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MARKETS AS AN INFORMATION AGGREGATION MECHANISM FOR DECISION SUPPORT

A Thesis in

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by

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Abstract

In almost all walks of life, predicting uncertain future events plays an essential role in decision-making processes. However, information related to future events frequently exists only as dispersed opinions, insights, and intuitions of individuals. Each individual only knows a little, but aggregating the dispersed information together may make considerable contribution to decision making. This is typical in many domains including business, politics, and entertainment. Therefore, how to aggregate such dispersed information for useful decision support is a crucial task.

Markets have shown great potential as one of the most effective mechanisms for gathering distributed information and generating accurate forecasts, often surpassing many existing methods in practice. This research studies information markets, markets that are specially designed for information aggregation and forecasting, from four different perspectives: theoretical examination, experimental evaluation, empirical analysis, and design.

With the ultimate goal of better understanding information markets as a forecasting device, this thesis makes four contributions to the field of information markets. The first contribution is a theoretical model of information markets that generalizes an existing model to situations with aggregate uncertainty, which is ubiquitous in the real world. It

helps answering the question of why information markets work, by modeling how information flows from traders to the market and back again, and characterizing convergence properties of information markets.

The second contribution is an experimental evaluation of several theoretical models of information markets. Because theoretical models often have to make simplified assumptions about human behavior for tractableness, we use human subject experiments to test them, while still maintaining close parallel settings with the theoretical models. Results of this part demonstrate whether and to what extent theoretical models are supported in a more realistic environment and point out important areas to be improved by theoretical models.

The third contribution is an initial attempt to compare the prediction accuracy of information markets and opinion pools using real-world market data. The results provide insights into the predictive performance of information markets, and the relative merits of selecting among various opinion pooling methods.

The last contribution of the thesis is a generic framework of information market development. Although evidence has shown that information markets can make accurate predictions, there are certainly cases that markets fail. How to design an information market for accurate predictions in practice remains an open question. To facilitate the development process, the proposed framework illustrates the life cycle of information market development and explains issues to be considered at each stage.

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As always, the greatest debt I owe is to my family, especially my parents. Thanks for keeping so close despite being so far away.

Chapter 1

Introduction

1.1 Problem Statement

Forecasting seems to be an ubiquitous endeavor in human societies. For instance, governments conjecture the effects of alternative policies; businesses project product sales revenue; meteorologists forecast future weather conditions; financial analysts predict stock price trends; and individuals bet on outcomes of sport games. In almost all walks of life, predicting uncertain future events plays a crucial role in decision support.

Many forecasting problems have at least two characteristics in common. First, the uncertainty of the problem changes as time goes by and new relevant information appears. Second, information related to the forecasting problems frequently only exists as the dispersed opinions, insights, and intuitions of individuals. Each individual knows very little, but aggregating the dispersed information together can make considerable contribution to decision-making. This is especially typical in situations such as supply chain management, business forecasting, new product development, policy analysis, and sports betting. Thus, how to make timely and accurate predictions that make use of such bits and pieces of information is very important, which is also the scope of the thesis.

1.2 Motivation

For decades, scientists have devoted themselves to developing and exploring various forecasting methods, which can be roughly divided into statistical and non-statistical approaches. Statistical methods, including econometric models and some machine learning techniques, are based on historical data. Non-statistical methods frequently rely on expert judgment and opinions. However, both of these approaches have limitations. Statistical methods require not only the existence of enough historical data but also that past data contain valuable information about the future event. Eliciting expert opinions means identifying experts, soliciting their participation, and determining how to combine different opinions when experts are not in agreement, which are often not easy [4, 6, 28, 35].

With the fast growth of the Internet, information markets have recently emerged as a promising alternative forecasting tool. Also called prediction markets, decision markets, event markets, or virtual stock markets, information markets are markets that are specially designed for aggregating information and making predictions about future events. Such markets are becoming very popular online. The Iowa Electronic Markets (IEM) [50] are real-money futures markets to predict economic and political events such as presidential elections. Hollywood Stock Exchange (HSX) [21] trades securities to forecast future box office proceeds of new movies. Tradesports.com [72], a betting exchange registered in Ireland, hosts markets for sports, political, entertainment, and financial events. Foresight Exchange (FX) [20] allows traders to wager on unresolved scientific questions or other claims of public interest. NewsFutures.com's World News Exchange [54] has very popular sports and financial betting markets. MIT's Innovation Futures [29] predict important business and technology trends. Tech Buzz Game [30] aims at both forecasting high-tech trends and testing a new market mechanism.

Information markets as a forecasting method have many advantages. Compared with statistical forecasting methods, information markets can incorporate real-time information, which was not contained in historical data. Compared with eliciting expert opinions, information markets are less constrained by space and time; they eliminate the effort of identifying experts and soliciting their participation, and hence are often less expensive in practice; and they do not need to deal with conflicting opinions. More importantly, information markets can potentially make real-time predictions that take advantage of the dispersed information, which are sometimes hard to capture using other forecasting methods.

Despite merits and popularity of information markets, why they work, how well they perform, and how to design effective information markets are still open questions to a large extent. If information markets are to be used to assist businesses, universities, and governments in making critical decisions in the real world, investigating these questions are imperative. The thesis is an effort on this track. It aims at providing a comprehensive understanding on properties and performance of information markets, through rigorous theoretical, experimental, and empirical examinations, and obtaining an initial framework to guide information market design and development.

1.3 Purposes of the Study

We investigate information markets from four related approaches: theoretical examination, experimental evaluation, empirical analysis, and design. Table 1.1 shows the general and specific research questions that we intend to address with each approach. Theoretical examination can help understand the general question of why information markets work. This is achieved through developing computational models of information markets. Experimental evaluation uses human subjects to test theoretical models in a controlled laboratory environment and identify where to improve. Empirical analysis using real-world datasets aims at investigating the actual predictive performance of information markets. Based on previous results, we then investigate issues of information market design and development. We will discuss our specific research questions in later sections when we actually address them.

1.4 Thesis Organization

The remainder of the thesis is organized as follow. Chapter 2 introduces basics of information markets and reviews related work. Chapter 3 covers the theoretical examination of information markets, in which we present a theoretical model of information markets, and discuss properties of information markets based on the model. Experiments to evaluate several theoretical models are the theme of Chapter 4. Chapter 5 compares information markets and various opinion aggregation methods in terms of prediction accuracy. Based

Table 1.1. Research Approaches and Research Questions

Research Approaches	General Research Questions	Specific Research Questions
Theoretical Examination	Why do information market work?	Will an information market converge to a consensus equilibrium? If yes, how fast is the convergence process? What is the best possible equilibrium? Will an information market always converge to it?
Experimental Evaluation	To what extent, are theoretical models of information markets valid?	Are properties derived from theoretical models supported by experiments? Are assumptions of theoretical models supported by experiments? Which aspects of theoretical models should be improved?
Empirical Analysis	How well do information market work?	How well do information markets perform compared with other forecasting methods, especially opinion pools?
Design & Development	How to develop an effective information markets?	When to choose information markets over other forecasting methods? What are issues to be considered when developing an information market? What are the options for each issue? Is there a generic framework for information market development?

on the evidence of our studies and previous research, Chapter 6 proposes a framework for information market development and identifies issues to be considered in each step of the development process. Chapter 7 concludes the thesis.

Chapter 2

Background

2.1 Fundamentals of Information Markets

Much of the enthusiasm of information markets stems from Hayek hypothesis [40] and efficient market hypothesis [22]. Hayek, in his classic critique of central planning in 1940's, claims that the price system in a competitive market is a very efficient mechanism to aggregate dispersed information among market participants. The efficient market hypothesis further states that, in an efficient market, price of a security almost instantly incorporates all available information. Market price summarizes all relevant information across traders, hence is market participants' consensus expectation about the future value of the security. Much evidence supports both hypotheses to a large extent [36, 41, 63]. Thus, when associating the value of a security with the outcome of an uncertain future event, market price, by revealing the consensus expectation of the security value, can indirectly predict the outcome of the event. This idea gives rise of information markets.

Using the 2004 U.S. Presidential election as an example, if we want to predict whether George W. Bush or John Kerry will win the election, an information market can trade a "Bush Security", a share of which pays the following after the election:

$$\begin{cases} \$1 & \text{if George W. Bush wins the election;} \\ \$0 & \text{otherwise.} \end{cases} \tag{2.1}$$

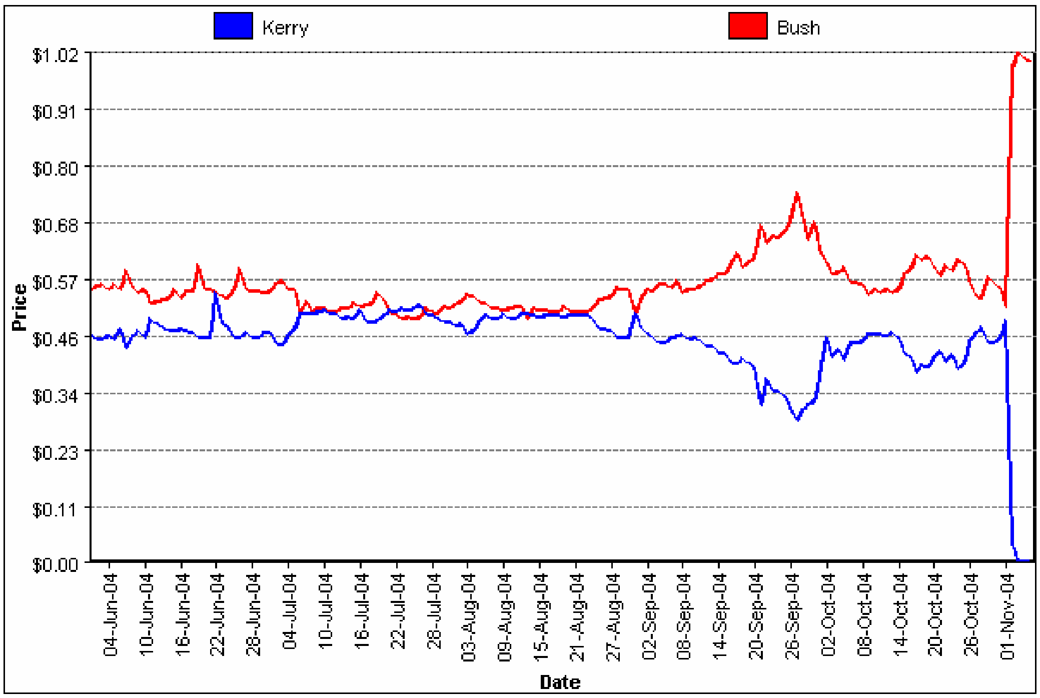
The security price should equal to the expected payoff of the security in theory for an efficient market. Time value of money usually can be ignored because the duration of most information markets is short. Let W represent the event that George W. Bush wins the election. Thus,

$$p = \mathbf{P}r(W) \times 1 + [1 - \mathbf{P}r(W)] \times 0, \tag{2.2}$$

where p is the price of the "Bush Security" and $\mathbf{P}r(W)$ is the probability that Bush will win the election. Observing the security price p before the election, we can derive $\mathbf{P}r(W)$, which is market participants' prediction about how likely Bush will win the election. For instance, if current price of the security is \$0.6, it means that market traders believe that with probability 0.6 Bush will beat Kerry. This probability is a consensus among market participants. If some market traders possess crucial information that leads them to believe that Bush only has half chances to win, they will sell their security holdings at the current price, which in turn drives down the price.

The above security is a *winner-takes-all* contract. It is used when the event to be predicted has only two possible disjoint outcomes. A winner-takes-all security pays off only when the specified outcome occurs. Thus, its price predicts the probability that a specific outcome will be realized. Iowa Electronic Markets (IEM) [50] trades winner-takes-all contracts to predict outcomes of presidential election. For the 2004 election, "Bush Security" pays off \$1 per share only if George W. Bush wins the election, while "Kerry Security" pays its holders \$1 per share only when Kerry wins the election. Figure 2.1 shows the prices of the two securities during June 4, 2004 and November 4, 2004, the date when the election outcome was announced. It was very impressive how security prices timely responded to relevant political events. Around the Democratic National Convention (July 26, 2004 to July 29, 2004), the price of the "Kerry Security" outgrew that of "Bush security". However, when the Republican National Convention was hold from August 30, 2004 to September 2, 2004, an increase of the "Bush security" price and an decrease of the "Kerry security" price were observed. After that, the price of the "Bush security" had been increasing until the first Bush-Kerry debate on September 30, 2004. Although, the price of the "Bush security" decreased around Bush-Kerry debates, the market clearly predicted that George W. Bush would win the election after the third debate on October 13, 2004.

When the outcome of a prediction problem can be any value in a continuous interval, we can design a security that pays its holder proportional to the realized value to predict the expectation of the outcome. This kind of security is what Wolfers and Zitzewitz [75] called *index* contract. The securities in the Presidential election vote share markets at IEM are examples of index contracts. In order to predict the percentage of votes that each candidate earn in the election, IEM trades a security for each candidate, payoff of which is \$1 times the candidate's share of the popular vote. If the market is efficient, price of such



Produced by: Iowa Electronic Markets, University of Iowa Tippie College of Business

Fig. 2.1. Bush versus Kerry
Souces: http://128.255.244.60/graphs/graph_Pres04_WTA.cfm

a security roughly equals the expected payoff of the security,

$$p = E(1 \times \textit{vote share of the candidate}) = E(\textit{vote share of the candidate}), \tag{2.3}$$

where $E(\textit{vote share of the candidate})$ is market's expectation of the vote share that the candidate will win. Thus, the security price offers a forecast of the mean value of the candidate vote share.

Many other aspects of a future event can be predicted in information markets by designing and trading different securities. Wolfers and Zitzewitz [75] provide a summary of the three main types of securities traded in information markets and what statistical properties they can predict. A winner-takes-all security predicts the probability that an event will occur. If defining many events, these winner-takes-all securities together can recover the probability distribution over the event space. An index security forecasts the mean value of a random variable. Generalizing it, it can be used to predict the mean value of a function of the random variable, if its payoff is proportional to the function value. A spread security can be used to estimate the median value of a random variable. More generally designed, it can be used to predict percentiles of the random variable.

2.2 Related Work

According to the research approaches used, we classify previous work that is related to information markets into four categories and review them separately as below.

2.2.1 Theoretical Examination

Theoretical work directly targeting information markets is still rare. But research on rational expectations equilibrium (REE) and common knowledge is closely related and provides a strong background for investigating information markets.

REE models have been the main approach to understanding and formalizing the efficient market hypothesis and Hayek hypothesis. They provide important explanations of certain macroeconomic and financial phenomena. REE models are an extension to general equilibrium models, but take account of potential informational feedbacks from market prices. At the *fully revealing REE*, equilibrium market price reveals information of all market traders. Traders' actions are based on all revealed information. Much work has been done to examine the existence and stability of REE [1, 2, 44, 66]. Jordan [43] provides a more detailed and complete review of REE models used in microeconomics. However, REE models are criticized for two paradoxes that they imply [19]. First, how can market traders take into account of the equilibrium price in making decisions when it is those decisions that determine the price? REE generally requires the simultaneous determination of equilibrium price and available information. It does not consider how information flows into the market. Second, there is no incentive for individuals to gather costly private information since it is going to be reflected in the price. Dubey et al. [19] proposed market games to overcome the difficulties of REE models.

Dating back to 1976, Aumann [5] presents the formal definition of common knowledge and studies how two people with asymmetric information can agree with each other. Aumann proves that if two people have the same priors, and their posteriors for some event are common knowledge, then these posteriors must be equal. However, it is very rare that two people can have common knowledge about their posteriors at the very beginning. Geanakoplos and Polemarchakis [31] extend Aumann's work by demonstrating that if two people with common priors successively announce their posteriors to each other, eventually this leads to a situation of common knowledge where their posteriors are equal. McKelvey and Page [51] generalize the previous results to n persons and only require successively announcing an aggregate statistic of individuals' posteriors. When this statistic eventually becomes common knowledge, all posteriors of n persons are equal. Nielsen et al. [55] contribute by extending the conditional probability (posterior) to the case of conditional expectation. The above mentioned papers study how people disagree with each other can eventually reach an agreement. This process is analogous to the process of information aggregation. Market traders with different information disagree with the expected value of the security at the beginning. By trading in the market, they gradually reach an agreement, which is represented by the market price. Thus, results from research on common knowledge provide useful tools to analyze information dynamics of information markets.

Feigenbaum et al. [23] appears to be the only work that explicitly deals with information markets rather than traditional markets. Using an innovative computational approach, they view the information aggregation process as a distributed computation.

Private information held by market traders are treated as inputs to a function. Equilibrium market price is the value or output of the function in ideal situations. Thus, an information market is modeled as attempting to correctly compute the value of the function. They prove that when the function takes a certain form, weighted threshold, the equilibrium market price is guaranteed to equal to the value of the function. The number of rounds for the market to converge to this equilibrium equals the number of traders in the market.

2.2.2 Experimental Studies

Early evidences from stock markets, futures markets, and options markets indicate that markets can aggregate less-than-perfect information. However, market structure and market traders can impact the preciseness and effectiveness of this aggregation. Laboratory experiments, by systematically controlling some of the market parameters, provide simplified environments for understanding information efficiency of markets.

Plott and Sunder [63] set up experiments to examine the issues of information aggregation when different traders have diverse information about an underlying state of nature. The information structure did not have aggregate uncertainty, which means that although no trader knows the state of nature, if traders pool their information together the state can be identified with certainty. Their results demonstrated that market structures are important for information aggregation. Only with an appropriate market structure, can a market aggregate diverse information. Lundholm [49] examined the effect of aggregate uncertainty and found that markets aggregate information less efficiently when there is greater aggregate uncertainty. Forsythe and Lundholm [24] studied the effect of trader's preferences on information aggregation. They found that if participants had heterogeneous preferences, experience of participants was a necessary condition for information aggregation. O'Brien and Srivastava [57] focused on the relationship between asset structure and information aggregation ability of the market. Their results showed that information aggregation ability decreased when asset structure of the market is sufficiently complex. Guarnaschelli, Kwasnica and Plott [37] demonstrate that, in markets with a continuum of possible states (i.e. the asset examined pays a real number dividend) and a large number of traders, the market price can be said to be converging to the aggregated information, but that there is significant variance in the observed market prices. Sunder [71] extensively summarized experimental work on information aggregation. He indicated that the difficulties of the state of research are to understanding what factors facilitate or prevent information aggregation.

2.2.3 Evidence from Online Information Markets

Outside the laboratory, there are many real world online information markets, providing test grounds for experimental and theoretical claims of information markets.

The Iowa Electronic Markets (IEM) [50] are real-money online futures markets, in which security payoffs depend on economic and political events such as elections. Presidential election markets of IEM are most extensively examined. Participants trade securities whose payoffs depend on outcomes of the presidential election. Analysis of trading data find that prices in these markets predict the election outcomes better than polls [8, 9, 26, 27].

While economic incentives of real-money markets might account for their accurate predictions, Servan-Schreiber et. al [67] find that money does not affect prediction accuracy of information markets in their empirical comparison of football game markets at TradeSports [72] and NewsFutures [54]. TradeSports, a betting exchange registered in Ireland, hosts real-money markets for sports, politics, entertainment, and financial events. NewsFutures's World News Exchange has very popular sports and financial play-money betting markets.

Some other online game markets include Hollywood Stock Exchange (HSX) [21], Foresight Exchange (FX) [20], Formula One Pick Six (F1P6) [69], and MIT's Innovation Futures [29]. HSX trades securities based on future box office proceeds of new movies. FX allows traders to bet on unresolved scientific questions or other claims of public interest. In F1P6, participants can predict Formula One International race car competition results. MIT's Innovation Futures [29] predicts important business and technology trends. Security prices in many of these markets are found to give as accurate or more accurate predictions than expert opinions [60, 61].

2.2.4 Information Market Design

Few work has discussed how to design an information market that will work. Wolfers and Zitzewitz [75] in their review paper lay out several security choices of information markets. Spann and Skiera [70] propose three steps for designing an information market: choice of forecasting goal, incentives for participation and information revelation, and financial market design. The choice of forecasting goal step is mainly to design the appropriate security according to the nature of the prediction problem as we discussed above. With a well designed security, an information market further needs to attract people that have relevant information to participate and reveal their information. Thus, in the second step, incentives for participation and information revelation, one needs to make decisions on how to reward well performing participants. Basically, there are two popular ways to create participation and information revelation incentives. The first is to ask participants to invest their own money. They gain or loss money through trading activities. This provides economic incentive as in traditional financial markets for informed individuals to participate and reveal their information. The second way is use virtual money in the market but reward participants with monetary or non-monetary prizes. Finally, in the financial market design step, trading mechanism and related issues must be specified. Currently, the most widely used trading mechanism in information markets is the double auction institution.

Chapter 3

Theoretical Properties of Information Markets

This chapter starts with some motivations for the theoretical examination of information markets. Then we describe our model in Section 3.2. Section 3.3 presents our results on convergence properties of information markets based on the model. Justifications of the model and comparison with a related work are discussed in Section 3.4. Section 3.5 is the conclusion of the chapter.

3.1 Overview

In order to understand the strengths and limitations of information markets, we take the theoretical approach to establish a model of information markets, based on which analysis of properties of information markets is then conducted. The focus of this chapter is to capture the information dynamics of markets. In other words, we emphasize the process of how dispersed information is incorporated into the market price.

There is a rich resource of prior work that examined the existence and stability of Rational Expectation Equilibrium (REE), as is shown in Chapter 2. But, the dynamic process of how markets achieve the REE and how information flows into markets has been less satisfactorily studied, because REE generally requires the simultaneous determination of equilibrium price and available information. As the main functionality of information markets is information aggregation, this dynamic process is probably one of the most crucial perspectives to be understood. Thus, this chapter attempts to examine whether information markets converge to the fully revealing REE. The specific questions that motivate our endeavor are:

1. Will an information market converge to some kind of consensus equilibrium?

   Before we can expect the equilibrium market price to aggregate dispersed information, the first question to ask is whether an information market can achieve equilibrium.

2. If yes, how fast is the convergence process?

   If the answer to the first question is ”yes”, it is natural to inquiry how long it takes an information market to get to the equilibrium without the arrival of new information.

3. What is the best possible equilibrium?

   Not every equilibrium that a market can achieve aggregates the same amount of information. Thus, we want to know what is the best equilibrium that an information market can possibly achieve.

4. Will an information market always converge to the best possible equilibrium?

   It is interesting to know whether an information market is guaranteed to converge to the best possible equilibrium.

Answers to these questions are crucial to understanding the accuracy and efficiency of information markets as a mechanism for information aggregation. Useful implications for designing information markets can be derived from results of the theoretical study.

3.2 Information Market Modeling

A generic model of information markets should include at least three indispensable components, information structure, market mechanism, and trader behavior, as shown in Figure 3.1. Our model generalizes Feigenbaum et al.'s model [23] to capture aggregate uncertainty. The market mechanism and assumptions on trader behavior in our model are basically the same as those of [23]. But the information structure of our model is different. In this section, we only lay out the three components of our model, and leave the justification of the model to Section 3.4.

3.2.1 Information Structure

Information structure of the market specifies what the state space of the world is, how much information traders know about the real state of the world, and how information of traders relates to the real state of the world. For example, in the presidential election markets at IEM, state space of the world might be who is going to be nominated by each party, nominees' stances on important policy issues, population demographics, current strength of economy, previous voting records, and etc. Traders might have some

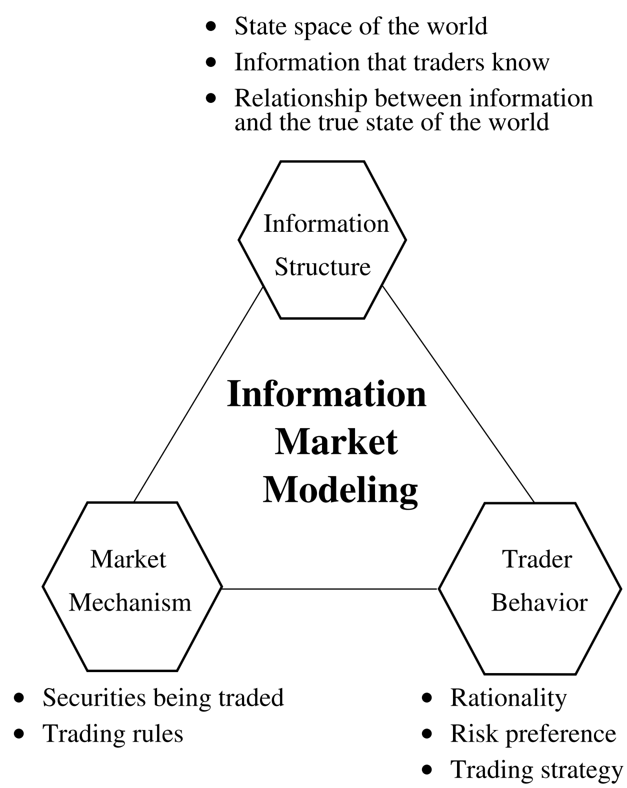


Fig. 3.1. Main Components of Information Market Modeling

information about the state of the world, such as their own regional demographics and local economy.

Usually information structure of markets is modeled using prior probability distributions of the state of the world and of the information that traders possess. Let S represents the state space of the world, where $s = (s_1, s_2, ..., s_m) \in S$ is a state vector of m dimensions. Assume there are n traders in the market, where all traders have a common prior probability distribution regarding to state of the world, $\mathcal{P}(s)$: $S \rightarrow [0, 1]$.

The trader's information space is X. Each trader $i = 1, ..., n$ gets a piece of information x_i about the state of the world, where $x = (x_1, x_2, ..., x_n) \in X$ is the information vector for all agents. Traders have common knowledge of the probability distribution of x, conditional on the state of the world s, $\mathcal{Q}(x|s)$: $X \times S \rightarrow [0, 1]$. For example, suppose we have a one dimensional state space (i.e., $s = s_1$). Conditional on $s = 1$ ($s = 0$), the probability to get $x_i = 1$ ($x_i = 0$) is 0.9, and the probability to get $x_i = 0$ ($x_i = 1$) is 0.1. If the trader i gets $x_i = 1$, although he does not know the value of s for certain, he knows that, with probability 0.9, s equals 1. This uncertainty in individual information introduces aggregate uncertainty to our model. The true state of the world is uncertain even with pooled information.

We make a further simplifying restriction: The state variables, s_i's, and the information variables, x_i's, can only take Boolean values 0 or 1. Thus, the state space of the world in our model is $S = \{0, 1\}^m$, $X = \{0, 1\}^n$ is the information space, the prior probability distribution of the state of the world is $\mathcal{P}(s)$: $\{0,1\}^m \rightarrow [0,1]$, and $\mathcal{Q}(x|s)$: $\{0,1\}^n \times \{0,1\}^m \rightarrow \{0,1\}$ is the conditional distribution of information.

In models of information markets without aggregate uncertainty, market traders hold accurate information about the state of the world s. For example, Feigenbaum et al.'s model [23] specifies that trader i knows s_i in an information market with n traders. Thus, pooling information of all traders together makes the state of the world, s, uniquely determined. Their model can be viewed as a special case of our model by setting $m = n$ and $x_i = s_i$ with probability 1.

3.2.2 Market Mechanism

Market mechanisms specify what securities are being traded and trading rules of the market. We model our market as predicting the value of a function $f(s)$. The value of the function is determined by the true state of the world, which will only be revealed some time in the future. One security is traded in the market, whose payoff is contingent on the value of $f(s)$. Specifically, the security pays off $\$f(s)$ in the future. The form of f is common knowledge to all traders. In our model, we restrict the value of function $f(s)$ to be Boolean. Thus, $f(s) : \{0,1\}^m \rightarrow \{0,1\}$.

To explain why we model the payoff of securities as related to a function, we go through the abstract process of setting up an information market. Suppose we have an event of interest to predict, we can turn it into a random variable, create a security whose payoff equals the realized value of the random variable, and bring a group of participants together via an Internet marketplace to let them trade shares of the security[1]. Typically, the random variable is a function of the underlying state of the world. Using the presidential election winner-takes-all market as an example, we are interested in predicting the event whether Democratic Party will win the presidential election. Turning the event into a random variable a, we have

$$a = \begin{cases} 1 & \text{if Democratic Party wins the presidential election;} \\ 0 & \text{if Republican Party wins the presidential election.} \end{cases} \tag{3.1}$$

The random variable a is determined by more fundamental variables such as who is going to be nominated by each party, what their stances on important policy issues are, and population demographics. These fundamental variables are characterized as state vector s in our model. Thus, a can be viewed as a function of s, (e.g. $a = f(s)$). Payoff of a share of the security for Democratic Party will be \$ a, which is the value of the function of $f(s)$.

Following Dubey et al. [19] and Feigenbaum et al. [23], we model the market mechanism as a *Shapley-Shubik market game* [68] with restrictions. The market game proceeds in rounds. In each round, each trader puts up quantities of the security to be sold and simultaneously puts up a positive amount of money to buy the security. For simplicity, we require the traders to offer selling one share of the security in each round, and assume that there are no restrictions on credit. Then, traders' bids can be represented as a vector

[1]In practice, transferring an event into a random variable and choosing appropriate market trading rules can be complicated, which deserves seperate discussion .

$b = (b_1, b_2, ..., b_n)$, where b_i is the amount of money trader i offers to buy securities. The market determines the price of the security by taking the average of all bids in a round, thereby clearing demand and supply. Thus, the price for a round is $p = \frac{\sum_{i=1}^{n} b_i}{n}$. Only this price p, not individual traders' bids, is publicly announced in each round. All trading occurs at the market price. At the end of the round, trader i holds the amount $a_i^{\prime} = \frac{b_i}{p}$ of the security. He or she profits p dollars through selling the security and loses b_i dollars from buying the security. Thus, net money gain (loss) of trader i is $(p - b_i)$ dollars. The market then enters a new round, where each agent has the same initial security holdings as previous rounds. The process continues until an equilibrium is reached, after which prices and bids do not change from round to round.

3.2.3 Trader Behavior

Modeling trader behavior can be achieved by specifying trader's risk preference, rationality, or trading strategy. In our model, we make the assumption that traders will always "tell the truth" rather than behave strategically. In other words, a trader will truthfully bid what he/she thinks the value of the security is in each stage of the market. This value is his/her expected payoff of a share of the security based on information available to him/her. The time value of money is ignored since information markets is usually alive only for a short period of time. Expectations are calculated based on probability distribution of the state of the world $\mathcal{P}(s)$, conditional probability distribution of information

$\mathcal{Q}(x|s)$, and information inferred from market prices. As market prices may contain extra information, traders revise their expectations as the market proceeds.

3.3 Convergence Properties of Information Markets

Based on the model of information markets in Section 3.2, we examine several convergence properties of information markets to answer the specific research questions we raised.

3.3.1 Price Convergence

As prices in information markets are predictors of future events, we desire that market prices are stable so long as no new information enters the markets. Thus, the first important question to ask is: Can an information market converge to an equilibrium, at which the price is stable if no new information arrives? With the aid of the results from McKelvey and Page [51] and Nielsen et al. [55], we present the answer to this question as Property 1.

Property 1. Without the new arrival of information, an information market converges to an equilibrium in finite steps. At equilibrium, all traders have the same expectation about the value of $f(s)$, which equals the equilibrium market price.

McKelvey and Page [51] and Nielsen et al. [55] studied how people who disagree with each other eventually reach an agreement. This process is analogous to the market trading process in our information market model. Roughly speaking, their results state that if the initial information partition of each trader is finite, and traders refine their information partition through an iterative process, in which a market statistic of traders' expectations of an event is made public in each period, then the market converges to an equilibrium in finite rounds. Further, if the market statistic satisfies some conditions, each trader's conditional expectation of the event must be identical at the equilibrium. We restate their results in our information market settings as Theorems 1 and Theorem 2, and apply them to obtain Property 1.

Let the initial information structure of an information market be as follows:

$$(\Omega, F, \rho) \quad \text{(a probability space)}, \tag{3.2}$$

$$P^0 = (P_1^0, ..., P_n^0) \quad \text{(initial information partitions)}, \tag{3.3}$$

$$h : \mathcal{R}^n \to \mathcal{R} \quad \text{(an aggregation function)} \tag{3.4}$$

For any individual i, P_i^0 is a finite partition of the probability space Ω. For any $\omega \in \Omega$, $P_i^0(\omega)$ denotes the element of P_i^0 that contains ω. The random variable that the market tries to predict is A. The market proceeds in rounds. Inductively, on round t, for each individual i and any state $\omega \in \Omega$, define

$$b_i^t(\omega) = E(A|P_i^t(\omega)) \tag{3.5}$$

to be individual i's expectation of the random variable A based on his current information partition.

$$b^t(\omega) = (b_1^t(\omega), ..., b_n^t(\omega)) \tag{3.6}$$

is the expectation vector for all agents.

Theorem 1. (McKelvey and Page [51] and Nielsen et al. [55]) Assume an initial information structure as in (3.2), (3.3), and(3.4). Assume the market proceeds in an iterative process such that:

(a) In every round t a market statistic $\Phi^t = h(b^t(\omega))$ is made public;

(b) Traders refine their information partitions according to the information brought by the market statistic;

(c) Traders revise their next round expectation b_i^{t+1}'s according to their new information partitions.

Then, for all $\omega \in \Omega$, there is a round T such that Φ^T is common knowledge at ω.

Theorem 2. (McKelvey and Page [51] and Nielsen et al. [55]) If the function h in (3.4) is stochastically regular, for any T, at which $\Phi^T = h(b^t(\omega))$ becomes common knowledge, and for all $\omega \in \Omega$, it must be the case that

$$b_1^T(\omega) = b_2^T(\omega) = ... = b_n^T(\omega) = \Phi^T. \tag{3.7}$$

Mckelvey and Page [51] define that a function $g : \mathcal{R}^n \rightarrow \mathcal{R}$ is stochastically regular, if it can be written in the form $g = l \circ g'$, where g' is stochastically monotone and l is invertible on the range of g'. According to Bergin and Brandenburger [10], a function $g : \mathcal{R}^n \rightarrow \mathcal{R}$ is stochastically monotone if it can be written in the form $g(\mathtt{x}) = \sum_{i=1}^{n} g_i(x_i)$, where each $g_i : \mathcal{R} \rightarrow \mathcal{R}$ is strictly increasing.

By mapping the settings of the theorems to our information market model, we find that all requirements of the theorems are met by our information market. First, the elements of the probability space (Ω, F, ρ) can be interpreted as: Ω includes both the state space S and the information space X, i.e. $\Omega = \{S, X\} = \{0,1\}^m \times \{0,1\}^n$; F is the measurable space of Ω; and ρ is the joint probability distribution of s and x, which can be derived from the prior distribution of s, $\mathcal{P}(s)$, and conditional distribution of x, $\mathcal{Q}(x|s)$. (3.2) is thus well-defined.

Second, the finite initial information partition requirement in (3.3) is met in our model, because the initial information partition for each trader i is simply a bi-partition of the sample space according to the trader's bit of information x_i, that is $P_i^0 = \{\{S, X|x_i = 0\}, \{S, X|x_i = 1\}\}$.

Third, in our model, the event to be predicted is the value of $f(s)$. In other words, it is the event that $f(s) = 1$. Since we assume that traders will truthfully bid their expectation of the function $f(s)$, for each trader i and for any state $\omega \in \Omega$, $E(f(s)|P_i^t(\omega))$ would be individual i's bid at period t. It is exactly $b_i^t(\omega)$ as defined in (3.5).

Forth, in our information market, market clearing price p^t is announced as the market statistic Φ^T in each round of trading. Thus, the aggregation function h in (3.4) is the function to calculate the market clearing price. It is the mean function of all traders' bids at round t, $p^t = \frac{\sum_{i=1}^{n} b_i^t(\omega)}{n}$. This mean function satisfies the stochastically regularity condition required by Theorem 2.

Hence, Theorem 1 and Theorem 2 are applicable to our information market. Applying Theorem 1, we conclude that at some round T, the market price p^T becomes common knowledge. Loosely speaking, common knowledge is the knowledge that can be inferred by every trader before it is observed from the market. It does not bring any new information to traders. Traders' information partitions can not be further refined. Thus, their bids won't change, and the market price will remain at the same level in later rounds. The market reaches its equilibrium at the round T. Theorem 2 tells us that, at equilibrium, all traders have the same expectation about the value of $f(s)$, which equals to the equilibrium market price.

3.3.2 Convergence Speed

Since an information market is guaranteed to converge to an equilibrium in finite steps, how fast does it converge? Property 2 answers this question.

Property 2. An information market converges to an equilibrium after at most n rounds of trading.

Derivation of Property 2 is based on the nature of common knowledge possibility sets. It uses similar technique as that of Feigenbaum et al. [23] in proving the convergence time bound for information markets without aggregate uncertainty. We describe our inference process for Property 2 below.

The knowledge of trader i at time t can be viewed as the set of states in the space Ω that trader i considers possible to be the true state at time t. We call this set *trader i's knowledge possibility set at time t*, and denote as S_i^t. Common knowledge of traders at time t can be described as the set of states in the space Ω that are considered possible to be the true state by an outside observer at time t who only observes market prices without possessing any private information. We use S^t to denote the *common knowledge possibility set at time t*.

Before the market starts, the common knowledge possibility set is simply the whole space Ω, i.e. $S^0 = \Omega$. Each trader i based on his knowledge possibility set S_i^0 to submit bid.

After observing the market clearing price of round 1, p^1, an outside observer can logically eliminate those states in Ω that are not possible to have resulted in p^1. Common knowledge possibility set after round 1 contains less or equal elements than the initial common knowledge possibility set, i.e. $|S^0| \geq |S^1|$. According to the common knowledge possibility set after round 1, each trader can eliminate impossible states from his own knowledge possibility set. Trader i's knowledge possibility set after round 1 is $S_i^1 = S_i^0 \bigcap S^1$. Traders can revise their expectation in the next round of trading based on their updated knowledge possibility sets. This process continues. We thus have a sequence of common knowledge possibility sets: S^0, S^1, S^2.... Since knowledge needs to be consistent,

$$|S^0| \geq |S^1| \geq |S^2| \geq ... \tag{3.8}$$

must be satisfied.

We can show that the inequality in (3.8) is strict before the market reaches its equilibrium. After the equilibrium, the common knowledge possibility sets remain the same. Suppose that for some round T, $|S^T| = |S^T + 1|$, it means that the market price after the $T + 1$ trading round does not provide any information to improve the common knowledge possibility set S^T. In later rounds, traders will behave the same as they were in trading round $T + 1$ because they gain no additional information from the market price in previous round. The market reaches its equilibrium, at which the market price becomes stable and common knowledge possibility sets in subsequent rounds equal to S^T. Thus, if the information market convergences to the equilibrium at round T, it must be the case that

$$|S^0| > |S^1| > ... > |S^T| = |S^{T+1}| = ... \tag{3.9}$$

The time for an information market to converge to its equilibrium equals the number of rounds that an observer of the market takes to improve the common knowledge possibility set from S^0 to S^T. The set S^0 is the whole sample space $\Omega = S \times X = \{0,1\}^m \times \{0,1\}^n$. Feigenbaum et al. has shown that for any round t, all elements that are possible to result in the price lie on a hyperplane in the sample space due to the linear price function of the Shapley-Shubik market game [23]. Thus, if S^{t-1} and S^t are not equal, S^t is the intersection of S^{t-1} with that hyperplane. Geometrically speaking, the dimension of S^t, i.e. the dimension of the smallest linear subspace of $\mathcal{R}^n$ that contains all the points in S^t, is at least one dimensional lower than that of S^{t-1} before the market equilibrium is reached. The dimension of S^0 is $m+n$. The dimension of S^T at equilibrium is at least m due to the aggregate uncertainty. Hence, the information market takes at most n rounds to converge to the equilibrium.

3.3.3 The Best Possible Prediction

Before we can evaluate the performance of an information market, we need a benchmark that defines what is the best possible prediction for information markets. This is given as Property 3. Knowing this will enable us to objectively analyze forecasting results of information markets and maybe suggest ways to improve the best possible forecast.

Property 3. The best possible prediction that an information market can make is the the forecast at direct communication equilibrium.

Property 3 is an intuitive result. Rather than only making a market statistic public, market traders can directly reveal their private information to each other. In this situation, an equilibrium can be reached immediately. This equilibrium is called *direct communication equilibrium* or *pooled information equilibrium* [31]. The equilibrium market price equals the expectation of the security payoff conditional on all available information, i.e. $E(f(s)|x)$. Since this prediction takes advantage of all information possessed by market traders, it is the best informed prediction in general. In other words, the best an information market can do is completely aggregate all private information that distributed among market traders. Direct communication equilibrium and fully revealing rational expectation equilibrium are the same in terms of information revelation and equilibrium price. We adopt the direct communication equilibrium in this chapter because it is more clear at how information is aggregated. Example 1 calculates the prediction at direct communication equilibrium for a simple two trader information market.

Example 1: Consider a simple information market, where there is only one state variable s_1 and two traders, $i = 1$ or 2. s_1 can take value 0 or 1, each with probability 0.5, which is common knowledge to both traders. The function that the market wants to predict value for is $f(s_1) = s$. Thus,

$$f(s_1) = \begin{cases} 1 & \text{when } s_1 = 1 \\ 0 & \text{when } s_1 = 0. \end{cases} \tag{3.10}$$

The security traded in the market pays off \$1 if $f(s_1) = 1$, and \$0 if $f(s_1) = 0$. The probability distributions of x_i conditional on s_1 for $i = 1$ and 2 are independent and identical as follow:

$$\begin{aligned} Pr(x_i = 0|s_1 = 0) = 0.8, \quad Pr(x_i = 1|s_1 = 0) = 0.2; \\ Pr(x_i = 0|s_1 = 1) = 0.2, \quad Pr(x_i = 1|s_1 = 1) = 0.8; \end{aligned} \tag{3.11}$$

Suppose the true state is $s_1 = 1$ and both traders' private information is 1, i.e. $x_1 = x_2 = 1$.

Using Bayes's rule, we can calculate the market price at the direct communication equilibrium:

$$\begin{aligned} E(f(s_1)|x_1 = 1, x_2 = 1) &= Pr(f(s_1) = 1|x_1 = 1, x_2 = 1) \\ &= Pr(s_1 = 1|x_1 = 1, x_2 = 1) \\ &= \frac{Pr(x_1 = 1, x_2 = 1|s_1 = 1)Pr(s_1 = 1)}{Pr(x_1 = 1, x_2 = 1)} \\ &= \frac{0.64 \times 0.5}{0.34} \\ &\approx 0.94. \end{aligned}$$

Thus, the best possible forecast of $f(s_1)$ is 0.94. It says that the true state of the world is very likely to be $s_1 = 1$, but there is uncertainty associated with the prediction.

The best possible prediction implies that the ability of information markets to make predictions is constrained by the amount of aggregate uncertainty. If the aggregate uncertainty is large, even if an information market aggregates all the information, the prediction result can still be poor. If we change the probability distribution of x_i conditional on s_1 for $i = 1$ and 2 in (3.3.3) of Example 1 to the followings:

$$\begin{aligned} &Pr(x_i = 0|s_1 = 0) = 0.2, \quad Pr(x_i = 1|s_1 = 0) = 0.8; \\ &Pr(x_i = 0|s_1 = 1) = 0.2, \quad Pr(x_i = 1|s_1 = 1) = 0.8; \end{aligned} \tag{3.12}$$

The expectation at the direct communication equilibrium would only be $E(f(s_1)|x_1 = 1, x_2 = 1) = 0.5$. It provides nothing better than simply knowing the prior distribution of s_1. This is an extreme case because of the independence of information x_i and the state s_1. Both trader 1 and trader 2 in this case don't have real information other than the prior probability distribution of s_1 regarding the future market situation. From this perspective, performance of information markets relies on the information quality of their participants. In other words, in order for an information market to make good predictions, there must be some knowledge in the market about the future event to be predicted.

3.3.4 Convergence to the Best Prediction Or Not

We have shown that an information market will converge to a consensus equilibrium, the convergence process takes at most n rounds of trading, and that the best possible prediction is the direct communication equilibrium. Our next question is: will an information market always converge to direct communication equilibrium? Unfortunately, the answer is “no”.

Property 4. An information market is not guaranteed to converge to direct communication equilibrium.

In the following, we provide two examples of information markets. Both markets trade the same security, but the probability distributions of traders' information are different. Example 2 does not converge to the direct communication equilibrium, while example 3 does.

Example 2: Consider an information market, where state of the world is $s = (s_1, s_2)$. There are two traders in the market, $i = 1, 2$. Value of s_j, $j = 1$ or 2, can be either 0 or 1. Suppose the common prior probability distribution of s is uniform, i.e. $s = (s_1, s_2)$ takes the values $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$ each with probability 0.25. The function that the market wants to predict is

$$f(s_1, s_2) = \begin{cases} 1 & \text{when } s_1 = s_2 \\ 0 & \text{otherwise.} \end{cases} \tag{3.13}$$

The security traded in the market pays off \$1 if $f(s_1, s_2) = 1$, and \$0 if $f(s_1, s_2) = 0$. The probability distributions of trader's information x_i conditional on s are independent and identical as follow:

$$\begin{aligned} &Pr(x_i = 0|s_1 = 0, s_2 = 0) = 0.9, \quad Pr(x_i = 1|s_1 = 0, s_2 = 0) = 0.1; \\ &Pr(x_i = 0|s_1 = 0, s_2 = 1) = 0.5, \quad Pr(x_i = 1|s_1 = 0, s_2 = 1) = 0.5; \\ &Pr(x_i = 0|s_1 = 1, s_2 = 0) = 0.5, \quad Pr(x_i = 1|s_1 = 1, s_2 = 0) = 0.5; \\ &Pr(x_i = 0|s_1 = 1, s_2 = 1) = 0.1, \quad Pr(x_i = 1|s_1 = 1, s_2 = 1) = 0.9. \end{aligned} \tag{3.14}$$

Suppose that the true state is $s = (1, 1)$, and both traders' private information is 1, i.e. $x_1 = x_2 = 1$.

According to Bayes' rule, trader i with information $x_i = 1$ would like to submit bid

$$\begin{aligned} b_i(1) &= E(f(s)|x_i = 1) \\ &= Pr(f(s) = 1|x_i = 1) \\ &= \frac{Pr(x_i = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_i = 1)} \\ &= \frac{0.5 \times 0.5}{0.5} \\ &= 0.5. \end{aligned}$$

Similarly, with information $x_i = 0$ trader i would like to submit bid

$$\begin{aligned} b_i(0) &= E(f(s)|x_i = 0) \\ &= Pr(f(s) = 1|x_i = 0) \\ &= \frac{Pr(x_i = 1|f(s) = 0)Pr(f(s) = 1)}{Pr(x_i = 0)} \\ &= \frac{0.5 \times 0.5}{0.5} \\ &= 0.5. \end{aligned}$$

Hence, no matter what value x_i is, trader i will always bid 0.5 in the first round of trading. When both traders bid $b_i = 0.5$, market clearing price is also 0.5. From the market clearing price, trader 1 can infer that trader 2 bid 0.5, but this gives him no information about trader 2's private information x_2. Trader 2 can do the same inference and also gains no additional information. Neither trader will change their bids in later rounds. Hence, the market reaches its equilibrium in the first round with equilibrium price equals 0.5. This is nothing better than simply using the $Pr(f(s) = 1)$ to make the prediction. However, pooling information directly can make better prediction. Under the direct communication equilibrium, market price should equals

$$
\begin{aligned}
E(f(s)|x_1 = 1, x_2 = 1) &= Pr(f(s) = 1|x_1 = 1, x_2 = 1) \\
&= \frac{Pr(x_1 = 1, x_2 = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_1 = 1, x_2 = 1)} \\
&= \frac{0.41 \times 0.5}{0.33} \\
&\approx 0.62.
\end{aligned}
$$

Thus, the direct communication equilibrium price indicates that, given both traders have information 1, the probability for the function $f(s)$ to be 1 is 0.62, which is a better prediction as opposed to 0.5.

Example 3: With all other conditions remain the same as in example 2, we change (3.14), the probability distribution of trader's information x_i conditional on s, to:

$$\begin{aligned}
&Pr(x_i = 0|s_1 = 0, s_2 = 0) = 0.9, \quad Pr(x_i = 1|s_1 = 0, s_2 = 0) = 0.1;\\
&Pr(x_i = 0|s_1 = 0, s_2 = 1) = 0.9, \quad Pr(x_i = 1|s_1 = 0, s_2 = 1) = 0.1;\\
&Pr(x_i = 0|s_1 = 1, s_2 = 0) = 0.5, \quad Pr(x_i = 1|s_1 = 1, s_2 = 0) = 0.5;\\
&Pr(x_i = 0|s_1 = 1, s_2 = 1) = 0.1, \quad Pr(x_i = 1|s_1 = 1, s_2 = 1) = 0.9.
\end{aligned} \tag{3.15}$$

We still suppose that the true state is $s = (1, 1)$ and both traders' private information is 1.

Under the condition of example 3, if trader i has information $x_i = 1$, his bid would be

$$\begin{aligned}
b_i(1) &= E(f(s)|x_i = 1)\\
&= Pr(f(s) = 1|x_i = 1)\\
&= \frac{Pr(x_i = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_i = 1)}\\
&= \frac{0.5 \times 0.5}{0.4}\\
&= 0.625.
\end{aligned}$$

If trader i has information $x_i = 0$, he would like to submit bid

$$\begin{aligned} b_i(0) &= E(f(s)|x_i = 0) \\ &= Pr(f(s) = 1|x_i = 0) \\ &= \frac{Pr(x_i = 1|f(s) = 0)Pr(f(s) = 1)}{Pr(x_i = 0)} \\ &= \frac{0.5 \times 0.5}{0.6} \\ &\approx 0.42. \end{aligned}$$

Thus, both traders will submit 0.625 as their bids in the first round of trading since they all have information 1. The market clearing price for round 1 would also be 0.625. Observing the clearing price, trader 1 can infer that trader 2 must have bid 0.625, which further means that trader 2's information is 1. Trader 1 thus gets to know both pieces of information. His bid in the second round will be

$$\begin{aligned} E(f(s)|x_1 = 1, x_2 = 1) &= Pr(f(s) = 1|x_1 = 1, x_2 = 1) \\ &= \frac{Pr(x_1 = 1, x_2 = 1|f(s) = 1)Pr(f(s) = 1)}{Pr(x_1 = 1, x_2 = 1)} \\ &= \frac{0.41 \times 0.5}{0.27} \\ &\approx 0.76. \end{aligned}$$

Similarly, trader 2 can infer from the market price that $x_1 = 1$, and bid $E(f(s)|x_1 = 1, x_2 = 1) \approx 0.76$ in the second round. Market price of round 2 will be 0.76, which incorporates private information of both traders. Thus, the information market reaches its equilibrium in the second round, at which it predicts that for probability 0.76 $f(s)$ will have value 1. This equilibrium is the same as direct communication equilibrium.

The reason that the information market in example 2 does not converge to the direct communication equilibrium seems to be the high degree of symmetry of traders bidding behavior. Even with different private information, a trader bids the same value. The market price is then unable to reveal trader's private information. Hence, if the prior probability distribution of the state of the world, $\mathcal{P}(s)$, and the conditional probability distribution of information, $\mathcal{Q}(x|s)$, of an information markets accidentally create this kind of symmetry, the information market might not be able to perform well in making predictions.

3.4 Discussions

3.4.1 Justification of the Model

We present a simple model of information markets with aggregate uncertainty in Section 3.2 without commenting on the reasonableness of the model. In this part, we will examine the validity and limitations of the model.

Information markets are molded as restricted Shapley-Shubik market games, in which traders only know how much money they are going to spend but don't know for sure how many shares of securities they can get. Trading rules of Shapeley-Shubik market games seem very different from those of commodity markets, where the price is fixed, and stock markets, where bids and offers include both price and quantities. But they are not too different. First, once an information market reaches its equilibrium, the equilibrium market price and information efficiency are usually not affected by trading rules. Second, in our analysis of convergence properties of information markets, the two key assumptions are that traders truthfully bid their expectations of the security value and that market price is a known stochastically regular function of traders' bids. For other market mechanisms, as long as they satisfy these assumptions, property 1, 3, and 4 hold. For example, these two assumptions are roughly satisfied with the *market scoring rule* mechanism for information markets, which was proposed by Hanson [39]. The limitation of using Shapley-Shubik market game to model information market is that property 2 of our analysis is not robust. It relies on the linear price function of the restricted Shapley-Shubik market games.

Traders are assumed to "tell the truth" rather than behave strategically in our model. This assumption seems reasonable when the number of traders in the market is large. When n is large, the effect of a single trader's bid on the market clearing price is relatively small or even ignorable. Thus, traders might not have enough incentive to deviate from their true expected values and bid strategically. In addition, solving optimal strategies of traders for a n-person game usually needs the assumption of symmetry among traders for computational reasons. Assuming that traders are symmetric in holding information, however, will make the information structure of the market too simple to be representative and interesting.

3.4.2 Comparison with Information Markets Without Aggregate Uncertainty

Our model captures aggregate uncertainty of information markets. In order to investigate the impact of aggregate uncertainty on information markets, we compare our model and results with those of Feigenbaum et al. [23], which does not consider aggregate uncertainty.

Table 3.4.2 briefly presents the comparison of both modeling and convergence properties of information markets with and without aggregate uncertainty. While market mechanism and trader behavior are the same for the two markets, information structures are different. For the market without aggregate uncertainty, trader i is informed of the value of s_i, which is part of the state vector s. But in the market with aggregate uncertainty, trader i only gets to know x_i, which relates to s_i with some uncertainty. Comparing the convergence properties of the two markets with different information structures, we can see that the first convergence property are the same regardless of aggregate uncertainty. An information market will always converge to a consensus equilibrium in finite rounds. At equilibrium, market traders' expectations of the security value are the same, which equal to the equilibrium market price. The second property is roughly equivalent for the two

Table 3.1. Comparisons of Information Markets With and Without Aggregate Uncertainty

Comparison Items		With Aggregate Uncertainty	Without Aggregate Uncertainty
Information Market Modeling	Market Structure	• *State of the world:* $s \in \{0,1\}^m$ with prior probability distribution $\mathcal{P}(s)$. • *Trader information:* n Traders; Trader i holds x_i; $x \in \{0,1\}^n$ with conditional probability distribution $\mathcal{Q}(x\|s)$.	• *State of the world:* $s \in \{0,1\}^n$ with prior probability distribution $\mathcal{P}(s)$. • *Trader information:* n traders; Trader i holds s_i.
	Market Mechanism	• The security pays off \$$f(s)$ in the future; • The market is a restricted Shapely-Shubik market game.	Same
	Trader Behavior	Bid expected payoff of a share of the security.	Same
Convergence Properties	Price Convergence	Converge to a consensus equilibrium in finite steps.	Same
	Convergence Speed	At most n rounds. n is the number of traders.	Same
	Best Possible Prediction	Direct communication equilibrium, where $p = E(f(s)\|x)$.	Direct communication equilibrium, where $p = f(s)$.
	Convergence to the Best Possible Prediction	Not guaranteed.	Guaranteed, if f is a weighted threshold function.

markets. The number of rounds for an information market to converge to a consensus equilibrium equals the number of traders in the market. The third property says that the direct communication equilibrium is the best possible prediction for both information markets. But, for information markets without aggregate uncertainty, pooling information together fully determines the true state of the world and hence market price at direct communication equilibrium computes the value of $f(s)$, while for information markets with aggregate uncertainty, price at direct communication equilibrium is the expectation of $f(s)$ conditional on pooled information. The most important difference between information markets with aggregate uncertainty and those without is the last property. Feigenbaum et al. proved that if $f(s)$ is a *weighted threshold function*, an information market without aggregate uncertainty is guaranteed to converge to direct communication equilibrium for any prior probability distribution of s [23]. The function f is a weighted threshold function if and only if there are real constants w_1, w_2, ... w_m such that

$$f(s) = 1 \ iif \ \sum_{i=1}^{m} w_i s_i \geq 1. \tag{3.16}$$

This neat guaranteed-convergence result is no longer valid when aggregate uncertainty is introduced into information markets. As shown in Example 2, the function f is a weighted threshold function, but the market does not converge to direct communication equilibrium. When there is aggregate uncertainty, the prior probability distribution of s and the conditional probability distribution of information have influence on whether an information market convergences to direct communication equilibrium, whatever the form of the function is.

3.5 Summary

This chapter provides a theoretical analysis on the information aggregation ability of information markets. By characterizing the uncertainty of market participants' private information, we incorporate aggregate uncertainty in our information market model. Based on the model, we examine some fundamental convergence properties of information markets, which answers the four research questions that we raised. Specifically, we have shown that (1) an information market is guaranteed to converge to an equilibrium, at which traders have consensus about the forecast; (2) it converges to the equilibrium in at most n rounds of trading, where n is the number of traders ; (3) the best possible prediction it can make is the direct communication equilibrium, at which price equals the expectation of the function value conditional on information of all traders; (4) but an information market is not guaranteed to converge to this best possible prediction.

Comparing these results with those of information markets without aggregate uncertainty, we find that differences brought by aggregate uncertainty lay in the third and fourth results. Although the best possible equilibrium is always the direct communication equilibrium, it accurately computes the value of the function f if there is no aggregate uncertainty, while it can only reveal the expected value of f conditional on all information when aggregate uncertainty exists. Without aggregate uncertainty, an information market is guaranteed to converge to the direct communication equilibrium if f is a weighted threshold function. But with aggregate uncertainty, this is no longer true. Since aggregate uncertainty is so common in the real world, the differences imply that in order for an information market to make good predictions, there must be some knowledge in the market. But knowledge is not sufficient for good predictions. How to design an information market that will converge to direct communication equilibrium is an important research question to be explored.

This chapter is an initial attempt to understanding the power of information markets. Several issues deserve further investigation:

- *The effect of aggregate uncertainty on information markets:* Our results have shown that aggregate uncertainty can negatively affect the power of information markets to various extends. It is important to measure or quantify the effect so that measures can be suggested to manage or reduce the effect.

- *Conditions of guaranteed convergence:* We have shown that with aggregate uncertainty, information markets may not converge to the best possible predictions. Finding those conditions under which such convergences are guaranteed to happen will greatly deepens our understanding of the information aggregation ability of markets.

- *Robustness of information markets:* Before information markets can be used to facilitating decision making, their robustness needs to be examined. For example, the prediction performance of information markets when there are manipulation incentives is of special importance.

Chapter 4

Experimental Evaluation of Information Markets

4.1 Overview

Supported by a long history of theory and evidence [36, 40, 41, 63], it has been well-recognized that markets can aggregate less-than-perfect information across market participants. However, information aggregation is usually mingled with other functions of markets such as capitalization and risk hedging, which makes it hard to further explore its properties and effectiveness. For example, in an efficient stock market, the price of a stock in theory reflects the underlying value of the company based on aggregated information. But since the underlying value of a company can hardly be measured objectively, it is difficult to assess how well the stock market aggregates dispersed information. The advent of the Internet, broadband, and other related information and communication technologies has given rise to a novel web-based information system, *information markets*, which separates the information aggregation ability of markets from other functions of markets.

Information markets, also known as prediction markets or decision markets, are web-based markets that are designed for the purpose of information aggregation and prediction. To achieve these goals, information markets associate payoffs of securities with outcomes of well-defined future events, and provide online marketplaces to trade the securities. For example, in an information market to predict whether the Democratic Party will win most votes in the next presidential election, the security pays a certain amount of money (e.g. $1) per share to its shareholders if and only if the Democratic Party gets most votes in the election. Otherwise, it pays nothing. Thus, before the election, the security price may reflect the expectations of market participants about how likely Democratic Party is to win the election. As the outcome of the presidential election will be revealed in the future, the information aggregation ability of such market can be evaluated and studied.

Information markets have proved to provide relatively accurate predictions. The Iowa Electronic Markets (IEM) [50] are real-money futures markets designed to predict economic and political events. IEM has correctly predicted the outcome of every U.S. presidential election since its inception in 1988. It also predicted the vote shares gained by presidential candidates more accurately than polls [8, 9, 26, 27]. On the eve of the 2004 U.S. presidential election, IEM predicted that George W. Bush would garner 50.45% of the popular vote, which only differed from the actual result, 51.54%, by 1.09%. The Hollywood Stock Exchange (HSX) [21] is a play-money market that has made predictions on openning-weekend box office proceeds of movies better than experts [61]. Hewlett-Packard uses an internal information markets to predict its printer sales; the market prediction beat its official managerial forecasts 15 out of 16 times [13].

Although the list of successful stories of information markets is much longer than those mentioned above, there are certainly cases that markets failed to make accurate predictions. As information aggregation becomes the primary function of information markets, it is important to understand what factors or conditions facilitate or prevent information aggregation.

Past studies in this direction are represented by a series of experimental works that utilized controlled human-subjects experiments. Plott and Sunder [62, 63] found that when participants had homogeneous preferences, which means that the value of the security given the state of the nature is the same for all participants, information was successfully aggregated in double auctions. Forsythe and Lundholm [24] further showed that if participants had heterogeneous preferences, experience of participants is a necessary condition for information aggregation. Kagel and Levin [45] experimented with sealed bid auctions where information was not aggregated in their settings. Guarnaschelli, Kwasnica, and Plott [37] used the same information structure as that of Kagel and Levin [45], but paired it with double auctions, and obtained somewhat more successful information aggregation. These studies partly revealed how individual preferences, structure of information, experience of participants, and market mechanism influenced the information aggregation ability of markets.

The research reported below is motivated by two objectives. First, we attempt to examine the effect of another factor, the design of securities, on information aggregation using laboratory experiments. Since information markets can have as novel securities as one can imagine, it is essential to know whether different designs of securities may affect the market's ability to aggregate information. We pay special attention to the model proposed by Feigenbaum et al. [23]. It is so far the only theoretical model that considers the relationship between information aggregation and security design. According to it, when the security traded in an information market satisfies certain conditions, information aggregation is guaranteed regardless of the structure of information. Otherwise, there exists some information structures such that information aggregation can not be achieved. Sensing that the model inevitably makes simplified assumptions, such as non-strategic behavior of market participants, we take an empirical approach to investigate to what extent the prominent predictions of the model can be observed in our experimental markets. Second, we intend to evaluate how well existing models capture the experimental market behavior and identify areas for improvements. In addition to Feigenbaum et al.'s model, two other well-known theoretical models, the rational expectation equilibrium model and the private information model, are examined against our experimental data.

Our main results are that all three models for information aggregation are substantially rejected. The effect of security design on information aggregation is observed, but is much less strong than what is projected by Feigenbaum et al.'s model. Evidence suguests that individual behavior does not conform to the assumptions made by any of the three models, which indicates room for improvements. We briefly discuss potential changes to the model that may produce a more realistic model.

The chapter is organized as follows. Section 4.2 contains the detailed design of the experimental markets created for this study. Section 4.3 outlines the three competing theoretical models and their projections for our experimental markets. Experimental results and analyses are reported in Section 4.4. Section 4.5 is a summary of our conclusions.

4.2 Design of Markets

Our market design closely follows the model of Feiganbaum et al. [23], which will be introduced in Section 4.3.3. We design two securities such that markets with one security should be able to aggregate information according to the theoretical model of Feiganbaum et al., while markets with the other security should not.

The markets were conducted as a series of trading periods. All periods in one experimental session were identical except for the information that participants received. There was one security traded in each market. The security paid a dividend of either 150 experimental dollars or 50 experimental dollars at the end of a period, contingent on which of the two states of nature, "Good" or "Bad", was realized. Each market consist of five participants. Each participant received a clue or signal about the nature of the state at the beginning of a period. The clue was either a "G" or a "B", and was provided by the computer through a random draw. It was equally likely for a participant to get a "G" clue or a "B" clue. Each participant knew his/her own private clue and the fact that other participants had independently drawn clues.

Two different securities that we examined were a majority security and a parity security. If a market traded a majority security, the state of nature was "Good" if and only if the majority of participants (i.e., three or more in a market with five subjects) got a "G" clue. Otherwise, the state of nature was "Bad." When a parity security was traded, the state of nature was "Good" if and only if there were odd number of "G" clues in the market (i.e., one, three, or five "G" clues). For both securities, the dividend was 150 experimental dollars when the state was "Good", and 50 experimental dollars when the state was "Bad".

Trading periods were designed as modified Sharpley-Shubik market games [68]. Each period consisted of at least 2 and up to 10 trading rounds each lasting a maximum of 2 minutes. [1] At the beginning of each period, each participant was given 1 unit of the security and 50 units of cash in experimental dollars. At each round, each participant was asked to enter a bid into the computer indicating the price at which the participant wanted to buy or sell a unit of the security. The bid was required to be between 50 and 150. A round ended when all traders had submitted bids or 2 minutes had elapsed. The market price for the round was then calculated by taking the average of all bids in a round. At the market price, total demand equaled total supply and the market cleared. All transactions happened at the market price. The net quantity traded for a participant was given by:

$$\text{Quantity Traded} = (\text{Bid of the participant - Market Price})/\text{Market Price}.$$

If the bid of the participant was higher than the market price, the participant would buy securities. If the bid of the participant was lower than the market price, the participant

[1] In a Sharpley-Shubik market game, a trading period ends when the market price or traders' bids do not change from round to round. Feigenbaum et al.'s model predicts that a trading period will end in the second or third round. Since conducting experiments that have infinite number of rounds was not practical, we set the maximum number of rounds in a trading period to be 10. Since 10 rounds is far greater than the predicted number of rounds for any theory, it was our feeling that it was more than sufficient to capture any convergence activity.

would sell securities. In general, the further a bid of a participant was from the market price, the more the participant bought or sold. Negative security or cash holdings were allowed. The trading period then proceeded to the next round where each participant could submit a new bid. After a minimum of 2 rounds, the trading period ended if any one of the following happened: (1) no traders submitted a bid in the last round, (2) everyone's bids are the same, (3) the market price did not change for two consecutive rounds, or (4) ten rounds were completed. At the end of a trading period, the true state of nature and true dividend were announced. Profit of a participant was the sum of his/her cash inventory and dividend from security holdings. [2]

Eleven sessions were conducted in the Laboratory for Economic Management and Auctions at The Pennsylvania State University. Five of the sessions used the majority security. The other six sessions used the parity security. The experiments are implemented using the z-Tree (Zurich Toolbox for Readymade Economic Experiments) software [77]. Each session had five Penn State undergraduates or graduates as subjects. All subjects did not have previous experience with this particular experimental setting, but some of them were familiar with other market institutions. All subjects were given the opportunity to familiarize themselves with the experimental procedure by participating in a practice trading period. Each experimental session consisted of eight trading periods and lasted for

[2] In a Shapley-Shubik market game, each participant is asked to put up a bid b_i and a quantity q_i in each round, where b_i is the amount of money that the participant want to spend on buying securities and q_i is the unit of security that the participant want to sell. Market price clears demand and supply of securities. Our modified Shapely-Shubik game is equivalent to always setting q_i as 1. In other words, participants are required to put 1 unit of the security for sale in each round.

about an hour and a half. A trading period was essentially a separate instance of market, with the clues of participants reseted.

All experiments were conducted using experimental dollars. At the end of each session, the profit of each participant was converted to U.S. dollars at the conversion rate of 80 experimental dollars per 1 U.S. dollar. This amount plus a $7 show-up fee were paid in private to the participant. A copy of the instructions is provided in Appendix.

4.3 Theoretical Models

Two models that represent two extremes are the fully revealing rational expectation equilibrium (RE) model and private information (PI) model. The fully revealing RE model assumes that market participants behave as if they know all available information in the market, while PI model makes the assumption that market participants only use their private information in trading. Both models are static in the sense that they only conjecture the equilibrium state of the market. As static models are usually subject to the criticism of not explaining how equilibrium is reached, Feigenbaum et al. [23] propose a dynamic process and establishes the connection between the two extremes. In the Feigenbaum et al.'s model, market participants at the beginning of the market use their private information in forming their expectation of the dividend, which is projected by the PI model. But market participants also learn from prices of previous rounds, and revise their expectation in later rounds. Under certain conditions, the market will eventually reach the RE equilibrium. We introduce the three models and examine the implications of these models for the outcomes in our experimental setting.

4.3.1 Rational expectations

In a market that participants have the same or symmetric information, a competitive equilibrium is a price and net trade, at which a market clears. In other words, a competitive equilibrium is reached when demand equals supply. The market price at a competitive equilibrium reflects the preferences and budget constrains of market participants. Rational expectation equilibrium [48] extends the concept of competitive equilibrium to situations where participants may have asymmetric information. With asymmetric information, future utilities of items traded are uncertain and participants may have different information about future state. Market activities may reflect the information of participants in addition to their preferences and budget constrains. Thus, market prices potentially provide informational feedback for market participants.

Suppose there are n traders in the market. Let s_i be trader i's private information. $s = (s_1, s_2, ..., s_n)$ is the information vector of all traders. Let u_i be the utility of trader i. Then, a rational expectation equilibrium is the price, p^*, and the demand, $y_i^*(p^*)$, satisfying, for each i,

$$\begin{array}{lll} y_i^*(p^*) & \text{maximizes} & E(u_i(y_i)|s_i, p^*) \\ \text{s.t.} & \sum_i y_i^*(p^*) = 0. & \end{array}$$

In a fully revealing rational expectations equilibrium, equilibrium price, p^*, reveals all available information s. Hence, all participants in equilibrium behave as if they know the pooled information of all participants in the market. Allen [1] and Jordan [42] have shown that fully revealing rational expectations equilibrium generically exist.

In our experimental setting, the fully revealing RE model implies that market participants know the total number of "G" clues in the market. Therefore, they know for certain what the dividend will be at the end of the period. The bid of each participant and hence the market price according to this model would be equal to the dividend of the security. Table 4.1 lists the equilibrium price predictions of the RE model when markets have different numbers of "G" clues. RE model is an extreme since it assumes that individual behavior of market participants is based on all information in the market.

Table 4.1. Equilibrium Price Predictions – RE Model

Security	Number of "G" Clues in a Trading Period					
	0	1	2	3	4	5
Majority	50	50	50	150	150	150
Parity	50	150	50	150	50	150

4.3.2 Private information

The private information model, also called the prior information model, is propsed by Plott and Sunder [63], while a similar model is used by Lintner [47] in analyzing US securities market. As opposed to the RE model, the PI model assumes that market participants do not condition expectations upon market price. In stead, market participants apply Bayes rule to update the likelihood of the future outcome upon receiving their private prior information, and act according to the derived probability.

According to the PI model, if a risk neutral trader gets clue "G" in a market with a majority security in our experiments, his/her expectation of the security payoff v is

$$E(v|G) = 150 \times P(v = 150|G) + 50 \times P(v = 50|G) = 118.75.$$

If the trader gets a clue "B", his/her expectation of the security payoff is

$$E(v|B) = 150 \times P(v = 150|B) + 50 \times P(v = 50|B) = 81.25.$$

Assume that market participants truthfully bid their expectations. If the market price clears the market, it will be the average of expected security payoffs of all participants.

With a parity security, a participant's expectation of the security payoff is always 100 in our experiments regardless of his/her private information, because the probability that the dividend is 150 is always 0.5, i.e.

$$P(v = 150|G) = P(\text{There are 0, 2, or 4 "G" clues in the rest of the market}) = 0.5, \text{and}$$

$$P(v = 150|B) = P(\text{There are 1, or 3 "G" clues in the rest of the market}) = 0.5.$$

Being the average of all bids, the market price for a parity security should always be 100 according to the PI model.

Table 4.2 shows the price predictions of PI model if market participants bid their expectations.

Table 4.2. Equilibrium Price Predictions – PI Model

Security	Number of "G" Clues in a Trading Period					
	0	1	2	3	4	5
Majority	81.25	88.75	96.25	103.75	111.25	118.75
Parity	100	100	100	100	100	100

4.3.3 Dynamic market model

Both the RE and PI models are static models that provide an equilibrium prediction, Feigenbaum et al.'s mode is a dyanmic model that connects the two static models.

The dynamic market model of Feigenbaum et al. [23] takes a computational approach. It treats a market as a computational device. In a market with n traders, each trader holds a piece of private information, whose value is either 0 or 1. Let $x \in \{0,1\}^n$ denote the information vector $(x_1, x_2, ...x_n)$. Then, x is the input to the market, and equilibrium price p is the output. The key component of the device is the security F whose payoff is a Boolean function of the private information, $f(x) : \{0,1\}^n \rightarrow \{0,1\}$. The market mechanism is modified Sharpley-Shubik games as described in Section 4.2.

Feigenbaum et al. studied under what conditions the market can aggregate the information of market traders and correctly compute the value of the function $f(x)$. With the assumption that in each round market participants truthfully bid their expectations conditional on their private information and market price of previous rounds, the main result of Feigenbaum et al. is that

> If the function f is a weighted threshold function, then the market will take at most n rounds of trading to reach an equilibrium, where the equilibrium price of F is equal to $f(x)$. If f is not a weighted threshold function, then there exists a prior probability distribution of x for which the price of the security F does not converge to the value of $f(x)$.

A function $f : \{0,1\}^n \rightarrow \{0,1\}$ is a weighted threshold function if and only if there are real constants $w_1, w_2, ..., w_n$ such that $f(x) = 1$ iff $\sum_{i=1}^{n} w_i x_i \geq 1$.

The majority security in our experiments is a linear transformation of a weighted threshold function, and the parity security is not. The implications of Feigenbaum et al.'s model in our experimental setting are that the price of the majority security will converge to its true dividend in the second round of trading, while the price of the parity security will not converge to its true dividend.

With a majority security, market participants bid their expectations of the security payoff only conditional on their private information in the first round of trading. As shown in Table 4.2, the correspondence between the total number of "G" clues and the market price based on private information is a one-to-one mapping. Hence, from the price of the first round, market participants can infer how many "G" clues there are in the market, and incorporate the information in forming their expectations in the second round. The market then reaches the fully revealing rational expectation equilibrium in the second trading round. With a parity security and the prior probability distribution of clues (equally likely to get a "G" clue or a "B" clue) in our experiments, market participants will not be able to get any extra information from the market price as shown in Table 4.2. They can only use their private information to bid in each round. Thus, the market does not aggregate information.

Table 4.3 shows the price predictions according to Feigenbaum et al.'s model. An equilibrium is reached within two rounds. Hence, prices in later rounds stay the same.

Table 4.3. Price Predictions – Feigenbaum et al.'s Model

Security	Round	Number of "G" Clues in a Trading Period					
		0	1	2	3	4	5
Majority	1	81.25	88.75	96.25	103.75	111.25	118.75
	2 and all other	50	50	50	150	150	150
Parity	1	100	100	100	100	100	100
	2 and all other	100	100	100	100	100	100

4.4 Results

Table 4.4 summarizes the actual number of "G" clues that subjects received in each trading period of our experiments. Aiming at examining to what extent the theoretical models are valid, we first compare aggregated behavior of our experimental markets with the conjectures of the models, and then study individual behavior of market participants to explain discrepancies.

4.4.1 Aggregated behavior

For each majority session, we plot the actual market prices of the last period (period 8) against the predicted prices of three models in Figure 4.1. Using the last period prices reduces errors caused by inexperience of traders, and was done by Plot and Sunder [63]. The first round price of Feigenbaum et al.'s model is the same as that of PI model, while

Table 4.4. Total Number of "G" Clues in Experiments

Security	Session	Trading Period							
		1	2	3	4	5	6	7	8
Majority	1	0	2	2	3	2	3	3	3
	2	3	3	3	1	3	3	3	4
	3	2	3	4	2	3	2	2	3
	4	2	3	1	2	3	2	3	2
	5	3	3	2	2	2	2	2	1
Parity	6	1	2	3	3	2	1	2	3
	7	4	0	2	2	4	1	3	3
	8	4	3	2	1	3	3	4	3
	9	3	3	1	4	5	2	4	2
	10	1	3	3	3	2	2	4	2
	11	3	3	1	3	3	3	2	2

prices in later rounds according the Feigenbaum et al's model are the same as those of RE model. Upon casual inspection, the actual market prices were clearly different from predictions of all three models most of the time, although in some cases (e.g. sessions 1 and 4) we can see clear trends of prices slowly moving toward the RE prices.

Result 1: With a majority security, actual market price is significantly different from the price predictions of all models.

Support:

If a model can accurately predict the market prices, the absolute difference between actual market prices and model predicted prices should roughly equal to zero. Let p_{ijk} be the market price for trading round k in trading period j of the experimental session i and m^l_{ijk} be the price predicted by model l for the same trading round, the average absolute difference between market prices and model l predicted prices for trading period j of the experimental session i is calculated as:

$$f^l_{ij} = \frac{\sum_k |p_{ijk} - m^l_{ijk}|}{\text{total number of rounds in period} j}. \tag{4.1}$$

For each majority experimental session i and each theoretical model l, we test the null hypothesis that the mean of f^l_{ij} (j across 8 trading periods) is equal to zero using a one-sample t test. Our test statistics indicate that, for all five majoriy sessions and all three

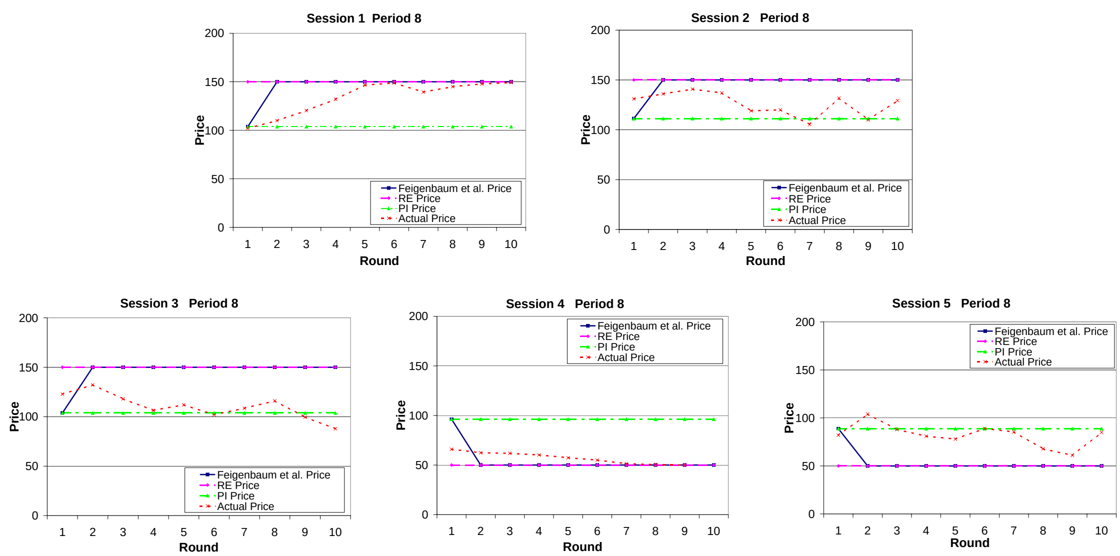


Fig. 4.1. Price Comparison for Markets with Majority Security (Period 8)

theoretical models, the mean of f_{ij}^{l} is statistically different from zero at the significance level of 0.05.

Figure 4.2 shows the actual market prices of the last period (period 8) of each parity session against the predicted prices of the three models. For markets with the parity security, the predictions of PI model and Feigenbaum et al.'s model are the same. We do not observe any obvious trends of market prices in Figure 4.2. Similar to result 1, result 2 shows that all models can be rejected in a statistical sense for the parity security.

Result 2: With a parity security, actual market price is significantly different from the price predictions of all models.

Support:

For each parity experimental session i and each theoretical model l, we test the null hypothesis that the mean of f_{ij}^{l} (across 8 trading periods) is equal to zero using a one-sample t test. f_{ij}^{l} is calculated according to equation 4.4.1. For all six parity sessions and all three models, the mean of f_{ij}^{l} is statistically different from zero at the significance level of 0.05.

The first two results indicate that none of the three models can be claimed as accurate in an absolute sense. This is typical in experimental studies, since simplified assumptions of theories are usually not completely conformed in laboratories. Plott and Sunnder [63] and Guarnaschelli, Kwasnica, and Plott [37] all found that market behavior

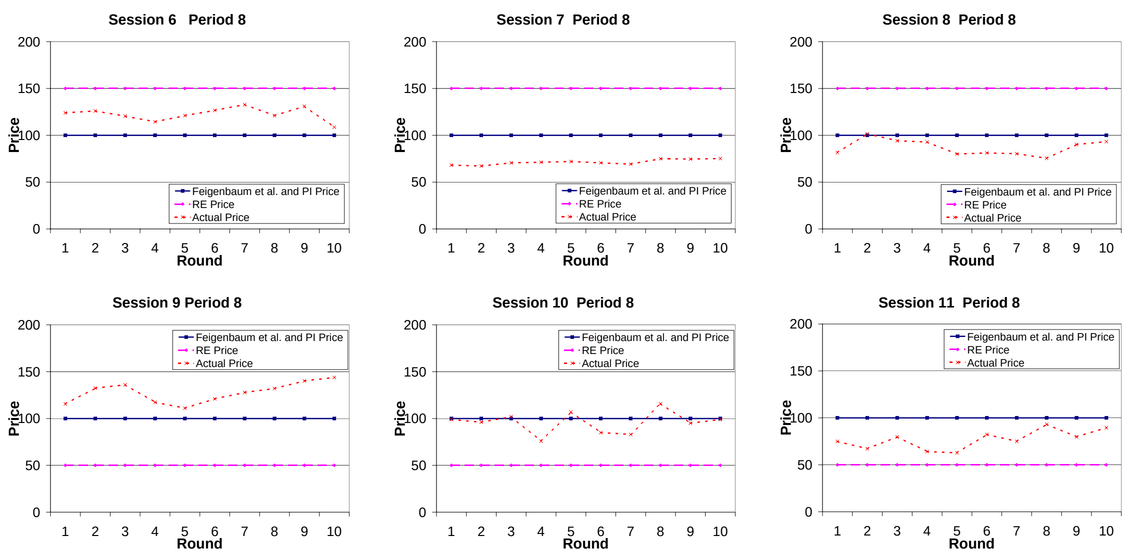


Fig. 4.2. Price Comparison for Markets with Parity Security

relative to predictions of theoretical models, including RE and PI, differed substantially. Hence, our subsequent results focus on to what extend the models are supported, and their relative accuracy. Results 3 and 4 show that although markets with majority security do not converge to the true dividend as fast and accurately as the Feigenbaum et al.'s model predicted, they do have less error and move toward the dividend more often than markets with parity security, which suggest that information aggregation is taking place in markets with majority security, although not to the degree of fully revealing.

Result 3: On average, markets with a majority security have less price errors than markets with a parity security.

Support: As before p_{ijk} be the market price for trading round k in trading period j of the experimental session i. Let d_{ij} be the dividend of trading period j of experimental session i. Price error for trading round k in trading period j of experimental session i is defined as the absolute difference between p_{ijk} and d_{ij}, i.e.

$$e_{ijk} = |p_{ijk} - d_{ij}|.$$

We calculate the price errors of every round and period in an experimental session i, and use the average,

$$e_i = \frac{\sum_j \sum_k e_{ijk}}{\text{total number of rounds in session } i},$$

to represent the price error for the experimental session i. Table 4.5 lists the average price error e_i for all experimental sessions i from 1 to 11. Applying the non-parametric Wilcoxon-Mann-Whitney test to the data in Table 4.5, we test the null hypothesis that the median of the average price errors for the majority sessions is equal to that for the parity sessions, with the alternative hypothesis being that the median for the majority sessions is less than that for the parity sessions. The resulted test statistic rejects the null hypothesis at the significance level of 0.05 (W = 19, p-value=0.0276). This implies that market prices deviate from true dividends less in majority sessions than in parity sessions.

Table 4.5. Average Absolute Difference of Price from Dividend

Security	Session	Average Price Error (e_i)
Majority	1	39.96
	2	46.38
	3	43.39
	4	28.24
	5	47.90
Parity	6	46.03
	7	44.36
	8	55.02
	9	52.61
	10	54.98
	11	56.18

Result 4: Prices in markets with the majority security converge toward the true dividend to a larger degree than those with the parity security.

Support:

The price error of a particular trading round, the absolute difference of market price from dividend, reflects how well information is aggregated at that time. If information is fully aggregated, price error should be 0. If no individual information is aggregated and the market price only incorporates the common prior of the outcome, the price error should be around 50 (which means that the price is around 100). Loosely speaking, the closer the price error is to 0, the better the information aggregation is. Thus, we examine both the first round and the last round price errors for all majority and parity trading periods to check whether the markets move toward the direction of reducing price errors.

Dividing the interval between 0 and 100 into four subsets, [0, 25], (25, 50], (50,75], and (75, 100], we count the total number of trading periods whose price errors fall into the same subset using first round and last round market prices respectively. The frequencies of price errors for all subsets are shown in Table 4.6 with majority markets and parity markets listed separately.

From Table 4.6, we can see that first round price errors for most majority and parity trading periods fall into the subsets (25, 50] and (50, 75]. This implies that both majority and parity markets start with a similar level of price errors. For the last round, however, 42.5% of majority trading periods have a last round price error that is less than or equal

Table 4.6. Frequency of Absolute Difference of Price from Dividend

Round	Period	Price Errors			
		[0,25]	(25,50]	(50, 75]	(75, 100]
First	Majority (out of 40)	6 (15%)	21 (52.5%)	9 (22.5%)	4 (10%)
	Parity (out of 48)	4 (8.33%)	17 (35.42%)	18 (37.5%)	9 (18.75%)
Last	Majority (out of 40)	17 (42.5%)	9 (22.5%)	11 (27.5%)	3 (7.5%)
	Parity (out of 48)	5 (10.42%)	19 (39.58%)	21 (43.75%)	3 (6.25%)

to 25, while only 10.42% of parity trading periods fall into the same range. Although the first round price errors for both majority and parity trading periods are similar, there are more majority trading periods that have less last round price errors.

We apply Chi-Square tests to examine whether the observed differences of frequencies are statistically significant. Table 4.7 displays the results of our four Chi-Square tests. Each table cell contains the value of the test statistic and p-value of a Chi-Square test for two rows of Table 4.6, indicated by the row and column names. The null hypothesis is that the frequencies of price errors for the two rows are the same. For example, the first table cell tests whether the observed differences of first round price errors between majority periods and parity periods as shown in the first two rows of Table 4.6 are significant. From the test results and frequencies of price errors in Table 4.6, we can conclude in a statistical sense that (1) majority trading periods and parity trading periods have the same level of price errors at the beginning of markets; (2) majority trading periods move toward the direction of reducing price errors, while parity trading periods do not; and (3) majority

Table 4.7. Results of Chi-Square Tests on Frequencies of Price Errors

	Majority (First Round)	Parity (Last Round)
Parity (First Round)	5.059 (0.168)	3.453 (0.327)
Majority (Last Round)	10.404 (0.015)	12.619 (0.006)

* The first number of each table cell is the value of the Chi-Square statistic. Numbers in parenthesis are the p-values of the tests.

trading periods have less price errors than parity trading periods at the end of the markets. These indicate that although prices of most majority trading periods do not converge to the true dividend as predicted my the Feigenbaum et al.'s model, they do move toward that direction to some degree, but prices of parity trading periods do not.

4.4.2 Individual behavior

The theoretical results of Feigenbaum et al.'s model, and the other models to some extent, rely on two critical assumptions about individual behavior. First, it is assumed that market participants are rational in the sense that they can correctly calculate conditional expectations and make inferences based on market prices. All participants know that all participants are rational, and know that all participants know this, and so forth ad infinitum. Second, the model assumes that participants are not strategic; they truthfully bid their conditional expectations in each rounds. Thus, in a market with the majority security, a participant bids 118.75 in the first round if his/her private clue is “G”, and bids 81.25 if his/her private clue is “B”. From the first round market price, rational participants can infer how many “G” clues and “B” clues are there in the markets, and get to know the true dividend. In the second round, participants will then bid the dividend. Hence the market price converges to the true dividend in the second round. On the other hand, in a market with the parity security, participants’ conditional expectations of the security value are always 100 regardless of their private clues. The market price does not reveal private information. Hence, the market does not converge to the true dividend.

However, the results of theoretical models, especially Feigenbaum et al.’s model, are not robust when these assumptions are violated. Even if there is only one participant who is not fully rational or does not bid truthfully, the market may fail to converge. The following two results are centered on observed individual behavior in our experimental markets. They show that these assumptions are substantially violated.

Result 5: For both majority and parity markets, most market participants do not bid the conditional expectations of the security value in first rounds of trading.

Support:

In a market with the majority security, a market participant’s conditional expectation of the security value is 81.25 if he/she receives a “B” clue, and 118.75 is he/she has a “G” clue. In a market with the parity security, a market participant’s conditional expectation of the security value is 100 regardless of the clue that the participant gets.

We examine whether market participants actually bid their conditional expectations of the security value in first rounds. For majority markets, we put some buffer around the expected security value – a participant is considered to bid his/her conditional expectation if his/her bid falls into the interval [77.5, 85] with a "B" clue, or the interval [115, 122.5] with a "G" clue.

In our experiments, there are 200 first round bids for majority markets, and 240 for parity markets. Only 13 out of 200 (6.5%) first round bids in majority markets are consistent with the buffered conditional expectation of the security value. 37 out of 240 (15.42%) first round bids in parity markets have the value 100. Most first rounds bids in both majority and parity markets deviate from the market participants' conditional expectations of the security value. These conclude that individual behavior assumptions of Feigenbaum et al.'s model are substantially violated, which might account for why the model is not fully supported by our experimental data.

Result 6: For majority markets, most market participants do not infer the direction of security value and bid accordingly in first rounds.

Support:

In majority markets, an individual may not correctly (or do not want to) calculate the conditional expectation of the security value, but he/she may know that the expected security value is lower than 100 if his/her first round clue is "B", and higher than 100 if his/her first round clue is “G”, and bid toward the expected direction. In order to capture such possible bidding behavior of market participants, we use the following exclusive categories to classify first round bids.

1. Bid 100 in the first round.

   In majority markets, biding 100 means that a participant ignores his/her private clue and only relies on the common prior. In parity markets, bidding 100 is the behavior based on the assumptions of Feigenbaum et al.’s model, but it can also be a result of ignoring private clues.

2. Bid lower than 100 with clue “B”.

   For majority markets, this and the next category imply some degree of rationality of participants. Although they are not fully rational as assumed, participants use their information to infer the expected direction of dividend.

3. Bid higher than 100 with clue “G”.

4. Bid lower than 100 with clue “G”.

   We consider both this category and the next one as against model assumptions on individual behavior.

5. Bid higher than 100 with clue “B”.

Table 4.8 presents the number and percentage of first round bids for each category. We notice that 47.5% of first round bids in majority markets get the direction right (categories 2 and 3). However, in parity markets, there are 42.08% of first round bids, which is approximately an equal proportion as in majority markets, fall into categories 2 and 3. Hence, there is no clear evidence to support that participants of majority markets use their information to infer the direction of dividend and bid accordingly in first rounds of trading.

Table 4.8. First Round Bidding Behavior of Market Participants

Behavior Category	Majority		Parity	
	Number	Percentage	Number	Percentage
1	46	23%	37	15.42%
2	60	30%	77	32.08%
3	35	17.5%	24	10%
4	38	19%	72	30%
5	21	10.5%	30	12.5%

Category 1: Bid 100 with "B" or "G"; *Category 2*: Bid lower than 100 with "B";
Category 3: Bid higher than 100 with "G"; *Category 4*: Bid lower than 100 with "G";
Category 5: Bid higher than 100 with "B";

Result 7: Market participants in general tend to bid lower than expected security value in first rounds of trading.

Support: From Table 4.8, we observe that there are more lower-than-100 bids than higher-than-100 bids (49% vs. 28% in majority markets and 62.08% vs. 22.5% in parity markets) in first rounds of trading. To take into accout of different clues that participants receive, we plot the empirical culmulative distributions of first round bids for different clues and different securities respectively in Figure 4.3.

The empirical cumulative probability distribution of participants with clue "G" in majority markets is the lowest curve in Figure 4.3, which implies that this group of participants in general have less low bids than other participants. However, even for this group of participants, more than 60% of the first round bids are lower than 100, the expected security value given only the prior information. For parity markets, where observing a clue "B" or a clue "G" does not help refining the prior probability of the state of nature, there are about 75% to 80% percent of first round bids that are lower than 100. These indicate that market participants are systematically bidding low in first rounds of trading.

Bidding lower than expectations could be a kind of strategic behavior of market participants. In a Shapley-Shubik market game, an individual's bid can affect market price. Hence, if an individual intents to buy the security, he/she has the incentive to bid lower to some degree so that the market price decreases and he/she can buy the security at a lower price. On the other hand, if an individual wants to sell, he/she also has the incentive to bid lower, since it will enable them to sell more units of the security. Another possible explanation of bidding low is the risk aversion of market participants. Assuming

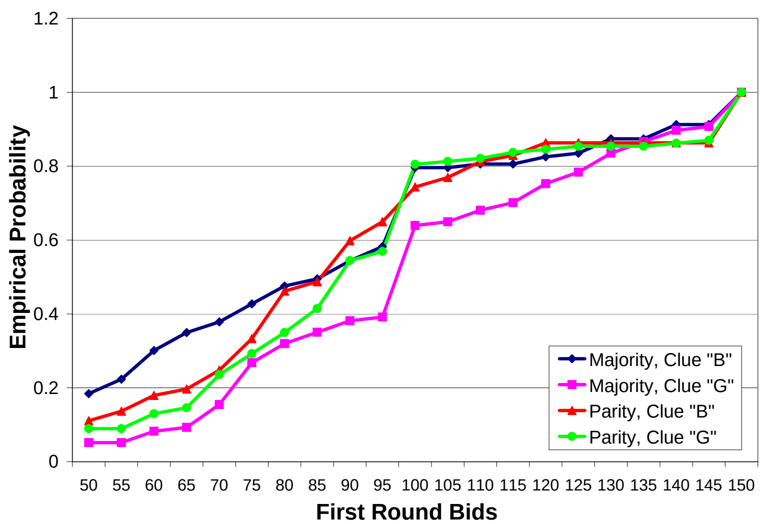


Fig. 4.3. Empirical Cumulative Probability Distribution of First Round Bids

others bid the same, bidding lower can result in less buying or more selling for a participant. In either case, it reduces the security holding of the participant, and hence is less risky.

Result 8: For both majority and parity markets, participants adjust their bids according to observed market prices.

Support:

Results 5 to 7 reveal that in our experimental markets, participants most often do not bid their conditional expectations of security value. This deviation makes market prices less informative and inferring the total number of "G" clues from the market price becomes very hard if not impossible. Based on this understanding, we start to examine whether market participants still appears to learn from prices.

Let b_t be the bid of an individual in round t, and p_t be the market price of round t, we calculate the correlation coefficients between bid adjustments, $b_t - b_{t-1}$, and the observed price-bid differences, $p_{t-1} - b_{t-1}$, for every participant of our experiments. The correlation coefficients are all positive. 45 out of 55 participants have a correlation coefficient that is greater than 0.5. The positive correlation means that market participants tend to adjust next round bids toward the direction of the observed previous round market prices. In other words, if a participant's bid is lower (higher) than the market price in the previous round, the participant will increase (decrease) his/her bid in the next round. Market participants still believe that market price contains some information of other participants and follow it. It is worth of noting that, price in a parity market may not be informative according to Feigenbaum et al.'s model, so following the price probably do not benefit market participants.

4.5 Summary

This work is an attempt to study the effect of security design on the information aggregation ability of markets in a controlled experimental market environment, in which the behavior of individuals are not well understood. Existing theoretical models on information aggregation inevitably make simplified assumptions on individual behavior. Examining them in a more realistic environment not only helps us validate theories but also facilitates developing better theories. We consider three existing theoretical models, Feigenbaum et al.'s model, RE model, and PI model, with the emphasis on Feigenbaum et al.'s model.

The basic results of our study are as follow:

1. None of the three models is supported in an absolute sense. Market prices in our experimental markets deviate from price predictions of all three models significantly.

2. Security design affects the information aggregation ability of markets. Some degree of information aggregation is observed in markets with the majority security, but not in markets with the parity security. However, the effect is much less strong than what is predicted by Feigenbaum et al.'s model.

3. Assumptions on individual behavior of theoretical models are substantially violated. Although demonstrating some degree of learning from prices, market participants do not behave as assumed.

This study implies that existing theoretical models, which are based on simplified assumptions of individual behavior, can not explain many market behavior when individual behavior is not as assumed. Theoretical models that can better capture realistic individual behavior or are robust at violations of assumptions are needed. Moreover, it is worthy of thinking about designing better mechanisms for information aggregation. With a majority security, fully aggregating information is possible, but only a small degree of information aggregation is achieved in our experimental markets.

Chapter 5

Empirical Analysis of Information Markets

5.1 Overview

Despite the popularity of information markets, one of the most important questions to ask is: how accurately can information markets predict? Previous research in general shows that information markets are remarkably accurate. The political election markets at IEM predict the election outcomes better than polls [24, 25, 26, 27]. Prices in HSX and FX have been found to give as accurate or more accurate predictions than judgment of individual experts [60, 61, 70]. However, information markets have not been calibrated against opinion pools, except for Servan-Schreiber et al. [67], in which the authors compare two information markets against arithmetic average of expert opinions. Since information markets, in nature, offer an adaptive and self-organized mechanism to aggregate opinions of market participants, it is interesting to compare them with existing opinion pooling methods, to evaluate the performance of information markets from another perspective. The comparison will provide beneficial guidance for practitioners to choose the most appropriate method for their needs.

This chapter contributes to the literature in two ways: (1) As an initial attempt to compare information markets with opinion pools of multiple experts, it leads to a better understanding of information markets and their promise as an alternative institution for obtaining accurate forecasts; (2) In screening opinion pools to be used in the comparison, we cast insights into relative performances of different opinion pools. In terms of prediction accuracy, we compare two information markets with several linear and logarithmic opinion pools (LinOP and LogOP) at predicting the results of NFL games. Our results show that at the same time point ahead of the game, information markets provide as accurate predictions as our carefully selected opinion pools. In selecting the opinion pools to be used in our comparison, we find that arithmetic average is a robust and efficient pooling function; weighting expert assessments according to their past performances does not improve the prediction accuracy of opinion pools; and LogOP offers bolder predictions than LinOP. The remainder of the chapter is organized as follows. Section 5.2 reviews popular opinion pooling methods. Section 5.3 introduces the basics of information markets. Data sets and our analysis methods are described in Section 5.4. We present results and analysis in Section 5.5, followed by conclusions in Section 5.6.

5.2 Review of Opinion Pools

Clemen and Winkler [17] classify opinion pooling methods into two broad categories: mathematical approaches and behavioral approaches. In mathematical approaches, the opinions of individual experts are expressed as subjective probability distributions over outcomes of an uncertain event. They are combined through various mathematical methods to form an aggregated probability distribution. Genest and Zidek [35] and French [28] provide comprehensive reviews of mathematical approaches. Mathematical approaches can be further distinguished into axiomatic approaches and Bayesian approaches. Axiomatic approaches apply prespecified functions that map expert opinions, expressed as a set of individual probability distributions, to a single aggregated probability distribution. These pooling functions are justified using axioms or certain desirable properties. Two of the most common pooling functions are the *linear opinion pool* (LinOP) and the *logarithmic opinion pool* (LogOP). Using LinOP, the aggregate probability distribution is a weighted arithmetic mean of individual probability distributions:

$$p(\theta) = \sum_{i=1}^{n} w_i p_i(\theta), \tag{5.1}$$

where $p_i(\theta)$ is expert i's probability distribution of uncertain event θ, $p(\theta)$ represents the aggregate probability distribution, w_i's are weights for experts, which are usually non-negative and sum to 1, and n is the number of experts. Using LogOP, the aggregate probability distribution is a weighted geometric mean of individual probability distributions:

$$p(\theta) = k \prod_{i=1}^{n} p_i(\theta)^{w_i}, \tag{5.2}$$

where k is a normalization constant to ensure that the pooled opinion is a probability distribution. Other axiomatic pooling methods often are extensions of LinOP [33], LogOP [34], or both [18]. Winkler [73] and Morris [52, 53] establish the early framework of Bayesian aggregation methods. Bayesian approaches assume as if there is a decision maker who has a prior probability distribution over event θ and a likelihood function over expert opinions given the event. This decision maker takes expert opinions as evidence and updates its priors over the event and opinions according to Bayes rule. The resulted posterior probability distribution of θ is the pooled opinion.

Behavioral approaches have been widely studied in the field of group decision making and organizational behavior. The important assumption of behavioral approaches is that, through exchanging opinions or information, experts can eventually reach an equilibrium where further interaction won't change their opinions. One of the best known behavioral approaches is the Delphi technique [46]. Typically, this method and its variants do not allow open discussion, but each expert has chance to judge opinions of other experts, and is given feedback. Experts then can reassess their opinions and repeat the process until a consensus or a smaller spread of opinions is achieved. Some other behavioral methods, such as the Nominal Group technique [7], promote open discussions in controlled environments.

Each approach has its pros and cons. Axiomatic approaches are easy to use. But they don't have a normative basis to choose weights. In addition, several impossibility results (e.g., Genest [32]) show that no aggregation function can satisfy all desired properties of an opinion pool, unless the pooled opinion degenerates to a single individual opinion, which effectively implies a dictator. Bayesian approaches are nicely based on the normative Bayesian framework. However, it is sometimes frustratingly difficult to apply because it requires either (1) constructing an obscenely complex joint prior over the event and opinions (often impractical even in terms of storage / space complexity, not to mention from an elicitation standpoint) or (2) making strong assumptions about the prior, like conditional independence of experts. Behavior approaches allow experts to dynamically improve their information and revise their opinions during interactions, but many of them are not fixed or completely specified, and can't guarantee convergence or repeatability.

5.3 How Information Markets Work

The idea of using information markets for prediction stems from Hayek hypothesis [40] and efficient market hypothesis [22]. Hayek, in his classic critique of central planning in 1940's, claims that the price system in a competitive market is a very efficient mechanism to aggregate dispersed information among market participants. The efficient market hypothesis further states that, in an efficient market, the price of a security almost instantly incorporates all available information. The market price summarizes all relevant information across traders, hence is the market participants' consensus expectation about the future value of the security. Empirical evidence supports both hypotheses to a large extent [36, 41, 63]. Thus, when associating the value of a security with the outcome of an uncertain future event, market price, by revealing the consensus expectation of the security value, can indirectly predict the outcome of the event.

For example, if we want to predict which team will win the NFL game between New England and Carolina, an information market can trade a security "$100 if New England defeats Carolina", whose payoff per share at the end of the game is specified as follow:

$$\begin{cases} \$100 & \text{if New England wins the game;} \\ \$0 & \text{otherwise.} \end{cases}$$

The security price should roughly equal the expected payoff of the security in an efficient market. The time value of money usually can be ignored because durations of most information markets are short. Assuming exposure to risk is roughly equal for both outcomes, or that there are sufficient effectively risk-neutral speculators in the market, the price should not be biased by the risk attitudes of various players in the market. Thus,

$$p = \Pr(\textit{Patriots win}) \times 100 + [1 - \Pr(\textit{Patriots win})] \times 0,$$

where p is the price of the security "\$100 if New England defeats Carolina" and $\Pr(\textit{Patriots win})$ is the probability that New England will win the game. Observing the security price p before the game, we can derive $\Pr(\textit{Patriots win})$, which is the market participants' collective prediction about how likely it is that New England will win the game.

The above security is a winner-takes-all contract. It is used when the event to be predicted is a discrete random variable with disjoint outcomes (in this case binary). Its price predicts the probability that a specific outcome will be realized. When the outcome of a prediction problem can be any value in a continuous interval, we can design a security that pays its holder proportional to the realized value. This kind of security is what

Wolfers and Zitzewitz [75] called an index contract. It predicts the expected value of a future outcome. Many other aspects of a future event such as median value of outcome can also be predicted in information markets by designing and trading different securities. Wolfers and Zitzewitz [75] provide a summary of the main types of securities traded in information markets and what statistical properties they can predict. In practice, conceiving a security for a prediction problem is only one of the many decisions in designing an effective information market. Spann and Skiera [70] propose an initial framework for designing information markets.

5.4 Design of Analysis

5.4.1 Data Sets

Our data sets cover 210 NFL games held between September 28th, 2003 and December 28th, 2003. NFL games are very suitable for our purposes because: (1) two online exchanges and one online prediction contest already exist that provide data on both information markets and the opinions of self-identified experts for the same set of games; (2) the popularity of NFL games in the United States provides natural incentives for people to participate in information markets and/or the contest, which increases liquidity of information markets and improves the quality and number of opinions in the contest; (3) intense media coverage and analysis of the profiles and strengths of teams and individual players provide the public with much information so that participants of information markets and the contest can be viewed as knowledgeable regarding to the forecasting goal.

Information market data was acquired, by using a specially designed crawler program, from TradeSports.com's Football-NFL markets [72] and NewsFutures.com's Sports Exchange [54]. For each NFL game, both TradeSports and NewsFutures have a winner-takes-all information market to predict the game outcome. We introduce the design of the two markets according to Spann and Skiera's three steps for designing an information market [70] as below.

- **Choice of forecasting goal:** Markets at both TradeSports and NewsFutures aim at predicting which one of the two teams will win a NFL football game. They trade similar winner-takes-all securities that pay off 100 if a team wins the game and 0 if it loses the game. Small differences exist in how they deal with ties. In the case of a tie, TradeSports will unwind all trades that occurred and refund all exchange fees, but the security is worth 50 in NewsFutures. Since the probability of a tie is usually very low (much less the 1%), prices at both markets effectively represent the market participants' consensus assessment of the probability that the team will win.

- **Incentive for participation and information revelation:** TradeSports and NewsFutures use different incentives for participation and information revelation. TradeSports is a real-money exchange. A trader needs to open and fund an account with a minimum of $100 to participate in the market. Both profits and losses can occur as a result of trading activity. On the contrary, a trader can register at NewsFutures for free and get 2000 units of Sport Exchange virtual money at the time of registration. Traders at NewsFutures will not incur any real financial loss. They can accumulate virtual money by trading securities. The virtual money can then be used to bid for a few real prizes at NewsFutures' online shop.

- **Financial market design:** Both markets at TradeSports and NewsFutures use the continuous double auction as their trading mechanism. TradeSports charges a small fee on each security transaction and expiry, while NewsFutures does not.

We can see that the main difference between two information markets is real money vs. virtual money. Servan-Schreiber et. al [67] have compared the effect of money on the performance of the two information markets and concluded that the prediction accuracy of the two markets are at about the same level. Not intending to compare these two markets, we still use both markets in our analysis to ensure that our findings are not accidental.

We obtain the opinions of 1966 self-identified experts for NFL games from the ProbabilityFootball online contest [64], one of several ProbabilitySports contests [65]. The contest is free to enter. Participants of the contest are asked to enter their subjective probability that a team will win a game by noon on the day of the game. Importantly, the contest evaluates the participants' performance via the quadratic scoring rule:

$$s = 100 - 400 \times Prob\_Lose^2, \tag{5.3}$$

where s represents the score that a participant earns for the game, and $Prob\_Lose$ is the probability that the participant assigns to the actual losing team. The quadratic score is one of a family of so-called *proper* scoring rules that have the property that an expert's expected score is maximized when the expert reports probabilities truthfully. For example, for a game team A vs. team B, if a player assigns 0.5 to both team A and B, his/her score for the game is 0 no matter which team wins. If he/she assigns 0.8 to team A and 0.2 to team B, showing that he is confident in team A's winning, he/she will score 84 points for the game if team A wins, and lose 156 points if team B wins. This quadratic scoring rule rewards bold predictions that are right, but penalizes bold predictions that turn out to be wrong. The top players, measured by accumulated scores over all games, win the prizes of the contest. The suggested strategy at the contest website is "to make picks for each game that match, as closely as possible, the probabilities that each team will win". This strategy is correct if the participant seeks to maximize expected score. However, as prizes are awarded only to the top few winners, participants' goals are to maximize the probability of winning, not maximize expected score, resulting in a slightly different and more risk-seeking optimization.[1] Still, as far as we are aware, this data offer the closest thing available to true subjective probability judgments from so many people over so many public events that have corresponding information markets.

[1] Ideally, prizes would be awarded by lottery in proportion to accumulated score.

5.4.2 Methods of Analysis

In order to compare the prediction accuracy of information markets and that of opinion pools, we proceed to derive predictions from market data of TradeSports and NewsFutures, form pooled opinions using expert data from ProbabilityFootball contest, and specify the performance measures to be used.

5.4.2.1 Deriving Predictions

For information markets, deriving predictions is straightforward. We can take the security price and divide it by 100 to get the market's prediction of the probability that a team will win. To match the time when participants at the ProbabilityFootball contest are required to report their probability assessments, we derive predictions using the last trade price before noon on the day of the game. For more than half of the games, this time is only about an hour earlier than the game starting time, while it is several hours earlier for other games. Two sets of market predictions are derived:

- NF: Prediction equals NewsFutures' last trade price before noon of the game day divided by 100.
- TS: Prediction equals TradeSports' last trade price before noon of the game day divided by 100.

We apply LinOP and LogOP to ProbabilityFootball data to obtain aggregate expert predictions. The reason that we do not consider other aggregation methods include: (1)

data from ProbabilityFootball is only suitable for mathematical pooling methods—we can rule out behavioral approaches, (2) Bayesian aggregation requires us to make assumptions about the prior probability distribution of game outcomes and the likelihood function of expert opinions: given the large number of games and participants, making reasonable assumptions is difficult, and (3) for axiomatic approaches, previous research has shown that simpler aggregation methods often perform better than more complex methods [17]. Because the output of LogOP is indeterminate if there are probability assessments of both 0 and 1 (and because assessments of 0 and 1 are dictatorial using LogOP), we add a small number 0.01 to an expert opinion if it is 0, and subtract 0.01 from it if it is 1.

In pooling opinions, we consider two influencing factors: weights of experts and number of expert opinions to be pooled. For weights of experts, we experiment with equal weights and performance-based weights. The performance-based weights are determined according to previous accumulated score in the contest. The score for each game is calculated according to equation 5.3, the scoring rule used in the ProbabilityFootball contest. For the first week, since no previous scores are available, we choose equal weights. For later weeks, we calculate accumulated past scores for each player. Because the cumulative scores can be negative, we shift everyone's score if needed to ensure the weights are non-negative. Thus,

$$w_i = \frac{cumulative\_score_i + shift}{\sum_{j=1}^{n}(cumulative\_score_j + shift)}. \tag{5.4}$$

where $shift$ equals 0 if the smallest $cumulative\_score_j$ is non-negative, and equals the absolute value of the smallest $cumulative\_score_j$ otherwise. For simplicity, we call performance-weighted opinion pools as weighted, and equally weighted opinion pools as unweighted. We will use them interchangeably in the remaining of the chapter.

As for the number of opinions used in an opinion pool, we form different opinion pools with different number of experts. Only the best performing experts are selected. For example, to form an opinion pool with 20 expert opinions, we choose the top 20 participants. Since there is no performance record for the first week, we use opinions of all participants in the first week. For week 2, we select opinions of 20 individuals whose scores in the first week are among the top 20. For week 3, 20 individuals whose cumulative scores of week 1 and 2 are among the top 20 are selected. Experts are chosen in a similar way for later weeks. Thus, the top 20 participants can change from week to week.

The possible opinion pools, varied in pooling functions, weighting methods, and number of expert opinions, are shown in Table 5.1. "Lin" represents linear, and "Log" represents Logarithmic. "n" is the number of expert opinions that are pooled, and "All" indicates that all opinions are combined. We use "u" to symbolize unweighted (equally weighted) opinion pools. "w" is used for weighted (performance-weighted) opinion pools. Lin-All-u, the equally weighted LinOP with all participants, is basically the arithmetic mean of all participants' opinions. Log-All-u is simply the geometric mean of all opinions.

Table 5.1. Pooled Expert Predictions

#	Symbol	Description
1	Lin-All-u	Unweighted (equally weighted) LinOP of all experts.
2	Lin-All-w	Weighted (performance-weighted) LinOP of all experts.
3	Lin-n-u	Unweighted (equally weighted) LinOP with n experts.
4	Lin-n-w	Weighted (performance-weighted) LinOP with n experts.
5	Log-All-u	Unweighted (equally weighted) LogOP of all experts.
6	Log-All-w	Weighted (performance-weighted) LogOP of all experts.
7	Log-n-u	Unweighted (equally weighted) LogOP with n experts.
8	Log-n-w	Weighted (performance-weighted) LogOP with n experts.

When a participant did not enter a prediction for a particular game, that participant was removed from the opinion pool for that game. This contrasts with the "Probability-Football average" reported on the contest website and used by Servan-Schreiber et. al [67], where unreported predictions were converted to 0.5 probability predictions.

5.4.2.2 Performance Measures

We use three common metrics to assess prediction accuracy of information markets and opinion pools. These measures have been used by Servan-Schreiber et. al [67] in evaluating the prediction accuracy of information markets.

1. $Absolute\_Error = Prob\_Lose$,

   where $Prob\_Lose$ is the probability assigned to the eventual losing team. Absolute error simply measures the difference between a perfect prediction (1 for winning team) and the actual prediction. A prediction with lower absolute error is more accurate.

2. $Quadratic\_Score = 100 - 400 \times (Prob\_Lose^2)$.

   Quadratic score is the scoring function that is used in the ProbabilityFootball contest. It is a linear transformation of squared error, $Prob\_Lose^2$, which is one of the mostly used metrics in evaluating forecasting accuracy. Quadratic score can be negative. A prediction with higher quadratic score is more accurate.

3. $Logarithmic\_Score = log(Prob\_Win)$,

where $Prob\_Win$ is the probability assigned to the eventual winning team. The logarithmic score, like the quadratic score, is a proper scoring rule. A prediction with higher (less negative) logarithmic score is more accurate.

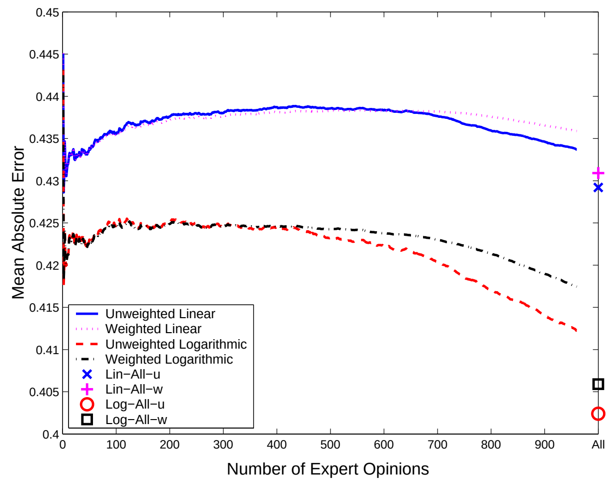
5.5 Empirical Results

5.5.1 Performance of Opinion Pools

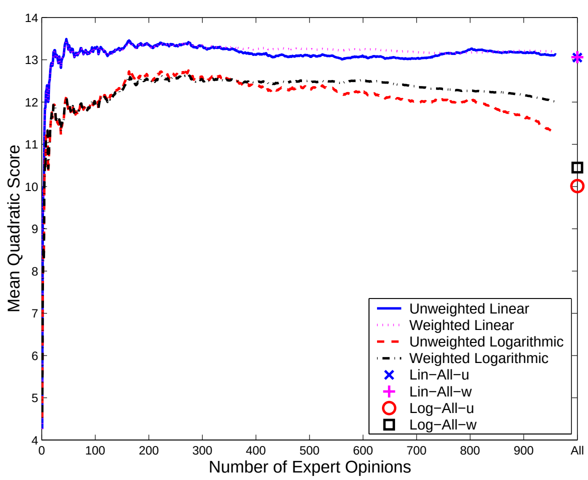
Depending on how many opinions are used, there can be numerous different opinion pools. We first examine the effect of number of opinions on prediction accuracy by forming opinion pools with the number of expert opinions varying from 1 to 960. In the ProbabilityFootball Competition, not all 1966 registered participants provide their probability assessments for every game. 960 is the smallest number of participants for all games. For each game, we sort experts according to their accumulated quadratic score in previous weeks. Predictions of the best performing n participants are picked to form an opinion pool with n experts.

Figure 5.1 shows the prediction accuracy of LinOP and LogOP in terms of mean values of the three performance measures across all 210 games. We can see the following trends in the figure.

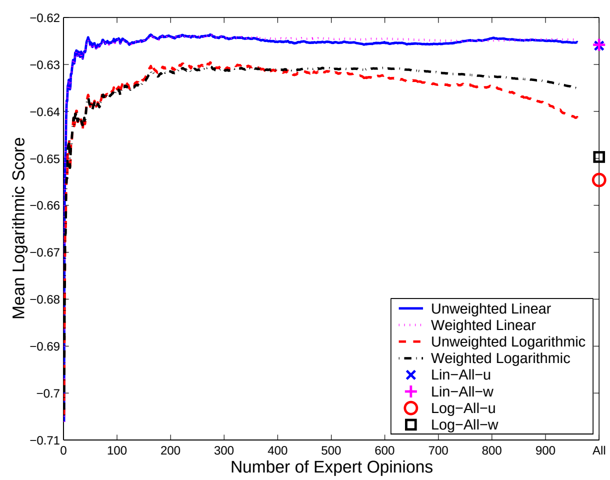
1. Unweighted opinion pools and performance-weighted opinion pools have similar levels of prediction accuracy, especially for LinOP.



(a) Mean Absolute Error



(b) Mean Quadratic Score



(c) Mean Logarithmic Score

Fig. 5.1. Prediction Accuracy of Opinion Pools

2. For LinOP, increasing the number of experts in general increases or keeps the same the level of prediction accuracy. When there are more than 200 experts, the prediction accuracy of LinOP is stable regarding the number of experts.

3. LogOP seems more accurate than LinOP in terms of mean absolute error. But, using all other performance measures, LinOP outperforms LogOP.

4. For LogOP, increasing the number of experts increases the prediction accuracy at the beginning. But the curves (including the points with all experts) for mean quadratic score, and mean logarithmic score have slight bell-shapes, which represent a decrease in prediction accuracy when the number of experts is very large. The curves for mean absolute error, on the other hand, show a consistent increase of accuracy.

The first and second trend above imply that when using LinOP, the simplest way, which has good prediction accuracy, is to average the opinions of all experts. Weighting does not seem to improve performance. Selecting experts according to past performance also does not help. It is a very interesting observation that even if many participants of the ProbabilityFootball contest do not provide accurate individual predictions (they have negative quadratic scores in the contest), including their opinions into the opinion pool still increases the prediction accuracy. One explanation of this phenomena could be that biases of individual judgment can offset with each other when opinions are diverse, which makes the pooled prediction more accurate.

The third trend presents a controversy. The relative prediction accuracy of LogOP and LinOP flips when using different accuracy measures. To investigate this disagreement, we plot the absolute error of Log-All-u and Lin-All-u for each game in Figure 5.2. When the absolute error of an opinion pool for a game is less than 0.5, it means that the team favored by the opinion pool wins the game. If it is greater than 0.5, the underdog wins. Compared with Lin-All-u, Log-All-u has lower absolute error when it is less than 0.5, and greater absoluter error when it is greater than 0.5, which indicates that predictions of Log-All-u are bolder, more close to 0 or 1, than those of Lin-All-u. This is due to the nature of linear and logarithmic aggregating functions. Because quadratic score and logarithmic score penalize bold predictions that are wrong, LogOP is less accurate when measured in these terms.

Similar reasoning accounts for the fourth trend. When there are more than 500 experts, increasing number of experts used in LogOP improves the prediction accuracy measured by absolute error, but worsens the accuracy measured by the other two metrics. Examining expert opinions, we find that participants who rank lower are more frequent in offering extreme predictions (0 or 1) than those ranking high in the list. When we increase the number of experts in an opinion pool, we are incorporating more extreme predictions into it. The resulting LogOP is bolder, and hence has lower mean quadratic score and mean logarithmic score.

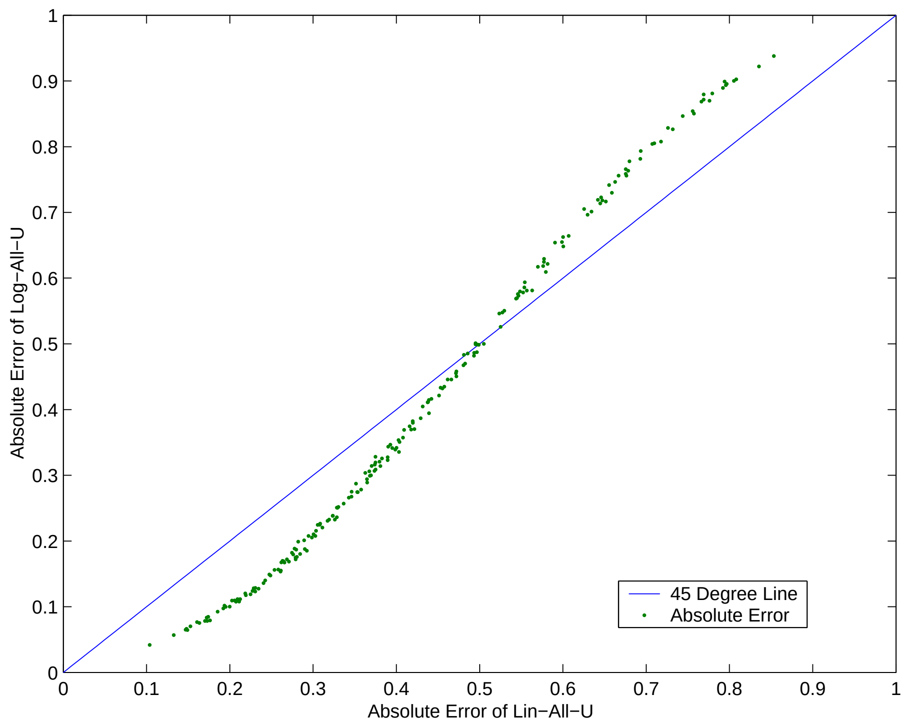


Fig. 5.2. Absolute Error: Lin-All-u vs. Log-All-u

5.5.2 Comparison of Information Markets and Opinion Pools

Through the first screening of various opinion pools, we select Lin-All-u, Log-All-u, Log-All-w, and Log-200-u to compare with predictions from information markets. Lin-All-u as shown in Figure 5.1 can represent what LinOP can achieve. However, the performance of LogOP is not consistent when evaluated using different metrics. Log-All-u and Log-All-w offer either the best or the worst predictions. Log-200-u, the LogOP with the 200 top performing experts, provides more stable predictions. We use all of the three to stand for the performance of LogOP in our later comparison.

If a prediction of the probability that a team will win a game, either from an opinion pool or an information market, is higher than 0.5, we say that the team is the predicted favorite for the game. Table 5.2 presents the number and percentage of games that predicted favorites actually win, out of a total of 210 games. All four opinion pools correctly predict a similar number and percentage of games as NF and TS. Since NF, TS, and the four opinion pools form their predictions using information available at noon of the game day, information markets and opinion pools have comparable potential at the same time point.

We then take a closer look at prediction accuracy of information markets and opinion pools using the three performance measures. Table 5.3 displays mean values of these measures over 210 games. Numbers in parentheses are standard errors, which estimate the standard deviation of the mean. To take into consideration of skewness of distributions,

Table 5.2. Number and Percentage of Games that Predicted Favorites Win

	NF	TS	Lin-All-u	Log-All-u	Log-All-w	Log-200-u
Number	142	137	144	144	143	141
Percentage	67.62%	65.24%	68.57%	68.57%	68.10%	67.14%

we also report median values of accuracy measures in Table 5.4. Judged by the mean values of accuracy measures in Table 5.3, all methods have similar accuracy levels, with NF and TS slightly better than the opinion pools. However, the median values of accuracy measures indicate that Log-All-u and Log-All-w opinion pools are more accurate than all other predictions.

We employ the *randomization test* [56] to study whether the differences in prediction accuracy presented in Table 5.3 and Table 5.4 are statistically significant. The basic idea of randomization test is that, by randomly swapping predictions of two methods numerous times, an empirical distribution for the difference of prediction accuracy can be constructed. Using this empirical distribution, we are then able to evaluate that at what confidence level the observed difference reflects a real difference. For example, the mean absolute error of NF is higher than that of Log-All-u by 0.0229, as shown in Table 5.3. To test whether this difference is statistically significant, we shuffle predictions from two methods,

Table 5.3. Mean of Prediction Accuracy Measures

	Absolute Error	Quadratic Score	Logarithmic Score
NF	0.4253 (0.0121)	**15.4352** (4.6072)	-0.6136 (0.0258)
TS	0.4275 (0.0118)	15.2739 (4.3982)	**-0.6121** (0.0241)
Lin-All-u	0.4292 (0.0126)	13.0525 (4.8088)	-0.6260 (0.0268)
Log-All-u	**0.4024** (0.0173)	10.0099 (6.6594)	-0.6546 (0.0418)
Log-All-w	0.4059 (0.0168)	10.4491 (6.4440)	-0.6497 (0.0398)
Log-200-u	0.4266 (0.0133)	12.3868 (5.0764)	-0.6319 (0.0295)

*Numbers in parentheses are standard errors.
*Best value for each metric is shown in **bold.**

Table 5.4. Median of Prediction Accuracy Measures

	Absolute Error	Quadratic Score	Logarithmic Score
NF	0.3800	42.2400	-0.4780
TS	0.4000	36.0000	-0.5108
Lin-All-u	0.3639	36.9755	-0.5057
Log-All-u	**0.3417**	**53.2894**	**-0.4181**
Log-All-w	0.3498	51.0486	-0.4305
Log-200-u	0.3996	36.1300	-0.5101

*Best value for each metric is shown in **bold.**

randomly label half of predictions as NF and the other half as Log-All-u, and compute the difference of mean absolute error of the newly formed NF and Log-All-u data. The above procedure is repeated 10,000 times. The 10,000 differences of mean absolute error results in an empirical distribution of the difference. Comparing our observed difference, 0.0229, with this distribution, we find that the observed difference is greater than 75.37% of the empirical differences. This leads us to conclude that the difference of mean absolute error between NF and Log-All-u is not statistically significant, if we choose the level of significance to be 0.05.

Table 5.5 and Table 5.6 are results of randomization test for mean and median differences respectively. Each cell of the table is for two different prediction methods, represented by name of the row and name of the column. The first lines of table cells are results for absolute error. The second and third lines are dedicated to quadratic score and logarithmic score respectively. We can see that, in terms of mean values of accuracy measures, the differences of all methods are not statistically significant to any reasonable degree. When it comes to median values of prediction accuracy, Log-All-u outperforms Lin-All-u at a high confidence level.

These results indicate that differences in prediction accuracy between information markets and opinion pools are not statistically significant. This may seem to contradict the result of Servan-Schreiber et. al [67], in which NewsFutures's information markets have been shown to provide statistically significantly more accurate predictions than the (unweighted) average of all ProbabilityFootball opinions. The discrepancy emerges in dealing

Table 5.5. Statistical Confidence of Mean Differences in Prediction Accuracy

	TS	Lin-All-u	Log-All-u	Log-All-w	Log-200-u
NF	8.92% 2.38% 2.99%	22.07% 26.60% 22.81%	75.37% 50.74% 59.35%	66.47% 44.26% 56.21%	7.76% 32.24% 33.26%
TS		10.13% 27.25% 32.35%	77.79% 53.65% 57.89%	68.15% 44.90% 60.69%	4.35% 28.30% 38.84%
Lin-All-u			82.19% 28.91% 44.17%	68.86% 23.92% 43.01%	9.75% 6.81% 17.36%
Log-All-u				11.14% 3.32% 5.25%	72.49% 18.89% 39.06%
Log-All-w					69.89% 18.30% 30.23%

*In each table cell, row 1 accounts for absolute error, row 2 for quadratic score, and row 3 for logarithmic score.

Table 5.6. Statistical Confidence of Median Differences in Prediction Accuracy

	TS	Lin-All-u	Log-All-u	Log-All-w	Log-200-u
NF	48.85% 45.26% 44.89%	47.3% 44.55% 46.04%	84.8% 85.27% 84.43%	77.9% 75.65% 77.16%	65.36% 66.75% 64.78%
TS		5.18% 5.37% 7.41%	94.83% 92.08% **95.62%**	94.31% 92.53% 91.09%	0% 0% 0%
Lin-All-u			**95.11%** **96.10%** **95.45%**	91.37% 92.69% **95.12%**	7.31% 9.84% 7.79%
Log-All-u				23.47% 26.68% 22.47%	**95.89%** 93.85% **96.42%**
Log-All-w					91.3% 91.4% 90.37%

*In each table cell, row 1 accounts for absolute error, row 2 for quadratic score, and row 3 for logarithmic score.
*Confidence above 95% is shown in **bold.**

with missing data. Not all 1966 registered ProbabilityFootball participants offer probability assessments for each game. When a participant does not provide a probability assessment for a game, the contest considers their prediction as 0.5.. This makes sense in the context of the contest, since 0.5 always yields 0 quadratic score. The ProbabilityFootball average reported on the contest website and used by Servan-Schreiber et. al includes these 0.5 estimates. Instead, we remove participants from games that they do not provide assessments, pooling only the available opinions together. Our treatment increases the prediction accuracy of Lin-All-u significantly.

5.6 Summary

With the fast growth of the Internet, information markets have recently emerged as an alternative tool for predicting future events. Previous research has shown that information markets give as accurate or more accurate predictions than individual experts and polls. However, information markets, as an adaptive mechanism to aggregate different opinions of market participants, have not been calibrated against many belief aggregation methods. In this chapter, we compare prediction accuracy of information markets with linear and logarithmic opinion pools (LinOP and LogOP) using predictions from two markets and 1966 individuals regarding the outcomes of 210 American football games during the 2003 NFL season. In screening for representative opinion pools to compare with information markets, we investigate the effect of weights and number of experts on prediction accuracy. Our results on both the comparison of information markets and opinion pools and the relative performance of different opinion pools are summarized as below.

1. At the same time point ahead of the events, information markets offer as accurate predictions as our selected opinion pools.

   We have selected four opinion pools to represent the prediction accuracy level that LinOP and LogOP can achieve. With all four performance metrics, our two information markets obtain similar prediction accuracy as the four opinion pools.

2. The arithmetic average of all opinions (Lin-All-u) is a simple, robust, and efficient opinion pool.

   Simply averaging across all experts seems resulting in better predictions than individual opinions and opinion pools with a few experts. It is quite robust in the sense that even if the included individual predictions are less accurate, averaging over all opinions still gives better (or equally good) predictions.

3. Weighting expert opinions according to past performance does not seem to significantly improve prediction accuracy of either LinOP or LogOP.

   Comparing performance-weighted opinion pools with equally weighted opinion pools, we do not observe much difference in terms of prediction accuracy. Since we only use one performance-weighting method, calculating the weights according to past accumulated quadratic score that participants earned, this might due to the weighting method we chose.

4. LogOP yields bolder predictions than LinOP.

   LogOP yields predictions that are closer to the extremes, 0 or 1.

An information markets is a self-organizing mechanism for aggregating information and making predictions. Compared with opinion pools, it is less constrained by space and time, and can eliminate the efforts to identify experts and decide belief aggregation methods. But the advantages do not compromise their prediction accuracy to any extent. On the contrary, information markets can provide real-time predictions, which are hardly achievable through resorting to experts. In the future, we are interested in further exploring:

- Performance comparison of information markets with other opinion pools and mathematical aggregation procedures.

  In this chapter, we only compare information markets with two simple opinion pools, linear and logarithmic. It will be meaningful to investigate their relative prediction accuracy with other belief aggregation methods such as Bayesian approaches. There are also a number of theoretical expert algorithms with proven worst-case performance bounds [11] whose average-case or practical performance would be instructive to investigate.

- Whether defining expertise more narrowly can improve predictions of opinion pools.

  In our analysis, we broadly treat participants of the ProbabilityFootball contest as experts in all games. If we define expertise more narrowly, selecting experts in certain football teams to predict games involving these teams, will the predictions of opinion pools be more accurate?

- The possibility of combining information markets with other forecasting methods to achieve better prediction accuracy.

  Chen, Fine, and Huberman [12] use an information market to determine the risk attitude of participants, and then perform a nonlinear aggregation of their predictions based on their risk attitudes. The nonlinear aggregation mechanism is shown to outperform both the market and the best individual participants. It is worthy of more attention whether information markets, as an alternative forecasting method, can be used together with other methods to improve our predictions.

Chapter 6

Issues on Information Market Development

6.1 Overview

Theoretical, experimental, and empirical results have shown that information markets can be very effective in making predictions. At the same time, research from all three perspectives has also demonstrated that there are situations in which information markets might not work. Thus, what are the important issues to be considered when designing an information market is worthy of studying if eventually information markets can be used to assist critical decision-makings. Very little previous research has addressed design issues of information markets. Wolfers and Zitzewitz [75] only indicate that there must be some real knowledge exist in order for information markets to work. Spann and Skiera [70] propose a three step framework for designing an information market: choice of forecasting goal, incentive for participation and information revelation, and financial market design. A more comprehensive framework that can guide the whole process of developing information markets to ensure effective forecasting is in great need.

The remaining of the chapter is organized as follow. Section 6.2 proposes a development life cycle for information markets. Section 6.3 to 6.9 then discuss every steps of the life cycle in details. These sections attempt to provide initial answers to the four specific design questions proposed in the Table 1.1 of Chapter 1. Section 6.10 concludes this chapter.

6.2 A Framework for Information Market Development

The current status quo of information market research is not capable of determining when information markets can succeed for certain. In this section, we garner previous evidence and attempt to propose a framework for developing information markets by identifying key issues that need to be considered for better chances of success. Taking a perspective of system analysis and design, we propose a framework of information market development in Figure 6.1.

The life cycle of information market development is similar to that of system development in the steps of design, implementation, and support. But it involves some more complicated steps such as mechanism design. The comparison and contrast of the two development life cycles are presented in Table 6.1.

6.3 Planning and Assessment

An information market development cycle starts with planning and assessment. Planning seeks to identify and prioritize those problems that will return the most value. If the problems identified require predicting outcomes of certain future events, assessment aims at choose the best methods for the given forecasting problems. Since information markets are not a panacea for all forecasting problems, the question of when to choose

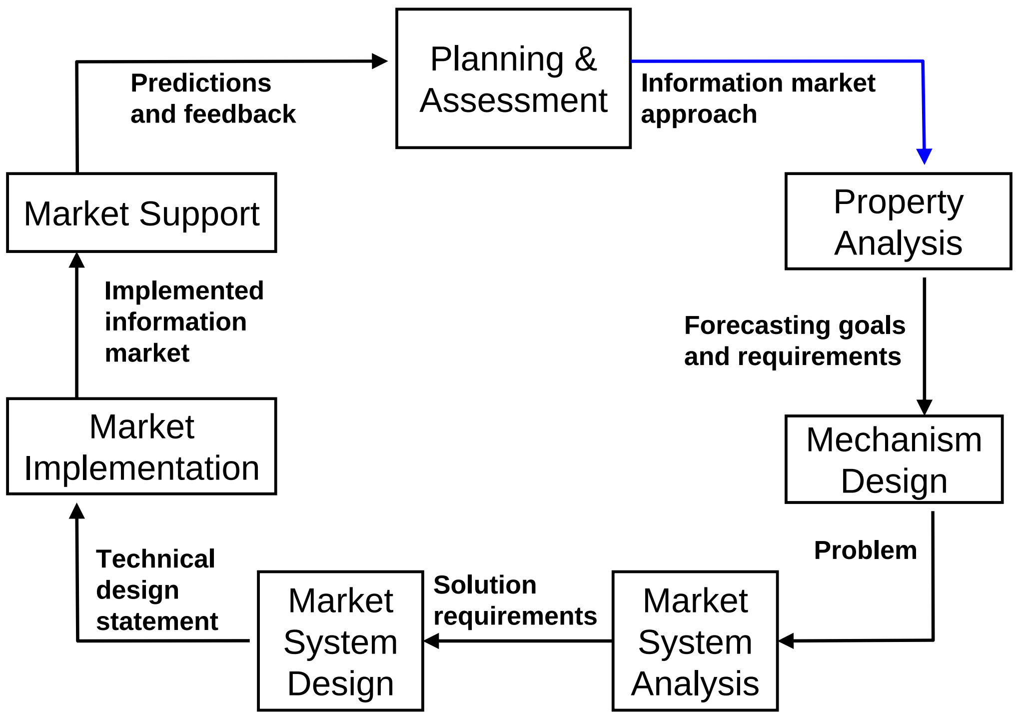


Fig. 6.1. Information Market Development Life Cycle

Table 6.1. Comparison of Information Market Development and System Development

Information Market Development	System Development
Planning and Assessment •Identify and prioritize problems; •Evaluate whether information markets are the most appropriate approach.	*System Planning* •Identify and prioritize problems.
Property Analysis •Define forecasting goals and requirements;	N/A
Mechanism Design •Use economic principles to design the market mechanism in order to achieve the forecasting goals and requirements.	N/A
Market System Analysis •Define system requirements.	*System Analysis* Same.
Market System Design •Evaluate alternative solutions; •Specify a detailed technical design.	*System Design* Same.
Market Implementation •Construction of the application; •Delivery of the application into operations.	*System Implementation* Same.
Market Support •On-going maintenance. •Promotion for the market.	*System Support* •On-going maintenance.

information markets over other forecasting methods is addressed in the first step. Some general guidelines are summarized as below.

- Historical data

  For a forecasting problem, if there is plenty of historical data that contain valuable information about the future event, statistical methods seem to be a low cost choice. If the condition can not be met, judgmental methods such as information markets should be considered.

- Nature of information

  Information markets are more suitable when information about future events is dispersed among an organization or society, especially when information only exists as tacit knowledge or those who have information tend to not reveal it. For example, the sales forecasting problem that Chen and Plott [13] studied is a typical problem that information markets are good at. Information about future sales level within a company usually spans several departments. People at marketing department probably possess the best information about future sales from their customer relations, but they tend to underestimate future sales level because their bonus is determined by how much the actual sale exceeds the estimated sale. Chen and Plott [13] set up internal information markets for Hewlett Packard to predict monthly printer sales. The predictions that these markets provided outperformed the official managerial predictions.

- Political and legal issues

  Some political and legal issues hinder the application of information markets in many domains. The Policy Analyst Market, a DARPA-funded research project that focus on forecasting military and political instability around the world and how US policies would effect such instability, was suddenly canceled amid a media storm on July 29, 2003 [3]. Although many researchers express their support to the Policy Analyst Market [76, 58], there are still concerns about using markets to get information about terrorism, which prevent such markets to be implemented. Legal issues of gambling make most public information markets in the United States only play-money [1]. The only exception is the Iowa Electronic Market (IEM) [50]. IEM agreed to limit positions to $500 to receive a "no action" notice from the Commodity Futures Trading Commision (CFTC). Thus, if a forecasting problem is about policy analysis, public information markets can hardly be used in the United States at least at this time. Play-money public information markets and internal information markets are the most common choice for information aggregation.

If after going through the above general guidelines information markets are selected as the forecasting tool, the development enters into the next step.

[1] TradeSports.com is registered in Ireland where gambling is allowed.

6.4 Property Analysis

Property analysis focuses on the economic aspects of the information market, and seeks to obtain forecasting goals and requirements that can be satisfied through mechanism design. Together with the mechanism design step, property analysis contributes to further defining the problem for later market system analysis phase.

The following issues are usually considered by a decision maker or market owner in this step.

- Desired information

  This is to select and prioritize the information that a market owner hope to get from the information markets. Every information market asks some questions to market participants through well-defined securities [38], depending on the desired information of the decision maker. The market owner needs to decide what questions to ask. For example, if the forecasting problem is to predict the outcome of the next US presidential election, the desired information or questions for the market owner can be as follow. What is the probability that the Democratic Party will win the election? What is the expected percentage of the vote share that the Democratic Party candidate will obtain in the election? How likely will the Democratic Party candidate win both Florida and Michigan? How likely will Howard Dean be nominated as the candidate for the Democratic Party? If Dean is nominated, what is the probability that the Republican Party will win the election? Answers to these questions are all useful information for the forecasting problem. However, asking too much questions in the market may decrease the liquidity of the market, which is essential for achieving information efficiency, and increase the cost. Thus, a market owner needs to ponder on what is the most important information to obtain for the forecasting problem.

- Cost of information markets

  Although designing, implementing, and supporting an information market all incur expenses, the cost of information markets in this phase only refer to the cost on creating incentives for participation. It seems to depend more on the number of questions asked than on which questions are asked [38]. Thus, a market owner needs to explore the trade-off between cost and desired information and set the budget.

- Feasibility analysis

  Feasibility analysis is used to investigate whether it is feasible to get desired information with the given budget using information markets. As is shown by the theoretical model in Chapter 3, the best an information market can achieve is to aggregate all available information in the market. Thus, whether there are people with relevant information, and whether these people can be attracted to participate are important for better predictions. However, counter-intuitively, only having informed people to participate is not enough. Wolfers and Zitzewitz [74] have pointed out that rational informed traders won't trade with each other according to the No Trade Theorem.

Attracting uninformed traders can increase the liquidity of the market and hence the accuracy of the predictions. Motivations for uninformed traders can be entertainment, overconfidence, and hedging. Feasibility analysis need to identify whether these conditions can possibly be met.

6.5 Mechanism Design

The mechanism design step is unique to information market development. It is arguably the most crucial part for developing an effective information markets. It uses economic principles to design the market mechanism in order to achieve the forecasting goals and requirements. Most existing research on information market design only focuses on this step. The three step framework of Spann and Skiera [70] is for this phase. Collecting evidence from existing information markets and previous research [70, 75], we summarize the market design issues and available options of each issue in Table 6.2.

6.5.1 Security

Selection of the security largely depends on desired information or questions to be asked. To predict how likely an event will happen (e.g. probability that the Democratic Party will win the next Presidential election), a winner-takes-all security should be used. If the desired information is the expected value of a continuous random variable (e.g. vote share of the Democratic Party candidate), we should choose the index security. Both the winner-takes-all and index securities have been introduced in Chapter 2. The spread

Table 6.2. Issues of Mechanism Design for Information Markets

Design Issues	Options
Security	Winner-takes-all Index Spread Conditional
Trading Mechanism	Continuous double auction Continuous double auction with market maker Market scoring rule Dynamic pari-mutuel market Other
Incentive	Real money Initial endowment Play money with rewards
Other Rules	Open to public or not Allow short trading or not Trading fee Duration and hours of the market Limits to positions or orders

security, according to Wolfers and Zitzewitz [74], has a fixed price and allows traders to bid on the cutoff that determines whether an event occurs. Such security can be used to predict percentile values of a continuous random variable. For example, for a security that costs \$1 and pays \$2 if the vote share of Democratic Party candidate exceeds $y^*\%$, traders specify the value of y^* when they trade the security. It predicts the median value of the Democratic Party candidate vote share. Conditional security is a security whose payoff is contingent on another specific event. The 2004 Presidential Election Markets at IEM traded securities that paid \$1 times the percentage of the vote share won by George W. Bush, given that a candidate (e.g. John Kerry or Howard Dean) wins the Democratic nomination race. Using several different securities together can reveal more aspects of the probability distribution of the future events. Whatever security is selected, the security must be clear, and easy to be understood.

6.5.2 Trading Mechanism

The most commonly used trading mechanism for information markets is the continuous double auction (CDA), which is also the typical mechanism for financial markets. In a CDA, market participants can place orders to buy or sell a security. The CDA constantly matches buy and sell orders to result in transactions. The owner or auctioneer of CDA takes on no risk, since CDA only matches orders. However, CDA suffers from illiquidity when the market is thin [39]. A thin market is a market with few buy or sell offers. It is usually characterized as huge bid-ask spreads or even empty order queues. Trading is often light because buyers or sellers may not find counterparts to conduct transactions, which hinders information aggregation.

Continuous double auction with market maker (CDAwMM) induces liquidity by introducing a market maker who is willing to accept a large number of buy and sell orders at specified prices. CDAwMM increases liquidity even when market is thin, but it also poses risk to the auctioneer or market owner. The auctioneer may lose considerable amounts of money depends on what happens in the future. The cost of the auctioneer is not bounded.

Hanson proposed a new mechanism, market scoring rule (MSR) [39], that combines scoring rules with CDA. In addition to running a CDA, a MSR maintains a probability distribution across all events. Anyone who believes that the probability distribution is wrong can change it at any time. The person then receives a payment, the amount of which is determined according to a scoring rule, and in return, agrees on to pay the next person who changes the distribution. The MSR requires some initial subsidy to pay the first person who makes changes, hence there is still some risk, but the maximum cost is bounded.

Dynamic pari-mutuel market (DPM) is another new mechanism that can be viewed as a hybrid of a pari-mutual market and a CDA [59]. As a pari-mutual market, a DPM allows people to place wagers on exclusive outcomes of future events. After the true outcome is revealed, all the money that is lost by those who bet on the incorrect outcome is redistributed to those who bet on the correct outcome. It has infinite liquidity since it does not require order matching. Unlike a pari-mutual market, where each dollar always buys an equal share of the payoff, each dollar that people wager in a DPM buys a variable share of the payoff depending on the state of the market. Thus, a DPM can continuously reflect the arrival of new information like a CDA. A DPM needs a pre-determined small amount of subsidy to start the market. After that, the owner of the market does not bear any risk of loss.

Pennock has given more detailed explanation of the advantages and disadvantages of the trading mechanisms when proposing the DPM [59]. The above four trading mechanisms can continuously incorporate new information, which is important for effective information aggregation. In Table 6.3, we briefly summarize the characteristics of these mechanisms according to liquidity and risk/cost, which are the main criteria for selecting a trading mechanism for an information market. Other than these four trading mechanisms, information markets can also take the mechanism of pari-mutuel markets or sports wagering markets such as the typical Las Vegas bookmakers.

6.5.3 Incentive

An information market with carefully selected security and trading mechanism does not necessarily attract informed people to participate. Hence, an important part of mechanism design is to design the incentives for participation. Spann and Skier [70] have given detailed review of the incentives for information markets. We provide high-level discussions of the three most frequently adopted reward structures: real money, initial endowment, and play money with prizes or rewards.

Table 6.3. Comparison of Trading Mechanisms

Trading Mechanism	Liquidity	Risk/Cost
CDA	Illiquidity when market is thin	No risk, only matching orders.
CDAwMM	Guaranteed liquidity	Market owner has risk, can incur unbounded cost.
MSR	Guaranteed liquidity	Market owner has limited risk, can incur bounded cost.
DPM	Guaranteed liquidity	Market owner needs a predetermined cost to start the market. No risk after the market is started.

Intuitively, the best incentive is monetary. A real money information market requires that participants invest their own money in the market. Market participants "put their money where there mouth is". This creates a strong incentive for informed people to participate in the market and perform well because their own money is at stake and they have chances to gain monetary reward with their information.

However, real money information markets are not always acceptable due to various concerns. For an internal information market that is used to predict some important events for a company, one of the managerial concerns is that requiring employees to invest their own money and bear the risk of losing it is inappropriate. Giving market participants some initial endowments can fix the problem. Microsoft ran several markets to predict schedule and bug count for projects. To not let market participants lose their own money but still create incentives for them to participate, Microsoft gave each participant $50 to trade in the market. Participants could profit from payoffs of their asset holdings and from the price differences of selling and buying. But the unused part of the $50 was faked.

For public information markets, legal and political concerns often prevent markets from using real money. The most common public information markets are play money markets. They create incentives for participation using prizes or non-monetary awards, often associated with performance. For example, participants of Newsfutures' prediction markets [54] can use the play-money they earned to buy some items in an online auction shop. Play money markets usually give participants some initial portfolios or play money to start with.

6.5.4 Other Rules of Markets

A designer of an information market also needs to make decisions on some other rules of the market as listed in Table 6.2. An information market can either open to public or selectively choose its participants. Internal information markets usually select their participants from relevant departments within a company. Some markets, usually play money markets, allow both long and short trading. Whether charging trading fees is another decision to be made. Generally, trading fees may result in low liquidity and hinder information aggregation, but they can also reduce the cost of market owners. Duration and hours of a market also need to be considered. Depending on the forecasting problem, an information markets can last for a couple of weeks to several years, and can open 24 hours a day or only several hours each day. Some information markets set special limits on positions, such as a maximum of x shares of the security, or maximum and minimum prices of orders. Such limitation is used to restrict the possible influence of a single trader on information aggregation.

After reviewing the mechanism design issues and options of each item, we summarize in Table 6.4 the design details of several popular information markets — Iowa Electronic Markets (IEM) [50], TradeSports [72], NewsFutures [54], Hollywood Stock Exchange [21], HP's internal sales forecasting markets, and Tech Buzz Game [30].

6.6 Market System Analysis

While mechanism design attempts to achieve forecasting goals using economic principles, it at the same time clearly defines the problem that needs to be solved through developing a network-based market system. Market system analysis is similar to the system analysis step in information system development process. The purpose is to understand the system requirements based on the selected market mechanism. Data models and process models can be used in this step to give an overview or stress on aspects of the whole system.

Due to characteristics of information markets, market system analysis needs to pay special attention to the following two aspects.

Table 6.4. Comparison of Information Markets Design

Markets	Mechanism Design Issues			
	Security	Trading Mechanism	Incentive	Other Rules
IEM	•Winner-takes-all; •index; •conditional	CDA	Real money	•Open to public; •Short positions not allowed; •No transaction fees; •Markets open continuously; •$500 investment limit.
TradeSports	Winner-takes-all	CDA	Real money	•Open to public; •Short positions allowed; •Charge trading fees and contract expiry fees; •Markets are halted approximately for one hour each day.
NewsFutures	Winner-takes-all	CDA	•Play money with rewards; •Initial endowment with play money.	•Open to public; •Short positions not allowed; •No trading fee; •Markets open continuously.
HSE	Index	CDAwMM	•Play money with rewards. •Initial endowment with play money.	•Open to public; •Short positions allowed; •Charge trading fees; •Markets open continuously; •Maximum of 50,000 shares position limit.
HP	Winer-takes-all	CDA	Initial endowment with portfolio and cash.	•Open only to selected participants; •Short positions not allowed; •No trading fees; •Markets open at lunch time and at evening for a week.
Tech Buzz	Index	DPM	•Play money with rewards. •Initial endowment with play money.	•Open to public; •Short positions not allowed; •Charge trading fees for selling but not for buying; •The markets close every Friday between 6pm and 9pm.

- Market interface

  The majority of users of an information market are market traders. They come to the market to observe market prices and trading histories, submit bids and asks, and conduct transactions. Although their interactions with the market system are limited to certain forms, they interact with the system at a high frequency. Thus, the market interface affects almost all users at almost all times in an information markets. It is essential to consider market traders' requirements and preferences in designing the trading interface.

- Scalability

  Scalability is more of a requirement for information markets, especially public information markets, than for traditional information systems. As the number of traders can increase very fast, the market system needs to be able to easily incorporate new traders.

6.7 Market System Design

Market system design is the step to evaluate alternative solutions and specify a detailed computer-based solution. It involves selecting a design target, acquiring the necessary hardware and software, and developing technical design specifications.

One of the advantages of information market design is that many market platforms are available as commercial softwares, such as NewsFutures's Prediction Trader V4 [54]. If the selected market mechanism is a popular one, it is more likely that some existing market platform can be customized to meet the requirements. Careful evaluations on portability are needed.

6.8 Market Implementation

Market implementation is the construction of the new information market and the delivery of the market into operation. As a standard system implementation step, it needs to build and test networks, databases, programs, and the whole market. Running a pilot market is usually needed to test the market system before put it into use.

One issue that needs to be considered at this stage is how to advertise/promote the established information market to targeted audiences, so that the market can have informed traders and enough liquidity. Most existing information markets, Iowa Electronic Markets [50] and TradeSports [72], only use "word of mouth" to promote their markets. But as the number of available information markets increases in the future, how to promote an information market might affect its prediction accuracy.

6.9 Market Support

After the information market is in operation, on-going market support is needed to correct any errors and assist market users. Problems that might arise at this stage include dispute over market trading rules and traded contracts, database or network problems, and system security issues. This step creates desired predictions and provides feedbacks for future applications.

6.10 Summary

From the perspective of system analysis and design, this chapter proposes a framework for information market development and discusses issues to be considered at each stage of the development process. As information markets can be considered as a kind of novel information systems, its development process goes through several similar steps as the system development process. However, as the functionality of information markets mainly comes from its economic properties, the development process also has some unique steps, such as mechanism design, which uses economic principles to design the market rules so that different forecasting goals can be achieved. In general, developing an information market is a complicated process, making deliberated decisions at every stage of the process can increase the chances for obtaining accurate predictions.

Chapter 7

Conclusions

7.1 Summary

Information markets have been proposed as an alternative tool for predicting future events. Many real world online markets are providing test grounds for information markets. Iowa Electronic Markets (IEM) [50] forecasts outcomes of political events. Hollywood Stock Exchange (HSX) [21] trades securities to predict future box office proceeds of new movies. TradeSports [72] hosts markets for sports, politics, entertainment, and financial events. Foresight Exchange (FX) [20] allows traders to bet on unresolved scientific questions or other claims of public interest. NewsFutures's World News Exchange [54] has very popular sports and financial betting markets. Although prices of securities in many of these markets were found to give as accurate or more accurate predictions than polls and expert opinions [8, 9, 26, 27, 60, 61], how and how well information markets work have not been fully explored. In this thesis, we systematically study information markets from four perspectives: theoretical examination, experimental evaluation, empirical analysis, and design.

Theoretical examination of information markets models information markets using a modified Sharpley-Shubik market game. It studies the information aggregation ability of information markets. By characterizing the uncertainty of market participants' private information, we incorporate aggregate uncertainty in our information market model. Based on the model, we examine some fundamental convergence properties of information markets. Specifically, we have shown that (1) an information market is guaranteed to converge to an equilibrium, at which traders have consensus about the forecast; (2) it converges to the equilibrium in at most n rounds of trading, where n is the number of traders ; (3) the best possible prediction it can make is the direct communication equilibrium, at which price equals the expectation of the value of the function based on information of all traders; (4) but an information market is not guaranteed to converge to this best possible prediction.

Experimental evaluation of information markets investigates to what extent models of information markets are valid in controlled laboratory environments. By maintaining a close parallel to settings of theoretical models, we tested Feigenbalm et al.'s model [23], rational expectations model (REE) [48], and private information model [63] using human subjects experiments. We find that although all models are not supported by the experimental data in an absolute sense, Feigenbaum et al.'s model is supported to some degree. A further examination on the assumptions made by theoretical models reveals that all assumptions on individual behavior are violated in the experiments. This implies that theoretical models can be and need to be improved by better capturing individual behavior in markets.

Empirical analysis compares information markets, as an adaptive mechanism to aggregate different opinions of market participants, with linear and logarithmic opinion pools. Our results show that at the same time point ahead of the event, information markets provide as accurate predictions as our carefully selected opinion pools. In selecting the opinion pools to be used in our comparison, we find that arithmetic average is a robust and efficient pooling function; weighting expert assessments according to their past performances does not improve prediction accuracy of opinion pools; and logarithmic opinion pools offer bolder predictions than linear opinion pools.

Finally, based on previous research on information markets, we propose a framework for developing information markets from the perspective of system analysis and design. Key design issues and options of each issue are identified and discussed.

7.2 Future Direction

While the potential of information markets as a forecasting device is well-recognized, there is a gap between theory and practice, which prevents widely applying information markets in many domains. In practice, most of existing information markets provide accurate predictions, often outperforming other forecasting methods. In theory, how markets achieve such accurate predictions is explained. However, the assumptions, on which theories are based, deviate from what are observed in practice. Hence, theories are questionable to some degree, and why and how information markets work are not fully understood. This thesis is among the endeavors to bridging the gap. Our future research bears the same goal. More specifically, we intend to pursue better information market models.

One possibility is to establish models for biased agents. We have observed in existing information markets and experiments that (1) market traders are biased; they tend to only use local information, which violates assumptions of most existing theoretical models; (2) they trade with each other, which deviates from the No Trade Theorem; and (3) market prices provide accurate predictions. We are interested in investigating why markets with biased agents can still create accurate forecasts.

Appendix A

Experiment Instruction

You are about to participate in an experiment in the economics of market decision making in which you will earn money based on the decisions you make. Your earnings are yours to keep and will be paid to you **in cash** at the end of the experiment. During the experiment all units of account will be in experimental dollars. Upon conclusion of the experiment, all experimental dollars earned will be converted into U.S. dollars at the conversion rate of **80** experimental dollars per U.S. dollar. Your earnings, plus a lump sum amount of $7, will be paid to you in private. You are not allowed to communicate with the other participants, except as permitted under the rules of the experiment. If you have any questions, please raise your hand and I will answer them in private. From this point forward, you will be referred to by your trader number, which is trader number[1] _____. There are four other traders in your market.

A.1 Earnings

Participants have the opportunity to conduct trades during a series of independent trading periods. At the beginning of the period you hold 1 unit of an asset. During each

[1] Put number 1, 2, 3, 4, or 5 here.

trading period, you will be able to purchase and/or sell units. The precise value of the asset at the time you make your decisions will be unknown to you. Instead, each of you will receive some information regarding the value of the asset which you may find useful in making your decisions.

At the end of the period, a single redemption value will be announced for all units that you hold. There are two possible outcomes: "Good" and "Bad". If the outcome "Good" occurs, then the redemption value for each unit you hold is 150 experimental dollars. If the outcome "Bad" occurs, then the redemption value for each asset you hold is 50 experimental dollars.

You earn money from buying and selling assets, and then at the end of each period, redeeming any assets you own. For example, suppose you hold 3 units at the end of the trading period. If the outcome "Good" is announced, you would receive 450 (3 times 150). On the other hands, if the outcome "Bad" is announced, your total redemption value is 150 (3 times 50), or

$$\text{Total Redemption Value} = (\#\text{ units}) \times (\text{redemption value}).$$

In order to determine your profits at the end of a trading period you will need to consider one other factor. You must add your cash inventory at the end of the period (given on your computer screen). At the beginning of the period, your cash inventory is 50. However, through the process of buying and selling units you may accumulate cash (your sale values exceeded your purchase values) or your cash may decline (your purchase values exceeded your sale values). In either case, this number should be added to your Total Redemption Value. Thus your final profit calculation is given by:

$$\text{Profit} = \text{Total Redemption Value} + \text{Cash}.$$

Let us continue the example above. Suppose you have a cash inventory of 250 experimental dollars. Then, if the outcome is "Good" your profit would be $450 + 250 = 700$. If the outcome is "Bad" your profit would be $150 + 250 = 400$.

You are allowed to have negative amounts of the asset and cash. If at the end of the trading period you own a negative quantity, then you must subtract the quantity held times the asset redemption value from your cash. For example, if you end the period with $-.5$ units of the asset and the outcome was "Good" then you must subtract $(.5)150 = 75$ experimental dollars from your cash. If the outcome was "Bad" then you would subtract $(.5)50 = 25$. If your cash was negative at the end of the period, then you would simply subtract that amount from the total redemption value to obtain your profits. For example, if you held 3 units but had cash holdings of -350, then depending upon the outcome your profit would be either $3(150) - 350 = 100$ ("Good") or $3(50) - 350 = -200$ ("Bad"). Notice that negative profits are possible in a given period. In that case, that amount will be subtracted from your earnings for the experiment. In the very unlikely event that your earnings for the experiment (including the $7 initial payment) fall to zero, you will not be allowed to continue with the experiment.

It is also possible to own fractional units of the asset. This does not change the profit calculations described above.

A.2 Information about Redemption Values

During the trading period, you will receive a single clue which will partially determine the outcome ("Good" or "Bad"). This clue will not tell you for certain which outcome is to occur but could provide valuable information in helping you make trading decisions. A clue will be either a "G" or a "B" and will not change during the period. In any given period, you are equally likely to see a "G" or a "B". All traders will be given a private clue about the outcome. While their clues are determined in the same manner, the actual clue ("G" or "B") will vary from trader to trader.

The clues determine whether the outcome is "Good" or "Bad". If a majority of the traders see a "G" clue then the outcome is "Good" and each unit of the asset you own will be worth 150 experimental dollars. Otherwise (the majority see a "B" clue), the outcome will be "Bad" and the asset will be worth 50 experimental dollars. Since there are a total of five participants in your market, the outcome will only be "Good" if three (3) or more traders see a "G" clue [2].

[2] This is for majority security. For a parity security, the outcome will only be "Good" if one (1), three (3), or five (5) traders see a "G".

A.3 Market Organization

The market is organized as follows. The market will be conducted in a series of trading periods. Trading periods consists of rounds each lasting a maximum of two (2) minutes. During a round, you may submit a price that you are willing to purchase or sell a unit by entering a bid into the computer. The bid must be between 50 and 150. A round ends when all traders have submitted bids or two minutes have elapsed. If you do not place a bid in the two minutes, you will not trade.

At the end of the round, the market price is determined. The market price is the per unit price at which all trades are conducted. For example, if the market price was 120, then for every unit of the asset you buy you would have to pay (lower your cash inventory by) 120, and for every unit of the asset you sell you would receive (increase your cash inventory by) 120. The market price is calculated by taking the average of all bids placed in a round. You will be told your own bid and the market price but not other players' individual bids. If your bid was higher than the market price, then you will buy units of the asset, and if your bid was lower than the market price then you will receive units of the asset. The amount you buy or sell depends upon how your bid price compares to the market price. In general, the further your bid is from the market price, the more you buy or sell. The actual amount you trade is determined by the taking the difference between your bid price and the market price and dividing by the market price, or

$$\text{Quantity Traded} = (\text{Your Bid Price} - \text{Market Price})/\text{Market Price}.$$

For example suppose we have two players and one player bids 120 and the other player bids 80. The market price is set at 100. The first player buys $(120-100)/100 = 0.2$ units of the asset at a cost of $.2(100) = 20$. The second player receives $.2(100) = 20$ in exchange for selling $(80 - 100)/100 = -0.2$ units of the security. After trading the first player will hold 1.2 units and the second player will hold .8 units.

After a short pause, trading will continue into the next round where you offer a bid which may be different than the bid or the market price of the previous round. You might find it useful to use the market price to help you choose a new bid for the next round. However, since the bid can change from round to round, the market might also change.

After a minimum of two (2) rounds, the trading period will end if any one of the following happens:

1. No traders submit a bid in the last round,
2. Everyone submits identical bids,
3. The market price does not change for two consecutive rounds, or
4. 10 rounds are completed.

The final outcome ("Good" or "Bad") will be announced and your earnings will be calculated. Your final earnings only depend upon the assets and cash you hold at the end of the trading period.

A.4 Computer Interface

All the information you need to participate in the market will be provided by the computer system. The computer also automatically completes all necessary calculations and trades according to the rules described above. However, it is important that you understand the processes involved since they help you determine how to earn money. Please refer to the screen shot included in your packet. The computer interface is divided into four main areas. The area in the upper left hand corner lists your current cash inventory and units of the asset. The upper right hand area lists the current round and displays the information you have received.

You place a bid in the lower right hand area. Type the price you are willing to buy or sell at in the purple box and press the "PLACE BID" button. The lower left hand box lists all your bids, trades and markets prices from earlier rounds (if any).

After a round is complete, you will be provided with information on the market price, and amount of the asset you have traded. After reviewing this information, please click "CONTINUE" to proceed.

A.5 Exercises

To make sure you understand the instructions, please complete the following exercise. Consider the following cost table when completing the exercise:

1. Suppose that you held 3 units of the asset and 400 experimental dollars in cash.

    a. If the outcome was "Good", what would be your earnings?

    b. If the outcome was "Bad", what would be your earnings?

2. Suppose you placed a bid of 75 for the asset and the market price is set at 70.

    a. Will you buy or sell units of the asset?

    b. At what price?

Period zero will be a practice period. You will receive no earnings for this practice period. To save time, period zero will last only two rounds. If you have any questions, please raise your hand and I will come by to answer your question(s).

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