any information to improve the common knowledge possibility set S^T. In later rounds, traders will behave the same as they were in trading round $T + 1$ because they gain no additional information from the market price in previous round. The market reaches its equilibrium, at which the market price becomes stable and common knowledge possibility sets in subsequent rounds equal to S^T. Thus, if the information market convergences to the equilibrium at round T, it must be the case that

$$|S^0| > |S^1| > \cdots > |S^T| = |S^{T+1}| = \cdots \quad (7)$$

The time for an information market to converge to its equilibrium equals the number of rounds that an observer of the market takes to improve the common knowledge possibility set from S^0 to S^T. The set S^0 is the whole sample space $\Omega = S \times X = \{0, 1\}^m \times \{0, 1\}^n$. Feigenbaum et al. [14] has shown that for any round t, all elements that are possible to result in the market price lie on a hyperplane in the sample space due to the linear price function of the Shapley-Shubik market game. Thus, if S^{t-1} and S^t are not equal, S^t is the intersection of S^{t-1} with that hyperplane. Geometrically speaking, the dimension of S^t, i.e. the dimension of the smallest linear subspace of $\mathcal{R}^n$ that contains all the points in S^t, is at least one dimensional lower than that of S^{t-1} before the market equilibrium is reached. The dimension of S^0 is $m + n$. The dimension of S^T at equilibrium is at least m due to the aggregate uncertainty. Hence, the information market takes at most n rounds to converge to the equilibrium.

4.3. The best possible prediction

Before we can evaluate the performance of an information market, we need a benchmark that defines what is the best possible prediction for information markets. This is given as Property 3. Knowing this will enable us to objectively analyze forecasting results of information markets and suggest possible ways to improve the best possible forecast.

Property 3. *The best possible prediction that an information market can make is the the forecast at direct communication equilibrium.*

Property 3 is an intuitive result. Rather than only making a market statistic public, market traders can directly reveal their private information to each other. In this situation, an equilibrium can be reached immediately. This equilibrium is called *direct communication equilibrium* or *pooled information equilibrium* [Geanakoplos and Polemarchakis, 22]. The equilibrium market price equals the expectation of the security payoff conditional on all available information, i.e. $E(f(s)|x)$. Since this prediction takes advantage of all information possessed by market traders, it is the best informed prediction in general. In other words, the best an information market can do is to completely aggregate all private information that is distributed among market traders. Direct communication equilibrium and fully revealing rational expectations equilibrium are the same in terms of information revelation and equilibrium price. We adopt the direct communication equilibrium in this paper because it is more clear at how information is aggregated. Example 1 calculates the prediction at direct communication equilibrium for a simple two trader information market.

Example 1. Consider a simple information market, where there is only one state variable s_1 and two traders, $i = 1$ or 2. s_1 can take value 0 or 1, each with probability 0.5, which is common knowledge to both traders. The function that the market wants to predict value for is $f(s_1) = s_1$. Thus,

$$f(s_1) = \begin{cases} 1 & \text{when } s_1 = 1 \\ 0 & \text{when } s_1 = 0. \end{cases}$$

The security traded in the market pays off \$1 if $f(s_1) = 1$, and \$0 if $f(s_1) = 0$. The probability distributions of x_i conditional on s_1 for $i = 1$ and 2 are independent and identical as follow:

$$Pr(x_i = 0|s_1 = 0) = 0.8, \quad Pr(x_i = 1|s_1 = 0) = 0.2;$$
$$Pr(x_i = 0|s_1 = 1) = 0.2, \quad Pr(x_i = 1|s_1 = 1) = 0.8;$$

Suppose the true state is $s_1 = 1$ and both traders' private information is 1, i.e. $x_1 = x_2 = 1$.

Using Bayes' rule, we can calculate the market price at the direct communication equilibrium:

$$\begin{aligned} E(f(s_1)|x_1 = 1, x_2 = 1) &= Pr(f(s_1) = 1|x_1 = 1, x_2 = 1) \\ &= Pr(s_1 = 1|x_1 = 1, x_2 = 1) \\ &= \frac{Pr(x_1 = 1, x_2 = 1|s_1 = 1)Pr(s_1 = 1)}{Pr(x_1 = 1, x_2 = 1)} \\ &= \frac{0.64 \times 0.5}{0.34} \\ &\approx 0.94. \end{aligned}$$

Thus, the best possible forecast of $f(s_1)$ is 0.94. It says that the true state of the world is very likely to be $s_1 = 1$, but there is uncertainty associated with the prediction.

The best possible prediction implies that the ability of information markets to make predictions is constrained by the amount of aggregate uncertainty. If the aggregate uncertainty is large, even if an information market aggregates all the information, the prediction result can still be poor. If we change the probability distribution of x_i conditional on s_1 for $i = 1$ and 2 in Example 1 to the followings:

$$Pr(x_i = 0|s_1 = 0) = 0.2, \quad Pr(x_i = 1|s_1 = 0) = 0.8;$$
$$Pr(x_i = 0|s_1 = 1) = 0.2, \quad Pr(x_i = 1|s_1 = 1) = 0.8;$$

The expectation at the direct communication equilibrium would only be $E(f(s_1)|x_1 = 1, x_2 = 1) = 0.5$. It provides nothing better than simply knowing the prior distribution of s_1. This is an extreme case because of the independence of information x_i and the state s_1. Both trader 1 and trader 2 in this case don't have real information other than the prior probability distribution of s_1 regarding the future market situation. From this perspective, performance of information markets relies on the information quality of their participants. In other words, in order for an information market to make good predictions, there must be some knowledge in the market about the future event to be predicted.

4.4. Convergence to the best prediction or not

We have shown that an information market will converge to a consensus equilibrium, the convergence process takes at most n rounds of trading, and that the best possible prediction is the direct communication equilibrium. Our next question is: will an information market always converge to direct communication equilibrium? Unfortunately, the answer is "no".

Property 4. *An information market is not guaranteed to converge to direct communication equilibrium.*

In the following, we provide two examples of information markets. Both markets trade the same security, but the probability distributions of traders' information are different. Example 2 does not converge to the direct communication equilibrium, while example 3 does.

Example 2. Consider an information market, where state of the world is $s = (s_1, s_2)$. There are two traders in the market, $i = 1, 2$. Value of s_j, $j = 1$ or 2, can be either 0 or 1. Suppose the common prior probability distribution of s is uniform, i.e. $s = (s_1, s_2)$ takes the values (0, 0), (0, 1), (1, 0), and (1, 1) each with probability 0.25. The function that the market wants to predict is

$$f(s_1, s_2) = \begin{cases} 1 & \text{when } s_1 = s_2 \\ 0 & \text{otherwise.} \end{cases}$$

The security traded in the market pays off \$1 if $f(s_1, s_2) = 1$, and \$0 if $f(s_1, s_2) = 0$. The probability distributions of trader's information x_i conditional on s are independent and identical as follow:

$$Pr(x_i = 0|s_1 = 0, s_2 = 0) = 0.9, \quad Pr(x_i = 1|s_1 = 0, s_2 = 0) = 0.1;$$

$$Pr(x_i = 0|s_1 = 0, s_2 = 1) = 0.5, \quad Pr(x_i = 1|s_1 = 0, s_2 = 1) = 0.5;$$

$$Pr(x_i = 0|s_1 = 1, s_2 = 0) = 0.5, \quad Pr(x_i = 1|s_1 = 1, s_2 = 0) = 0.5;$$

$$Pr(x_i = 0|s_1 = 1, s_2 = 1) = 0.1, \quad Pr(x_i = 1|s_1 = 1, s_2 = 1) = 0.9.$$

Suppose that the true state is $s = (1, 1)$, and both traders' private information is 1, i.e. $x_1 = x_2 = 1$.

According to Bayes' rule, trader i with information $x_i = 1$ would like to submit bid

$$\begin{aligned} b_i(1) &= E(f(s)|x_i = 1) \\ &= Pr(f(s) = 1|x_i = 1) \\ &= \frac{Pr(x_i = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_i = 1)} \\ &= \frac{0.5 \times 0.5}{0.5} \\ &= 0.5. \end{aligned}$$

Similarly, with information $x_i = 0$ trader i would like to submit bid

$$\begin{aligned} b_i(0) &= E(f(s)|x_i = 0) \\ &= Pr(f(s) = 1|x_i = 0) \\ &= \frac{Pr(x_i = 0|f(s) = 1)Pr(f(s) = 1)}{Pr(x_i = 0)} \\ &= \frac{0.5 \times 0.5}{0.5} \\ &= 0.5. \end{aligned}$$

Hence, no matter what value x_i is, trader i will always bid 0.5 in the first round of trading. When both traders bid $b_i = 0.5$, market clearing price is also 0.5. From the market clearing price, trader 1 can infer that trader 2 bid 0.5, but this gives him no information about trader 2's private information x_2. Trader 2 can do the same inference and also gains no additional information. Neither trader will change their bids in later rounds. Hence, the market reaches its equilibrium in the first round with equilibrium price equals 0.5. This is nothing better than simply using the $Pr(f(s) = 1)$ to make the prediction. However, pooling information directly can make better prediction. Under the direct communication equilibrium, market price should equal

$$\begin{aligned} E(f(s)|x_1 = 1, x_2 = 1) &= Pr(f(s) = 1|x_1 = 1, x_2 = 1) \\ &= \frac{Pr(x_1 = 1, x_2 = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_1 = 1, x_2 = 1)} \\ &= \frac{0.41 \times 0.5}{0.33} \\ &\approx 0.62. \end{aligned}$$

Thus, the direct communication equilibrium price indicates that, given both traders have information 1, the probability for the function $f(s)$ to be 1 is 0.62, which is a better prediction as opposed to 0.5.

Example 3. With all other conditions remain the same as in example 2, we change the probability distribution of trader's information x_i conditional on s to:

$$\begin{aligned} &Pr(x_i = 0|s_1 = 0, s_2 = 0) = 0.9, \quad Pr(x_i = 1|s_1 = 0, s_2 = 0) = 0.1; \\ &Pr(x_i = 0|s_1 = 0, s_2 = 1) = 0.9, \quad Pr(x_i = 1|s_1 = 0, s_2 = 1) = 0.1; \\ &Pr(x_i = 0|s_1 = 1, s_2 = 0) = 0.5, \quad Pr(x_i = 1|s_1 = 1, s_2 = 0) = 0.5; \\ &Pr(x_i = 0|s_1 = 1, s_2 = 1) = 0.1, \quad Pr(x_i = 1|s_1 = 1, s_2 = 1) = 0.9. \end{aligned}$$

We still suppose that the true state is $s = (1, 1)$ and both traders' private information is 1.

Under the condition of example 3, if trader i has information $x_i = 1$, his bid would be

$$\begin{aligned} b_i(1) &= E(f(s)|x_i = 1) \\ &= Pr(f(s) = 1|x_i = 1) \end{aligned}$$

$$
\begin{aligned}
&= \frac{Pr(x_i = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_i = 1)} \\
&= \frac{0.5 \times 0.5}{0.4} \\
&= 0.625.
\end{aligned}
$$

If trader i has information $x_i = 0$, he would like to submit bid

$$
\begin{aligned}
b_i(0) &= E(f(s)|x_i = 0) \\
&= Pr(f(s) = 1|x_i = 0) \\
&= \frac{Pr(x_i = 0|f(s) = 1)Pr(f(s) = 1)}{Pr(x_i = 0)} \\
&= \frac{0.5 \times 0.5}{0.6} \\
&\approx 0.42.
\end{aligned}
$$

Thus, both traders will submit 0.625 as their bids in the first round of trading since they all have information 1. The market clearing price for round 1 would also be 0.625. Observing the clearing price, trader 1 can infer that trader 2 must have bid 0.625, which further means that trader 2's information is 1. Trader 1 thus gets to know both pieces of information. His bid in the second round will be

$$
\begin{aligned}
E(f(s)|x_1 = 1, x_2 = 1) &= Pr(f(s) = 1|x_1 = 1, x_2 = 1) \\
&= \frac{Pr(x_1 = 1, x_2 = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_1 = 1, x_2 = 1)} \\
&= \frac{0.41 \times 0.5}{0.27} \\
&\approx 0.76.
\end{aligned}
$$

Similarly, trader 2 can infer from the market price that $x_1 = 1$, and bid $E(f(s)|x_1 = 1, x_2 = 1) \approx 0.76$ in the second round. Market price of round 2 will be 0.76, which incorporates private information of both traders. Thus, the information market reaches its equilibrium in the second round, at which it predicts that for probability 0.76 $f(s)$ will have value 1. This equilibrium is the same as direct communication equilibrium.

The reason that the information market in example 2 does not converge to the direct communication equilibrium seems to be the high degree of symmetry of traders bidding behavior. Even with different private information, a trader bids the same value. The market price is then unable to reveal trader's private information. Hence, if the prior probability distribution of the state of the world, $\mathcal{P}(s)$, and the conditional probability distribution of information, $\mathcal{Q}(x|s)$, of an information market accidentally create this kind of symmetry, the information market might not be able to perform well in making predictions.

5. Discussions

5.1. Justification of our model

We present a simple model of information markets with aggregate uncertainty in Section 3 without commenting on the reasonableness of the model. In this part, we will examine the validity and limitations of the model.

Information markets are molded as restricted Shapley-Shubik market games, in which traders only know how much money they are going to spend but don't know for sure how many shares of securities they can get. Trading rules of Shapeley-Shubik market games seem quite different from those of commodity markets, where the price is fixed, and stock markets, where bids and offers include both price and quantities. But they are not too different. First, once an information market reaches its equilibrium, the equilibrium market price and information efficiency are usually not affected by trading rules. Second, in our analysis of convergence properties of information markets, the two key assumptions are that traders truthfully bid their expectations of the security value and that market price is a known stochastically regular function of traders' bids. For other market mechanisms, as long as they satisfy these assumptions, properties 1, 3, and 4 hold. For example, these two assumptions are roughly satisfied with the *market scoring rule* mechanism for information markets, which was proposed by Hanson [25]. The limitation of using Shapley-Shubik market game to model information markets is that property 2 of our analysis is not robust. It relies on the linear price function of the restricted Shapley-Shubik market games.

Traders are assumed to "tell the truth" rather than behave strategically in our model. This assumption seems reasonable when the number of traders in the market is large. When n is large, the effect of a single trader's bid on the market clearing price is relatively small or even ignorable. Thus, traders might not have enough incentive to deviate from their true expected values and bid strategically. In addition, solving optimal strategies of traders for a n-person game usually needs the assumption of symmetry among traders for computational reasons. Assuming that traders are symmetric in holding information, however, will make the information structure of the market too simple to be representative and interesting.

5.2. Comparison with information markets without aggregate uncertainty

Our model captures aggregate uncertainty of information markets. In order to investigate the impact of aggregate uncertainty on information markets, we compare our model and results with those of Feigenbaum et al. [14], which does not consider aggregate uncertainty.

Table 1 briefly presents the comparison of both modeling and convergence properties of information markets with and without aggregate uncertainty. While market mechanisms and trader behaviors are the same for the two markets, information structures are different. For the market without aggregate uncertainty, trader i is informed of the value of s_i, which is part of the state vector s. But in the market with aggregate uncertainty, trader i only gets to know x_i, which relates to s_i with some uncertainty.

Comparing the convergence properties of the two markets with different information structures, we can see that the first convergence property are the same regardless of aggregate uncertainty. An information market always converges to a consensus equilibrium in finite rounds. At equilibrium, market traders' expectations of the security value are the same, which equal to the equilibrium market price. The second property is roughly equivalent for the two markets. The number of rounds for an information market to converge to a consensus equilibrium equals the number of traders in the market. The third property says that the di-

Table 1 Comparisons of information markets with and without aggregate uncertainty

Comparison Items		With aggregate uncertainty	Without aggregate uncertainty
Information Market Modeling	Market Structure	• *State of the world:* $s \in \{0, 1\}^m$ with prior probability distribution $\mathcal{P}(s)$. • *Trader information: n* Traders; Trader i holds x_i; $x \in \{0, 1\}^n$ with conditional probability distribution $\mathcal{Q}(x\|s)$.	• *State of the world:* $s \in \{0, 1\}^n$ with prior probability distribution $\mathcal{P}(s)$. • *Trader information: n* traders; Trader i holds s_i.
	Market Mechanism	• The security pays off $\$f(s)$ in the future; • The market is a restricted Shapely-Shubik market game.	Same
	Trader Behavior	Bid expected payoff of a share of the security.	Same
Convergence Properties	Price Convergence	Converge to a consensus equilibrium in finite steps.	Same
	Convergence Speed	At most n rounds. n is the number of traders.	Same
	Best Possible Prediction	Direct communication equilibrium, where $p = E(f(s)\|x)$.	Direct communication equilibrium, where $p = f(s)$.
	Convergence to the Best Possible Prediction	Not guaranteed.	Guaranteed, if f is a weighted threshold function.

rect communication equilibrium is the best possible prediction for both information markets. But, for information markets without aggregate uncertainty, pooling information together fully determines the true state of the world and hence market price at direct communication equilibrium computes the value of $f(s)$, while for information markets with aggregate uncertainty, price at direct communication equilibrium is the expectation of $f(s)$ conditional on pooled information. The most important difference between information markets with aggregate uncertainty and those without is the last property. Feigenbaum et al. [14] proved that if $f(s)$ is a *weighted threshold function*, an information market without aggregate uncertainty is guaranteed to converge to direct communication equilibrium for any prior probability distribution of s. The function f is a weighted threshold function if and only if there are real constants $w_1, w_2, \ldots w_m$ such that

$$f(s) = 1 \;\; iif \;\; \sum_{i=1}^{m} w_i s_i \geq 1. \tag{8}$$

This neat guaranteed-convergence result is no longer valid when aggregate uncertainty is introduced into information markets. The function f in example 2 is not a weighted threshold function. However, this is not the reason (at least, not the only reason) for its non-convergence to the direct communication equilibrium. We can redefine the function f in example 2 to be a weighted threshold function as below:

$$f(s_1, s_2) = \begin{cases} 1 & \text{when } s_1 = s_2 = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Using the same inference as in example 2, it can be shown that the market still does not converge to the direct communication equilibrium, given the following conditional probability distribution of traders' information and with all other conditions remain the same,

$$Pr(x_i = 0|s_1 = 0, s_2 = 0) = 0.8, \quad Pr(x_i = 1|s_1 = 0, s_2 = 0) = 0.2;$$

$$Pr(x_i = 0|s_1 = 0, s_2 = 1) = 0.5, \quad Pr(x_i = 1|s_1 = 0, s_2 = 1) = 0.5;$$

$$Pr(x_i = 0|s_1 = 1, s_2 = 0) = 0.5, \quad Pr(x_i = 1|s_1 = 1, s_2 = 0) = 0.5;$$

$$Pr(x_i = 0|s_1 = 1, s_2 = 1) = 0.6, \quad Pr(x_i = 1|s_1 = 1, s_2 = 1) = 0.4.$$

Thus, when there is aggregate uncertainty, the conditional probability distribution of information and the prior probability distribution of s have influence on whether an information market converges to direct communication equilibrium, whatever the form of the function is.

6. Conclusions

Information markets have been proposed as an alternative tool for predicting future events. Many real world online markets are providing test grounds for information markets. In addition to Iowa Electronic Markets (IEM) [31], Hollywood Stock Exchange (HSX) [12], and TradeSports [44] that we mentioned in Section 1, Foresight Exchange (FX) [11] allows traders to bet on unresolved scientific questions or other claims of public interest, NewsFutures's World News Exchange [33] has very popular sports and financial betting markets, and MIT's Innovation Futures [20] predicts important business and technology trends. Although prices of securities in many of these markets were found to give as accurate or more accurate predictions than polls and expert opinions [Berg et al., 6; Berg and Rietz, 7; Forsythe et al., 17; Forsythe, Rietz, and Ross, 18; Pennock et al., 36–37], how and why information markets work have not been fully explored.

This study provided a theoretical analysis on the information aggregation ability of information markets. By characterizing the uncertainty of market participants' private information, we incorporated aggregate uncertainty in our information market model. Based on the model, we examined some fundamental convergence properties of information markets, which answers the four research questions raised in Section 1. Specifically, we have shown that (1) an information market is guaranteed to converge to an equilibrium, at which traders have consensus about the forecast; (2) it converges to the equilibrium in at most n rounds of trading, where n is the number of traders ; (3) the best possible prediction it can make is the direct communication equilibrium, at which price equals the expectation of the value of the function based on information of all traders; (4) but an information market is not guaranteed to converge to this best possible prediction.

Comparing these results with those of information markets without aggregate uncertainty, we found that differences brought by aggregate uncertainty lay in the third and fourth results. Although the best possible equilibrium is always the direct communication equilibrium, it accurately computes the value of the function f if there is no aggregate uncertainty, while it can only reveal the expected value of f conditional on all information when aggregate uncertainty exists. Without aggregate uncertainty, an information market is guaranteed to converge to the direct communication equilibrium if f is a weighted threshold function. But with aggregate uncertainty, this is no longer true. Since aggregate uncertainty is so common in the real world, the differences imply that in order for an information market to make good predictions, there must be some knowledge in the market. But knowledge is not sufficient for good predictions. How to design an information market that will converge to direct communication equilibrium is an important research question to be explored.

This work is an initial attempt to understanding the power of information markets. Several issues deserve further investigation:

- *The effect of aggregate uncertainty on information markets:* Our results have shown that aggregate uncertainty can negatively affect the power of information markets to various extends. It is important to measure or quantify the effect so that measures can be suggested to manage or reduce the effect.
- *Design issues of information markets:* Our results have shown that information markets might not always converge to the direct communication equilibrium, where all information is aggregated. How to design information markets to ensure good predictions is an area that needs more effort.
- *Robustness of information markets:* Before information markets can be used to facilitating decision making, their robustness needs to be examined. For example, the prediction performance of information markets when there are manipulation incentives is of special importance.

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